Practical Astronomy

By G. L. HOSMER

Navigation

Geodesy

By C. B. BREED and G. L. HOSMER

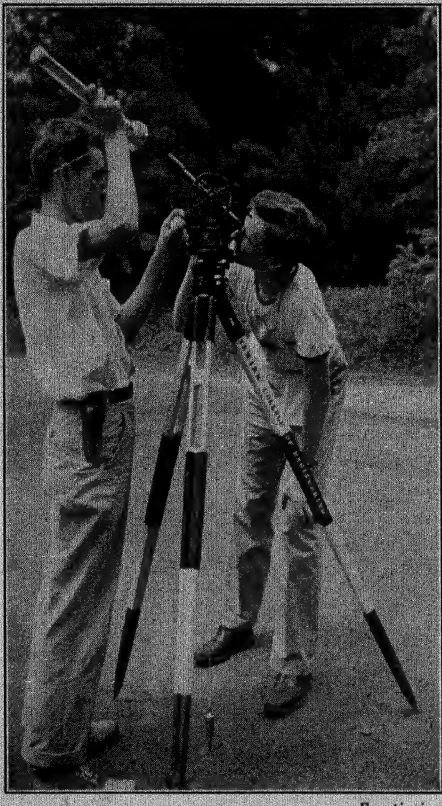
The Principles and Practice of Surveying

Volume I. Elementary Surveying. *Eighth Edition*

Volume II. Higher Surveying. *Sixth Edition*

By G. L. HOSMER and J. M. ROBBINS

Practical Astronomy. *Fourth Edition*



Frontispiece

Observation on *Polaris* for Azimuth

Practical Astronomy

TEXTBOOK FOR ENGINEERING SCHOOL AND A MANUAL OF FIELD METHODS

BY

GEORGE L. HOSMER

LATE PROFESSOR OF GEODESY
MASSACHUSETTS INSTITUTE OF TECHNOLOGY

Fourth Edition

REVISED AND REWRITTEN BY

JAMES M. ROBBINS

PROFESSOR IN CIVIL ENGINEERING
NEWARK COLLEGE OF ENGINEERING

JOHN WILEY & SONS, INC., NEW YORK
CHAPMAN & HALL, LTD., LONDON

FOURTH EDITION

Fifth Printing, May, 1956

PRINTED IN THE UNITED STATES OF AMERICA

Preface

TO THE FOURTH EDITION

The years which have elapsed since the publication of the third edition of this textbook have witnessed a number of developments which indicated the desirability of a complete revision of the book. I am most happy to have had the opportunity to carry on the late Professor Hosmer's work in this revision, for my associations with him were close and very pleasant for many years.

The basic order of presentation has, in general, been maintained, but significant changes have been made throughout. New concepts and new instruments have received attention. All chapters have been rewritten to the extent necessary to incorporate recent developments in the field and to include those changes dictated by many years teaching of the subject. Many new problems have been added.

The student is required to become familiar with one or more of the various ephemerides by the omission from the statements of many problems of data which can be found in the current *Ephemeris* or *Nautical Almanac.* This has seemed wise even though certain inconsistencies result from the fact that, when observations made in past years are to be reduced using ephemeris data for the current year, the celestial body will no longer have precisely the stated altitude at the indicated instant of time in the current year.

Former chapters on observations for time and for longitude have been combined into one because of the intimate relationship existing between them. Certain seldom-used observations and others requiring geodetic instruments have been deleted. The chapter on nautical astronomy has been eliminated. A full discussion of modern celestial navigation would greatly enlarge the text and would depart from the basic concept of a book that is useful to the civil engineer and surveyor. Part I of the text is fundamental to both the navigator and the surveyor, but the applications in Part II have been confined to the needs of the surveyor.

The appendix on the tides has been eliminated and that on spherical trigonometry has been expanded to meet the needs of the student with no previous instruction in the subject. Table IX of the previous edition has been deleted, and an arc-time conversion table has taken its place.

I have spared no pains to eliminate errors, but I cannot hope that my efforts in this direction have been entirely successful. I will therefore greatly appreciate notification of any errors which are found.

I wish to acknowledge my indebtedness to all who have assisted in the preparation of the book. C. L. Berger and Sons; W. and L. E. Gurley; Pioneer Instrument; Fairchild Aviation Corporation; Professor Lawrence Perez of Pennsylvania State College; and Professor Charles O. Roth, Jr., of Newark College of Engineering have contributed many suggestions and much illustrative material. The Director of the United States Naval Observatory has supplied many data. I am under particular obligation to Professor Herman J. Shea of the Massachusetts Institute of Technology for his thorough criticism of the entire manuscript, for his many contributions, and for his whole-hearted helpfulness throughout.

J.M.R.

NEWARK, NEW JERSEY
June, 1947

Preface

TO THE FIRST EDITION

The purpose of this volume is to furnish a text in practical astronomy especially adapted to the needs of civil engineering students who can devote little time to the subject and who are not likely to take up the advanced study of astronomy. The book deals chiefly with the class of observations which can be made with surveying instruments; the methods applicable to astronomical and geodetic instruments are treated only briefly. It has been my intention to produce a book which is intermediate between the textbook written for the student of astronomy or geodesy and the short chapter on the subject generally given in textbooks on surveying. The subject has therefore been treated from the standpoint of the engineer, who is interested chiefly in obtaining results, and those refinements have been omitted which are beyond the requirements of the work which can be performed with the engineer's transit. This has led to the introduction of some rather crude mathematical processes, but it is hoped that these are presented in such a way as to aid the student in gaining a clearer conception of the problem without conveying wrong notions as to when such short-cut methods can properly be applied. The elementary principles have been treated rather elaborately but with a view to making these principles clear rather than with a view to the introduction of refinements. Much space has been devoted to the measurement of time because this subject seems to cause the student more difficulty than any other branch of practical astronomy. An attempt has been made to arrange the text so that it will be a convenient reference book for the engineer who is doing field work.

I wish to acknowledge my indebtedness to those who have assisted in the preparation of the book, especially to Professor A. G. Robbins and Mr. J. W. Howard of the Massachusetts Institute of Technology and to Mr. F. C. Starr of the George Washington University for their valuable suggestions and criticisms of my manuscript.

G.L.H.

BOSTON, MASSACHUSETTS
June, 1910

Contents

Part I

Fundamental Principles of Practical Astronomy

Chapter 1 The Celestial Sphere — Real and Apparent Motions

CHAPTER 5 MEASUREMENT OF TIME

CHAPTER 6 EPHEMERIDES — STAR CATALOGUES — INTERPOLATION

CHAPTER 7 THE EARTH'S FIGURE — CORRECTIONS TO OBSERVED ALTITUDES

Chapter 8 Description of Instruments

Chapter 9 The Constellations — Star Identification

Part II

Engineering Astronomy

Chapter 10 Observations for Latitude

Chapter 11 Observations for Time and Longitude

Chapter 12 Observations for Azimuth

Tables

Part I
Fundamental Principles of Practical Astronomy

1

The Celestial Sphere
Real and Apparent Motions

1 Practical Astronomy

Practical astronomy treats the theory and use of astronomical instruments and the methods of computing the results obtained by observation. Aviators, navigators, and surveyors make use of the same basic principles of this applied science in obtaining, from celestial observations, geographic positions and bearings. The observations involved are those for the determination of *latitude, longitude, time, and azimuth.* In solving these problems the observer makes angular measurements of the *directions* of the sun, moon, stars, and other heavenly bodies; he is not concerned with the distances to these objects, with their actual motions in space, nor with their physical characteristics. He regards them solely as a number of visible objects of known positions upon which he can observe and thereafter compute the desired results.

2 The Celestial Sphere

Since it is only the directions of these objects that are required in practical astronomy, it is found convenient to regard all heavenly bodies as being projected onto the surface of an imaginary sphere of infinitely large radius whose center

is at the eye of an observer considered to be at the earth's center. The apparent position of any object on the sphere is found by imagining a line drawn from the eye to the object and by prolonging it until it pierces the sphere. For example, the apparent position of S_1, on the sphere (Fig. 1) is at S_1', which is supposed to be at an infinite distance from C; the position of S_2 is S_2', etc. By means of this imaginary

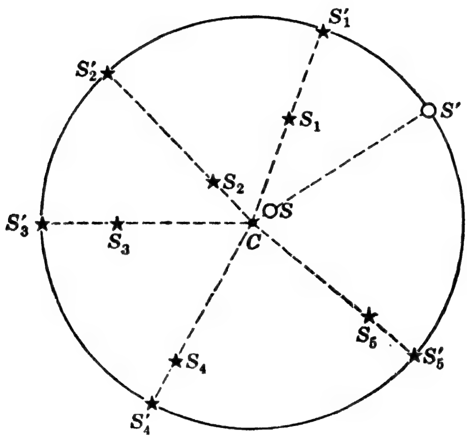


FIG. 1. Apparent Positions on the Sphere

sphere all problems involving the angular distances between points, and angles between planes through the center of the sphere, may readily be solved by applying the formulae of spherical trigonometry. This device is not only convenient for mathematical purposes, but it is also perfectly consistent with what we see because all celestial objects are so far away that they appear to the eye to be at the same distance and consequently on the surface of a great sphere. The fact that the center of the celestial sphere is considered to coincide with the center of the earth, whereas observations must be made from the earth's surface or from the air above, will cause no actual inconvenience. With respect to the stars, the

astronomical distances involved are so large in comparison with the radius of the earth that the error caused by considering directions measured at the earth's surface to be the same as if measured at the center is insignificant when directions are measured with the transit or the sextant. The mean radius of the earth is approximately 3960 miles. The nearest star, *Proxima Centauri,* is 4.16 light years away, or about 24.5 million, million miles. The most distant stellar system so far known is some 150 million light years away. Only with observations on the nearer bodies of the solar system, such as the sun and the moon, do small corrections have to be applied to reduce the measurements to the center of the earth.

Since the radius of the celestial sphere is infinite, all the lines in a system of parallels will pierce the sphere in the same point, and parallel planes at any finite distance apart will cut the sphere in the same *great circle* (see Chapter 2). This must be kept constantly in mind when representing the sphere by means of a sketch, in which minute errors will necessarily appear to be very large. The student should become accustomed to thinking of the appearance of the sphere both from the inside and from an outside point of view. It is usually easier to understand the spherical problems by studying a small globe, but when celestial objects are actually observed they are necessarily seen from a point inside the sphere.

3 Apparent Motion of the Celestial Sphere

If a person watches the stars for several hours he will see that they appear to rise in the east and to set in the west and that their paths are arcs of circles. By facing to the north (in the northern hemisphere) it will be found that the circles traced by the stars become smaller to the northward and that they all appear to be concentric about a certain point in the sky called the *pole;* if a star were exactly at this point it would have no apparent motion. In other words, the

whole celestial sphere appears to be rotating about an axis.* This apparent rotation is caused by the actual rotation of the earth about its axis (from west to east) in the opposite direction to that in which the stars appear to move.

4 Real and Apparent Motion of the Stars

In the preceding article we have recognized that the apparent westerly movement of the stars is caused by the actual easterly rotation of the earth on its axis. We are likely to consider the positions of the stars to be fixed on the celestial sphere while we on earth are rotating at its center. Indeed we commonly speak of the *fixed stars*, to distinguish them from the bodies of the solar system whose projected positions on the sphere are rapidly changing. We find, however, that the positions of the stars do change very slowly. The largest increment in this change is caused by the movement of the *vernal equinox*, the point on the celestial sphere from which we measure the spherical coordinates of the stars. The reasons for this movement are considered in Art. 9. In addition, the entire solar system, including the earth, is moving through space with a velocity exceeding 12 miles per second, and the stars themselves are moving at high velocities in various directions. When we have accounted for the change in position of a star caused by the movement of the earth we still find a change in its position caused by its own motion through space. This change may be divided into two components: its *proper motion*, the rate of change in the direction of the star (this will represent the change in position on the celestial sphere); and its *radial velocity*, the speed with which the star is approaching or receding from the sun. The amount of the proper motion in angular measure will

* This apparent rotation may be easily demonstrated by taking a photograph of the stars near the pole and exposing the plate for several hours. The result is a series of concentric arcs all subtending the same angle. If the camera is pointed southward and high enough to photograph stars near the equator, the star trails appear as straight lines.

be controlled by the distance of the star and its linear velocity. Although the linear velocities are high the distances are so great that the angular velocities are small. The largest proper motion known is about 10″ per year. In most instances it is a small fraction of a second per year so that

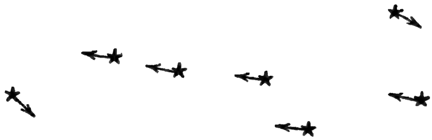


FIG. 2. Relative Proper Motions of Stars in *Ursa Major*

the relative change in the positions of the stars is not apparent during the lifetime of man. Figure 2 shows the directions of the proper motions of the stars forming the Big Dipper, the length of the arrows indicating the amounts of these motions over a period of 50,000 years.

5 The Solar System — Real and Apparent Motions

The solar system is composed of the sun, the planets and their satellites, and the comets, planetoids, and other secondary bodies. In practical astronomy we are concerned with the motions of the earth, the sun, the moon, and the brightest planets. Physical observations of the motions of the planets permit the deduction of the "laws" which control their movements. These were first expressed by Kepler and may be summarized as follows:

1. The orbit of each of the planets is an ellipse with the sun at one of its foci.

2. Every planet revolves so that the line between it and the sun's center (its radius vector) sweeps over equal areas in equal time intervals.

3. The squares of the periods of revolution of any two planets are in the same ratio as the cubes of their mean distances from the sun.

These laws were followed by the publication of Newton's law of gravitation. From it, Kepler's laws may be derived, and by its application we can account for the *perturbations* or small departures of the planets from simple elliptical paths

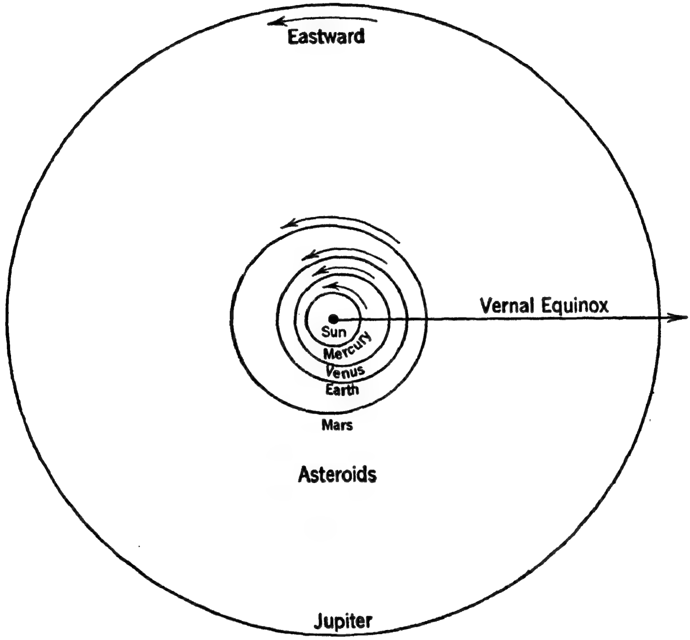


FIG. 3. Diagram of the Solar System within the Orbit of Saturn

since the motion of a planet results not only from the effect of mutual gravitational attraction between it and the sun but also from the smaller gravitational effects of the other bodies of the solar system as well.

Figure 3 shows a diagram of the solar system as far as the orbit of Jupiter. Beyond, in order, lie the orbits of Saturn, Uranus, Neptune, and Pluto. On the same scale Pluto would lie approximately 13 inches from the sun. The nearest star,

α Centauri*, would be somewhat over 7000 feet away on this scale.

Table A shows the mean distances of the planets in terms of the mean distance from the sun to the earth (one astronomical unit), the periods of revolution about the sun in years, the eccentricities of the orbits, the inclinations of the planes of the orbits to that of the earth, and the mean diameters of the planets in miles. The diameters of the sun and moon are also shown. Data are from the *American Ephemeris and Nautical Almanac.*

TABLE A

Planet	Mean Distance	Period (Years)	Eccentricity	Inclination	Mean Diameter (Miles)
Mercury	0.3871	0.24085	0.2056	7° 00′ 13″.3	3,009
Venus	0.7233	0.61521	0.0068	3 23 38 .6	7,575
Earth	1.0000	1.00004	0.0167	....	7,918
Mars	1.5237	1.88089	0.0934	1 51 00 .2	4,216
Jupiter	5.2028	11.86223	0.0484	1 18 22 .7	86,728
Saturn	9.5388	29.45772	0.0557	2 29 26 .2	72,430
Uranus	19.1910	84.01529	0.0472	0 46 22 .6	30,878
Neptune	30.0707	164.78829	0.0086	1 46 30 .5	32,932
Pluto	39.4574	247.6968	0.2485	17 08 35 .5	....
Sun					864,392
Moon					2160

If an observer were to view the solar system from a point far outside, looking from the north toward the south, he would see that all of the planets (including the earth) revolve about the sun in elliptical orbits which are nearly circular, the direction of the motion being *counterclockwise* or left-handed. He would also see that the planets rotate on their axes in a counterclockwise direction. Thus we have, for the earth, an *annual revolution* about the sun and a *daily rotation* about its axis, both in the same direction. With most of the planets the period of revolution is long; the period of rota-

* The Greek alphabet is given on p. 313.

tion is short. The period of rotation of Mercury is uncertain. It is possible that, like the moon, the periods of rotation and revolution are equal. Our satellite, the moon, revolves about the earth in an orbit which is not so nearly circular, but the motion is in the same (counterclockwise) direction.

The apparent motions resulting from these actual motions are as follows: the whole celestial sphere, carrying with it all the stars, sun, moon, and planets, appears to rotate about the earth's axis once per day in a *clockwise* (right-handed) direction. The stars change their positions so slowly that they appear to be fixed in position on the sphere whereas all objects within the solar system rapidly change their apparent positions with reference to the stars. The sun appears to move eastward among the stars at a rate of about 1° per day and to make one revolution about the earth in just one year. The moon also travels eastward among the stars but at a considerably faster rate; it moves an amount equal to its own diameter in about an hour and completes one revolution around the earth in about $27\frac{1}{3}$ days. Figures 4*a* and 4*b* show the daily motions of the sun and moon respectively, as indicated by their plotted positions when passing through the constellation *Taurus*. It should be observed that the motion of the moon eastward among the stars is an actual motion, not merely an apparent one like that of the sun. The planets all move eastward among the stars, but since we ourselves are on a moving object the motion we see is a combination of the real motions of the planets around the sun and an apparent motion caused by the earth's revolution around the sun; the planets consequently appear at certain times to move westward (i.e., backward), or to *retrograde*. Figure 5*a* shows the loop in the apparent path of the planet Jupiter caused by the earth's motion around the sun during 1947. It will be seen that the apparent motion of the planet was *direct* from December 1946 to the middle of March 1947, was *retrograde* motion from March 1947 to the middle

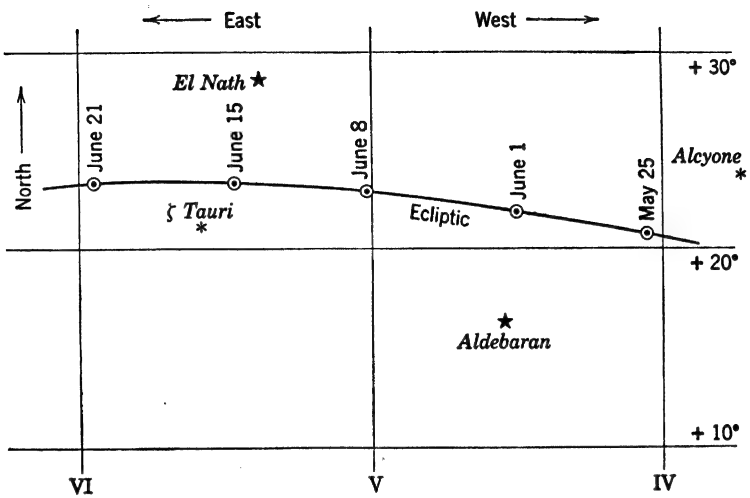


FIG. 4*a*. Sun's Apparent Position at Greenwich 0^h, May 25 to June 21, 1947

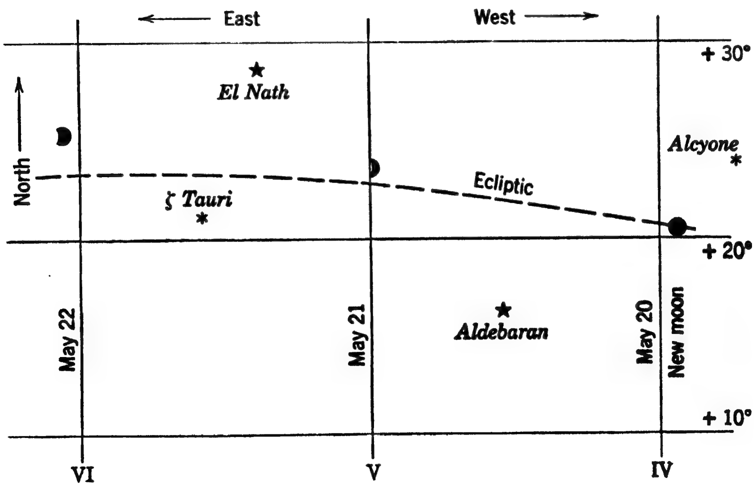


FIG. 4*b*. Moon's Apparent Position at Greenwich 18^h, May 20, 21, 22, 1947 (Note that moon's apparent motion is much more rapid than that of sun)

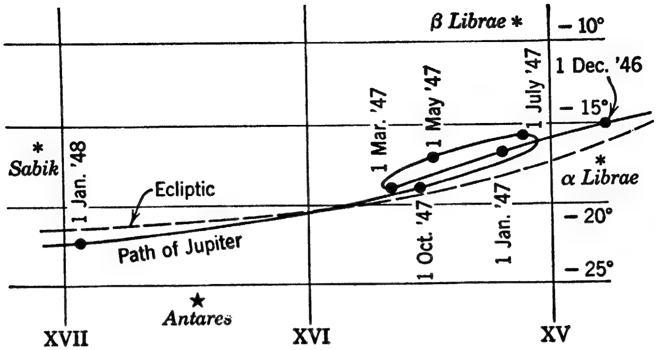


Fig. 5*a*. Apparent Path of Jupiter, December 1946 to January 1948

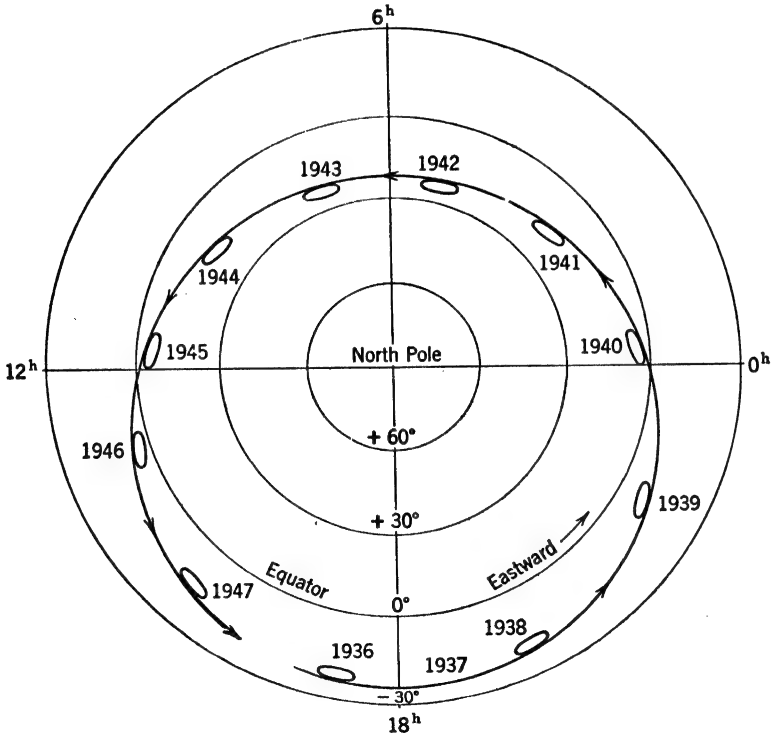


Fig. 5*b*. Apparent Motion of Jupiter, 1936 through 1947

of July 1947, and then again became direct. Figure 5*b* shows the apparent path of Jupiter during one revolution about the sun, from 1936 through 1947, and indicates eleven periods of retrograde motion during this cycle.

6 Meaning of Terms East and West

In astronomy the terms " east " and " west " cannot be taken to mean the same as they do when dealing with directions in one plane. On a plane " east " and " west " may be considered to mean the directions perpendicular to the north-south line. If a person in longitude 0° (that of Greenwich, England) and another person on the 180° meridian

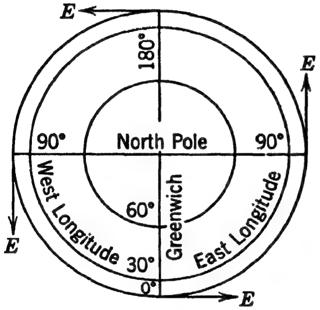


FIG. 6. Arrows All Point Eastward

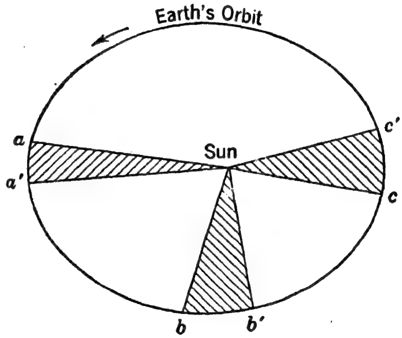


FIG. 7. The Earth's Orbital Motion

should both point due east, they would actually be pointing to opposite points of the sky. In Fig. 6 all four of the arrows are pointing east at the places shown. It will be seen from this figure that the terms " east " and " west " must therefore be taken to mean *directions of rotation.*

7 The Earth's Orbital Motion — the Seasons

The earth, as we have seen, moves eastward around the sun once a year in an orbit which lies (very nearly) in one plane and whose form is that of an ellipse, the sun being at

one of the foci. Since the earth is maintained in its position by the force of gravitation, it moves, as a consequence, in accordance with Kepler's second law, at such a speed in each part of its path that the line joining the earth and sun moves over equal areas in equal times. In Fig. 7 all the shaded areas are equal, and the arcs aa', bb', and cc' represent the distances passed over in the same number of days.*

The axis of rotation of the earth is inclined to the plane of the orbit at an angle of about $66°\frac{1}{2}$; that is, the plane of the earth's equator is inclined at an angle of about $23°\frac{1}{2}$ to the plane of the orbit. This latter angle is known as the *obliquity of the ecliptic.* The direction of the earth's axis of rotation is nearly constant, and it therefore points nearly to the same place in the sky year after year.

The changes in the seasons are a direct result of the inclination of the axis and the fact that the axis remains nearly parallel to its mean position. When the earth is in that part of the orbit where the northern end of the axis is pointed away from the sun (Fig. 8) it is winter in the northern hemisphere. The sun appears to be farthest south about December 22, and at this time the days are shortest and the nights are longest. This position of the earth is known as the *winter solstice.* About 10 days later the earth passes the end of the major axis of the ellipse and is at the point of nearest approach to the sun, or *perihelion.* Although the earth is really nearer to the sun in winter than in summer, this has only a small effect upon the seasons; the chief reasons why it is colder in winter are that the days are shorter and that the rays of sunlight strike the surface of the ground more obliquely. The sun appears to be farthest north about June 22, the *summer solstice,* at which time summer begins in the northern hemisphere and the days are longest and the nights

* The eccentricity of the ellipse shown in Fig. 7 is exaggerated for the sake of clearness; the earth's orbit is in reality much more nearly circular, the variation in the earth's distance from the sun being only about 3 per cent as shown in Table A.

shortest. When the earth passes the other end of the major axis of the ellipse it is farthest from the sun, or at *aphelion.* On March 21 the sun is in the plane of the earth's equator, and day and night are of equal length at all places on the earth (Fig. 8). On September 23 the sun is again in the plane

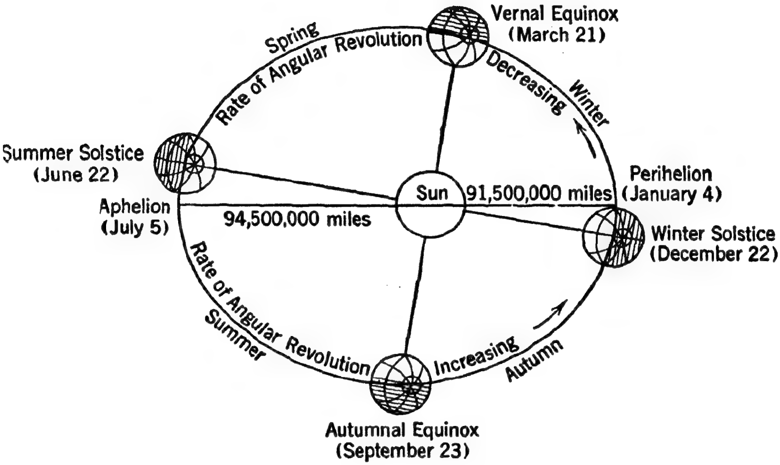


FIG. 8. The Seasons (Dates shown here are for 1947)
Note that there are 186 days in spring and summer and 179 days in autumn and winter.

of the equator, and day and night are everywhere equal. These two times are called the *equinoxes* (vernal and autumnal, respectively), and the points in the sky where the sun's center appears to be at these two dates are called the *equinoctial points,* or more commonly the *equinoxes.*

8 The Sun's Apparent Position at Different Seasons

The apparent positions of the sun on the celestial sphere corresponding to these different positions of the earth are shown in Fig. 9. As a result of the sun's apparent eastward motion from day to day along a path which is inclined to the equator, the angular distance of the sun from the equator is

continually changing. Half of the year the sun is north of the equator, and half of the year it is south. On June 22 the sun is in its most northerly position and is visible more than half of a 24-hour day to a person in the northern hemisphere (*J*, Fig. 9). On December 22 it is farthest south of the equator and is visible less than half the day (*D*, Fig. 9).

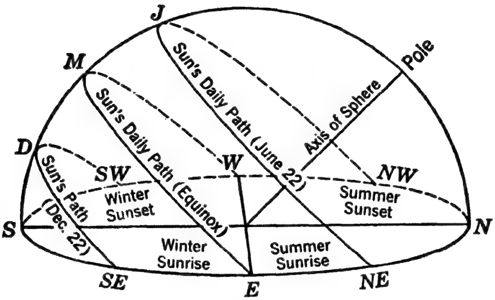


FIG. 9. Sun's Apparent Position at Different Seasons

In between these two extremes it moves back and forth across the equator, passing it about March 21 and September 23 each year. The apparent motion of the sun is therefore a helical motion about the axis of rotation; that is, the sun, instead of following the path which would be taken by a fixed star, gradually increases or decreases its angular distance from the pole at the same time that it appears to revolve once a day around the earth. *The sun's apparent motion eastward on the celestial sphere, caused by the earth's orbital motion, is not noticed until the sun's position is carefully observed with reference to the stars.* If a record is kept for a year showing which *constellations* (named groups of stars) are visible in the east soon after sunset, it will be found that these change from month to month, each being replaced in turn by constellations lying farther to the east, until at the end of a year the one first seen will again appear in the east. Since the sun appears to the observer in approximately the same position at each successive sunset, while the constella-

tions have apparently moved westward with the passage of time, and since we know that the positions of the stars are relatively fixed on the celestial sphere, it should be obvious that the actual apparent motion of the sun has been eastward with respect to the stars and that it has apparently completed a circuit of the heavens in one year.

9 Precession and Nutation

The direction of the earth's rotational axis undergoes a very slow, progressive (*secular*) change known as *precession* and a rapid, though smaller, *periodic* change known as *nutation*. Precession results from the following factors. The earth is spheroidal rather than spherical in form; that is, the equatorial diameter is greater than the polar diameter. The plane of the equator makes an angle of about $23°\frac{1}{2}$ with the plane of the orbit (*the ecliptic*). If the earth were not rotating, the gravitational attraction of the moon and the sun, both nearly in the plane of the ecliptic, would act on the equatorial bulge to bring the plane of the earth's equator into coincidence with the plane of the ecliptic. But, since the earth is actually rotating at high velocity and resists this attraction, the actual effect is not to change permanently the inclination of the equator to the orbit but to cause the earth's axis to describe a cone about the axis joining the poles of the ecliptic. The movement of the axis along this conical surface is westward (opposite to the direction of rotation); the period is about 25,800 years. The movement of the pole, if constant, would describe a circular path on the celestial sphere. This is indicated in Fig. 10 where CD indicates the plane of the ecliptic, A and B are the poles of the ecliptic, and P, P' and P'' are successive positions of the north celestial pole.

The phenomenon can be readily demonstrated by spinning a top about an inclined axis. Gravity, acting on the bulge of the top, combined with the effect of rotation, will cause the axis to describe a cone of revolution, with the direction

of revolution of the axis opposite to the direction of rotation. When rotation ceases, gravity alone will act, and the top will topple over.

Precession has been described as a slow westward gyration of the earth's axis. Since the equator is perpendicular to the polar axis its plane will tip slowly with the movement

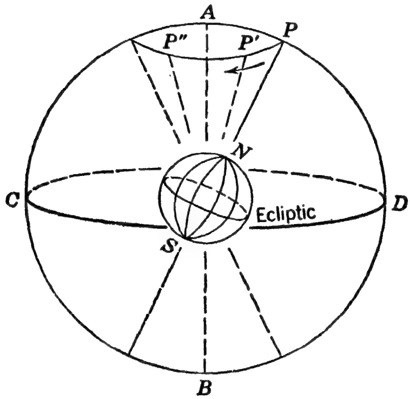


Fig. 10. Precession of the Pole

of the pole. The celestial equator will therefore move relative to the ecliptic so that the points of their intersection, the equinoxes, move slowly westward along the ecliptic. This is called the *precession of the equinoxes.* It is indicated in Fig. 11 where A and B represent the poles of the ecliptic and CVD represents the ecliptic. P is one position of the north celestial pole. The celestial equator corresponding to this position of the pole (EVQ) intersects the ecliptic at V, the vernal equinox. Several centuries later the pole has moved westward to P', the equator has tipped to the position $E'V'Q'$, and its intersection with the ecliptic is now V'. The arc VV' represents the westward precession of the equinox during the period. This precession amounts to $50''.2$ per year on the average. It is of importance because the vernal equinox is used as a reference point for

the determination of the positions of stars and other heavenly bodies. Since both the positions of the equinox and the plane of the equator shift slowly, the spherical coordinates of these bodies will also gradually change. A considerable variation in the rate of precession is caused by the somewhat erratic behavior of the moon. Much nearer to the earth than the sun, it has about twice the effect on precession that the sun has. Its total gravitational attrac-

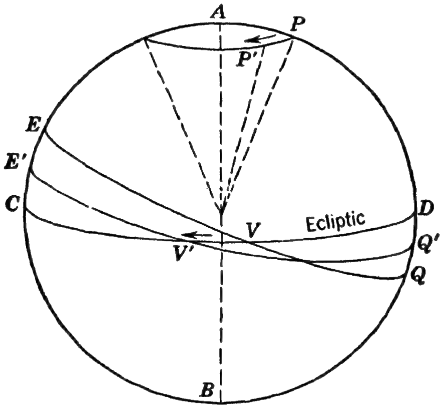


FIG. 11. Precession of the Equinoxes

tion is less, but the difference in attraction for different parts of the earth's surface is greater. The moon's orbit is inclined about 5° to the ecliptic, and therefore the inclination of this orbit to the plane of the equator varies between 18°½ and 28°½. The points of intersection of the moon's orbit and the ecliptic are known as the *nodes*. These are equivalent to the equinoxes. Because the sun tends to pull the moon into the plane of the ecliptic, just as both of them pull on the earth, the nodes move westward or *regress* in exactly the same manner that the equinoxes *precess*. The period is, however, much shorter, amounting to about 19 years. It is the total change of 10° in the obliquity of the moon's orbit to the equator, occurring over

a period of 19 years, which causes nutation. The result is that the celestial pole has a small elliptical motion around its mean position in the circular path caused by precession, taking 19 years to revolve in this elliptical path. Thus the pole wobbles or nods slightly in its essentially circular path. The maximum value of this nutation is about 9″.2

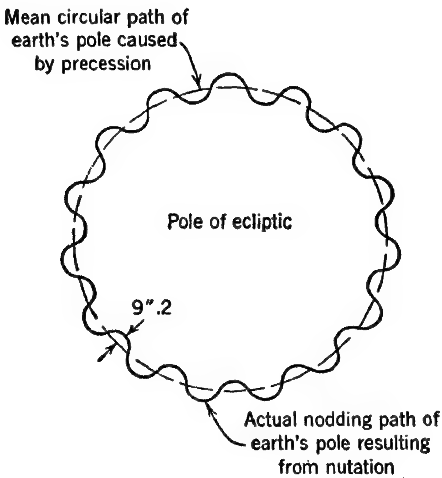


Fig. 12. Nutation of the Pole. (Not to scale)

on each side of the mean circular path. The effect of nutation on the path of the pole is indicated in Fig. 12. Other smaller variations result from the effects of the planets on the precession of the pole and a gradual variation in the obliquity of the ecliptic over a long period. It is sufficient to point out here that because of these effects all of the heavenly bodies will gradually change their positions with reference to the plane of the equator and the position of the vernal equinox. This necessitates periodic revision of star catalogs and ephemerides which tabulate star positions for definite instants of time. The stars themselves have only a very slight angular motion, their apparent

change in position being caused almost entirely by these changes in the positions of the circles of reference.

10 Aberration of Light

Another apparent displacement of the stars, caused by the earth's motion, is that known as aberration. Because of the rapid motion of the earth through space, the direction in which a star is seen by an observer is a result of the combined velocities of the observer and of light from the star. The star always appears to be slightly displaced in the direction in which the observer is actually moving. In Fig. 13*a*, if light moves from C to B in the same length of time that the observer moves from A to B, C would

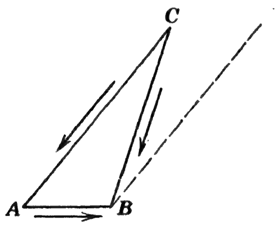


Fig. 13*a*

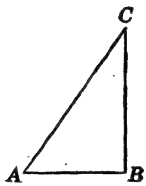


Fig. 13*b*

appear in the direction AC. This may be more clearly understood by using the familiar illustration of the falling raindrop. If a raindrop is falling vertically (CB, Fig. 13*b*) and while it is falling a person moves from A to B, then, considering only the two motions, it appears to the person that the raindrop has moved toward him in the direction CA. If a tube is to be held in such a way that the raindrop shall pass through it without touching the sides it must be held at the inclination of AC. The apparent displacement of a star due to the observer's motion is similar to the change in the apparent direction of the raindrop.

There are two kinds of aberration, *annual* and *diurnal.* Annual aberration is that produced by the earth's motion

in its orbit and is the same for all observers. Diurnal aberration is caused by the earth's daily rotation about its axis. It is different in different latitudes because the speed of a point on the earth's surface is greatest at the equator and diminishes toward the pole.

If v represents the velocity of the earth in its orbit and V the velocity of light, then when CB is at right angles to AB the displacement is a maximum, and

$$\tan \alpha_0 = \frac{v}{V}$$

where α_0 is the angular displacement and is called the "constant of aberration." Its value is about $20''.5$. If CB is not perpendicular to AB,

$$\sin \alpha = \frac{v}{V} \sin A$$

or approximately

$$\tan \alpha = \sin \alpha = \frac{v}{V} \sin B$$

where α is the angular displacement, and B is the angle ABC.

Problem

1. Prepare a tabulation for the planets listed in Table A which will provide a check of Kepler's third law.

2

Definitions—Points and Circles of Reference

11 Definitions

In the preceding chapter many terms, strange to the everyday vocabulary, have been italicized for emphasis, and their meanings have been made clear. Many others appear in this and in succeeding chapters. The difficulty of many students in obtaining a sound working knowledge of practical astronomy may be traced directly to their failure to understand the "language of astronomy." The definitions of these terms, uncommon to many of us, must be learned so thoroughly that, whenever one is mentioned, the mind will at once visualize a definite point, arc, angle, or circle of reference on the celestial sphere or on the earth.

Great Circle. If a plane is passed through the center of a sphere its resulting intersection with the spherical surface will form a *great circle.* Through any two points upon the surface of a sphere a great circle can be drawn since a plane can be passed through the two points and the center of the sphere, and this plane cuts the surface in a great circle. In general, only one great circle can be passed through any two points on a sphere. If, however, the two points lie at the ends of a diameter of the sphere, an infinite number of great circles can be drawn through them. It can be shown

that the shortest distance between any two points on a spherical surface is the arc of a great circle, the length of the arc being not greater than a semicircle.

Spherical Triangle. A *spherical triangle* is that portion of the surface of a sphere bounded by the arcs of three *great circles.*

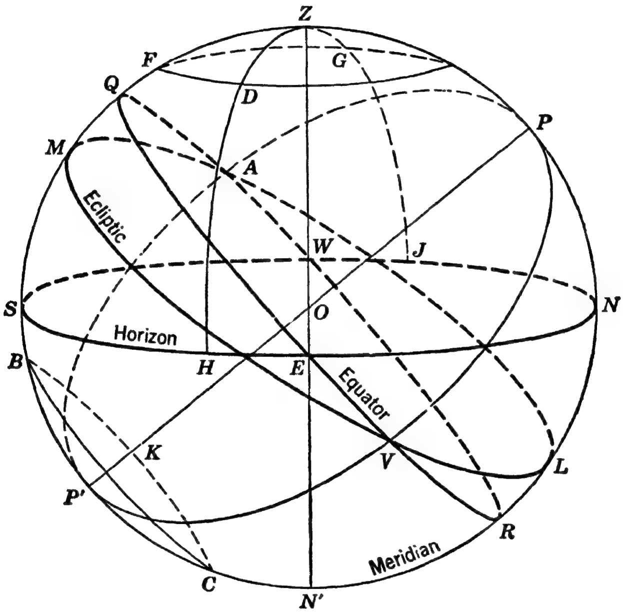


Fig. 14. The Celestial Sphere

Small Circle. If a plane which does not pass through the center of a sphere cuts the spherical surface the resulting intersection forms a *small circle.* Care must be taken that the arc of a small circle is not erroneously used in the solution of a supposed spherical triangle.

The foregoing definitions are general. Those following refer in the main to the celestial sphere only.

Vertical Line. A *vertical line* at any point of the earth's surface is the direction of gravity at that point and is shown

by the plumb line or indirectly by means of the spirit level (ZON', Fig. 14).

Zenith — Nadir. If the vertical at any point is prolonged upward it will pierce the sphere at a point called the *zenith* (Z, Fig. 14). This point is of great importance because it is the point on the sphere which indicates the position of the observer on the earth's surface. The point where the vertical prolonged downward pierces the sphere is called the *nadir* (N', Fig. 14).

Horizon. The *horizon* is the great circle on the celestial sphere that is cut by a plane through the center of the earth perpendicular to the vertical ($NESW$, Fig. 14). The horizon is everywhere 90° from the zenith and the nadir. It is evident that a plane through the observer perpendicular to the vertical cuts the sphere in this same great circle. The *visible horizon* is the circle where the sea and sky seem to meet. Projected onto the sphere it is a small circle below the true horizon and parallel to it. Its distance below the true horizon depends upon the height of the observer's eye above the surface of the water.

Vertical Circles. Vertical circles are great circles passing through the zenith and nadir. They all cut the horizon at right angles (HZJ, Fig. 14).

Almucantars. Parallels of altitude, or *almucantars,* are small circles parallel to the horizon (DFG, Fig. 14).

Poles. If the earth's axis of rotation is produced indefinitely it will pierce the sphere in two points called the north and south celestial *poles* (P and P', Fig. 14).

Equator. The *celestial equator,* or *equinoctial,* is a great circle of the celestial sphere cut by a plane through the center of the earth perpendicular to the axis of rotation ($QWRE$, Fig. 14). It is everywhere 90° from the poles. A parallel plane through the observer cuts the sphere in the same circle.

Hour Circles. Hour circles are great circles passing through the north and south celestial poles (PVP', Fig. 14).

Parallels of Declination. Small circles parallel to the plane of the equator are called *parallels of declination* (*BKC*, Fig. 14).

Meridian. The *meridian* is the great circle passing through the zenith and the poles (*SZPL*, Fig. 14). It is both an hour circle and a vertical circle. It is evident

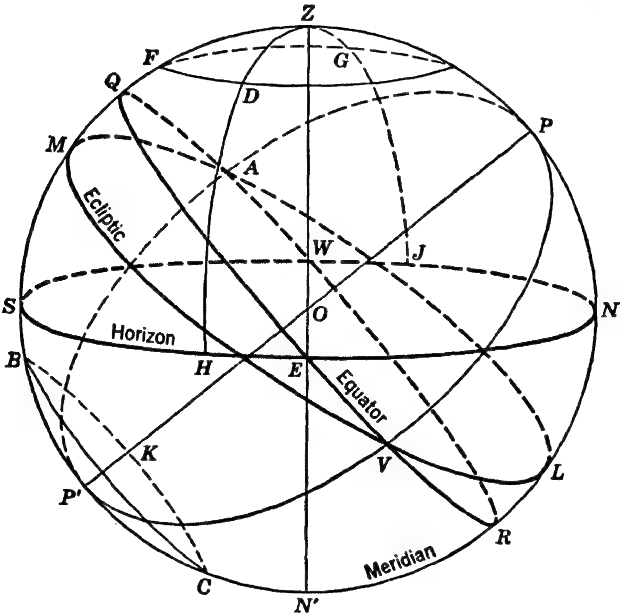


Fig. 14. The Celestial Sphere

that different observers will in general have different meridians. The meridian cuts the horizon in the north and south points (*N*, *S*, Fig. 14). The intersection of the plane of the meridian with the horizontal plane through the observer is the meridian line (or north-south line) through the observer. The *upper branch* of the meridian is that half extending from pole to pole which contains the zenith. The *lower branch* is that half which contains the nadir.

Prime Vertical. The *prime vertical* is the vertical circle

whose plane is perpendicular to the plane of the meridian (*EZW*, Fig. 14). It cuts the horizon and the equator at the *east* and *west* points of the horizon (*E*, *W*, Fig. 14).

Six-Hour Circle. The *6-hour circle* is the hour circle whose plane is perpendicular to that of the meridian. It also cuts the equator and the horizon at the east and west points.

Equinoctial Colure. The *equinoctial colure* is the hour circle passing through the equinoxes (*PVP'A*, Fig. 14).

The ecliptic is a circle of reference, and the equinoxes are points of reference already defined in Chapter 1. The vernal equinox is also known as the *first point of Aries* and is indicated by the symbols *V* and ♈.

Questions

1. What common imaginary circle or circles, if any, on the surface of the earth correspond to the circles on the celestial sphere listed below?

(*a*) Vertical circles.	(*f*) Hour circles.
(*b*) Ecliptic.	(*g*) Equator.
(*c*) Meridian.	(*h*) Horizon.
(*d*) Almucantars.	(*i*) Parallels of declination.
(*e*) Prime vertical.	(*j*) Six-hour circle.

2. On what parallels of latitude do the Tropics of Cancer and of Capricorn and the Arctic and Antarctic Circles fall? What determines the position of these imaginary circles? Within what latitudes will the sun be in the zenith sometime during the year? Within what latitudes may the sun remain below the horizon for at least 24 consecutive hours during the year?

3

Systems of Coordinates on the Sphere

12 Spherical Coordinates

The direction of a point in space may be defined by means of two spherical coordinates, that is, by two angular distances measured on a sphere along arcs of two great circles

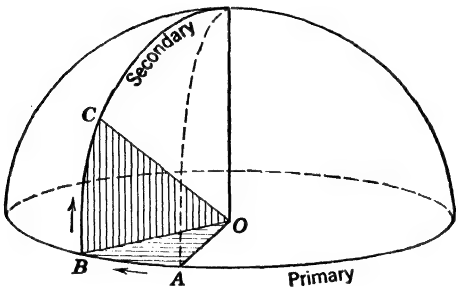


FIG. 15. Spherical Coordinates

which cut each other at right angles. Suppose that it is desired to locate C (Fig. 15) with reference to the plane OAB and the line OA, O being the *origin* of coordinates. Pass a plane OBC through OC perpendicular to OAB; these planes will intersect in the line OB. The two angles which fix the position of C, or the spherical coordinates, are BOC

and AOB. These may be regarded as the angles at the center of the sphere or as the arcs BC and AB. In every system of spherical coordinates the two coordinates are measured, one on a great circle called the *primary* and the other on one of a system of great circles at right angles to the primary called *secondaries.* There are an infinite number of secondaries, each passing through the two poles of the primary. The coordinate measured from the primary is an arc of a secondary circle; the coordinate measured between the secondary circles is an arc of the primary. In practical astronomy we require at least two systems of spherical coordinates: one, a system dependent on the position of the observer; the second, a system independent of the observer's position. The first, *the horizon system,* is necessitated by the fact that we can measure only horizontal and vertical angles with the engineer's transit and that horizontal and vertical angles are commonly measured by means of the compass and sextant, respectively. Obviously angles to a star measured in this way are dependent on the location of the observer on the earth's surface. On the other hand, it is necessary to publish, in star catalogues, almanacs, or ephemerides, positions of heavenly bodies referred to spherical coordinates which are independent of the observer's position so that these positions apply to observers anywhere on the earth. The *independent equatorial system* is used for this purpose. At times it is convenient to use a primary circle which is independent of the observer and a secondary system dependent on his position. This is therefore a *dependent equatorial system. All problems of practical astronomy devolve about the relationships existing between these three systems.*

13 The Horizon System

In this system the primary circle is the horizon, and the secondaries are vertical circles. The first coordinate of a point is called its *azimuth,* Z. The azimuth is the angular

distance along the horizon measured from the meridian to the foot of the vertical circle through the point. It is also equal to the angle at the zenith between the meridian and this vertical circle. Since it is possible to measure azimuth from either the upper or lower branch of the meridian and either to the east or to the west the student must be cautious in his use of this term and indicate clearly in the

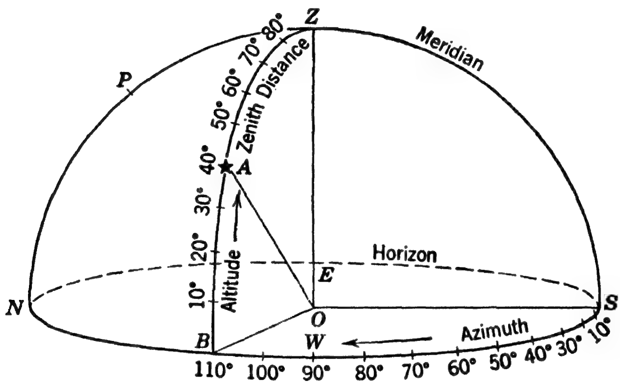


FIG. 16. The Horizon System

solution of problems exactly how it has been measured. The surveyor customarily measures azimuth from the upper branch of the meridian, at the south point, in a westward direction from 0° to 360°. This practice will be followed in this book. On the other hand, navigation practice is to reckon azimuth, also designated as *course* or *bearing*, from the north point clockwise from 000° to 360°. Furthermore, problems involving the determination of azimuth involve the computation of the angle Z of the spherical triangle that may be drawn to connect points P, Z, and A of Fig. 16. This azimuth angle, as computed or taken from tables, will be from either the north or south and either toward the east or west, depending upon the formula used and whether the body observed is east or west of the meridian. This angle will then have to be adjusted to conform

to either the surveying or the navigational practice as described above.

The second coordinate in this system is the *altitude*, h, the angular distance above the horizon, measured on the vertical circle through the point. The complement of the altitude is called the *zenith distance*, ζ. In Fig. 16 the azimuth of the star A (from the south point clockwise) is SB; it is also the angle at the zenith, SZB; its altitude is BA; and its zenith distance is ZA. These coordinates involve vertical and horizontal angles which can be measured with a transit, or by means of a sextant and compass, or which can be deduced from such measurements.

14 The Equator Systems

1. *Independent Equatorial System.* The circles of reference in both equatorial systems are the equator and hour circles. In the independent equatorial system the first coordinate of a point is called its *right ascension*, R.A.* or α. Right ascension is the angular distance along the equator measured from the vernal equinox *eastward* to the foot of the hour circle through the point in question; it may be measured in degrees, minutes, and seconds of arc or in hours, minutes, and seconds of time. The right ascension of a body is also equal to the angle at the pole from the equinoctial colure eastward to the hour circle through the body. For certain purposes it is more convenient to use the *sidereal hour angle*, S.H.A., of a point instead of its right ascension for the first coordinate. The sidereal hour angle is the arc of the equator measured *westward* from the vernal equinox to the foot of the hour circle through the point. It is usually measured in units of arc and is, therefore, 360° minus the right ascension.

The second coordinate of a point in both equatorial systems is its angular distance north or south of the equator, measured on an hour circle; it is called the *declination*, δ.

* See p. 314 for a list of abbreviations and symbols used in this book.

Declination is considered positive when the body is north of the equator and negative when it is south. The complement of the declination is called the *polar distance, p.* Normally the polar distance will be measured on the hour circle from the celestial pole that is nearer to the body. In Fig. 17 the right ascension of the star S is the arc of the equator VA or the angle at the pole VPA; the sidereal

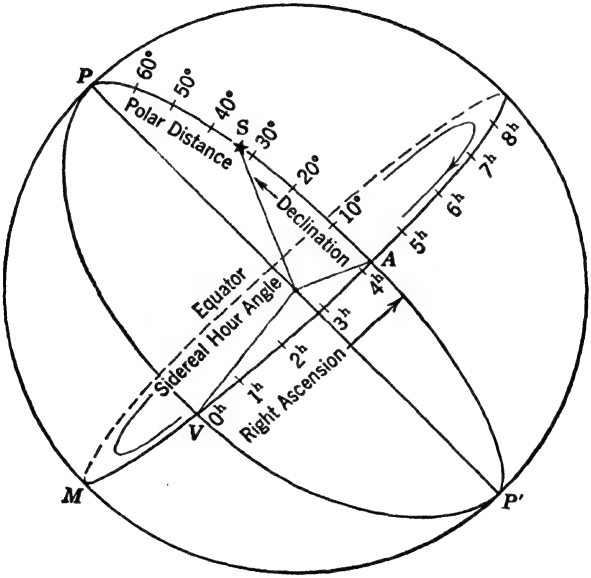


Fig. 17. The Independent Equatorial System

hour angle is the equatorial arc VMA; the declination is the arc AS; the polar distance is the arc PS.

2. Dependent Equatorial System. Instead of locating a point by means of right ascension and declination it is sometimes more convenient to use the dependent equatorial system involving *hour angle* and declination. The hour angle, t, or local hour angle, L.H.A., of a point is the angular distance along the equator measured *westward* from the observer's meridian to the foot of the hour circle through

the point. It is also equal to the angle at the pole between the meridian and the hour circle through the point in question. It may be measured either in units of arc or in units of time. In Fig. 18 the hour angle of the star S is the arc of the equator MA; it is also equal to the angle at the pole MPA; and the declination is the arc AS (negative). For the measurement of time the hour angle may be counted

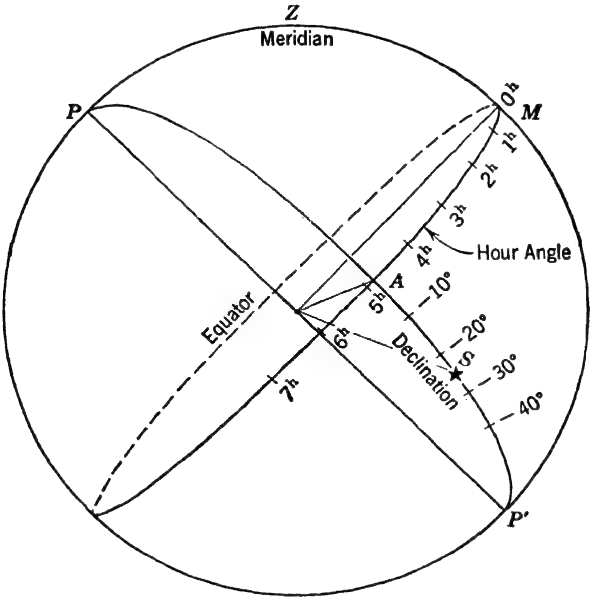


FIG. 18. The Dependent Equatorial System

from either the upper or lower branch of the meridian. Hour angle, like azimuth, may cause some confusion in the mind of the student for it may be measured from either the upper or lower branch of the meridian, and, while it is defined as being measured westward from the meridian, we frequently find it expedient to measure the hour angle to the east of the meridian when the body observed is in the east. Furthermore, there are in common use certain astro-

nomical tables, used both in surveying and in celestial navigation, which consider the hour angle as being taken either east or west of the meridian, on whichever side the heavenly body happens to lie. To eliminate this difficulty, other tables use the term *meridian angle,* M.A., which is defined as the angle at the celestial pole between the meridian and the hour circle of the body, measured either eastward or westward from the meridian from 0^h to 12^h (or from 0° to 180°). It is marked E or W, depending on whether the body is east or west of the meridian.

To permit the concept of hour angle to be used in an independent equatorial system of coordinates, the hour angle of celestial bodies referred to the meridian of Greenwich is tabulated in almanacs as the *Greenwich hour angle,* G.H.A. The use of sidereal hour angle also conforms to this concept.

These three systems of coordinates are shown in Table B. In the first system, both coordinates depend upon the observer's position; in the second, both are independent of his position; in the third, one coordinate is independent while the second is dependent. The terrestrial system of *longitude,* λ and *latitude,* ϕ is included for comparison.

15 Celestial Latitude and Longitude

There is another system which is employed in some branches of astronomy but which will not be used in this book. The coordinates are called *celestial latitude* and *celestial longitude;* the primary circle is the ecliptic. Celestial latitude is measured from the ecliptic just as declination is measured from the equator. Celestial longitude is measured eastward along the ecliptic from the equinox just as right ascension is measured eastward along the equator. The student should be careful not to confuse celestial latitude and longitude with terrestrial latitude and longitude. Terrestrial latitude and longitude are used in the problems discussed in this book.

TABLE B

Systems of Coordinates

Name of System	Reference Circles: Primary Circle	Reference Circles: Secondary Circles	Primary Coordinate (Measured Along Primary Circle to Secondary Circle through Body): Name	Measured from	Direction	Limits	Secondary Coordinate (Measured on Secondary Circle from Primary to Body): Name	Limits
Horizon	Horizon	Vertical circles	Azimuth (Z)	South point of horizon (*surv.*); north point of horizon (*nav.*)	Westward (clockwise)	0° to 360°	Altitude (h)	0° to +90° (*positive toward zenith*)
Independent equatorial	Celestial equator (*equinoctial*)	Hour circles	Right ascension (R.A. or α)	Vernal equinox (*first point of Aries*) (V or ♈)	Eastward	0^h to 24^h or 0° to 360°	Declination (δ or d)	0° to +90° (N) or 0° to −90° (S)
			Sidereal hour angle (S.H.A.)	Vernal equinox	Westward	0° to 360°	Declination	0° to ±90°
			Greenwich hour angle (G.H.A.)	Projected meridian of Greenwich	Westward	0° to 360°	Declination	0° to ±90°
Dependent equatorial	Celestial equator (*equinoctial*)	Hour circles	Hour angle (t or L.H.A.)	Intersection of meridian and equator	Westward	0^h to 24^h or 0° to 360°	Declination	0° to ±90°
			Meridian angle (M.A.)	Intersection of meridian and equator	Eastward or westward	0° to 180° E or 0° to 180° W (*time units also used*)	Declination	0° to ±90°
Terrestrial	Terrestrial equator	Meridians of Longitude	Longitude (λ)	Meridian of Greenwich	Eastward or westward	0° to 180° E 0° to 180° W (*time units also used*)	Latitude (ϕ)	0° to 90° N or 0° to 90° S

16 Coordinates of the Observer

The observer's position is located by means of his latitude and longitude. The latitude, which on the earth's surface is the angular distance of the observer north or south of the equator, may be defined astronomically as the declination of the observer's zenith. In Fig. 19, the terrestrial latitude

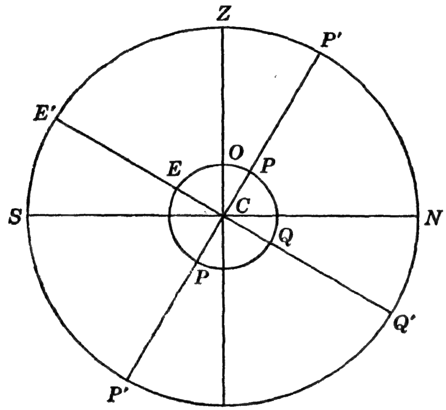


Fig. 19. The Observer's Latitude

is the arc EO, EQ being the equator and O the observer. The point Z is the observer's zenith so that the latitude on the sphere is the arc $E'Z$, which evidently will contain the same number of degrees as EO. The complement of the latitude is called the *colatitude.* The terrestrial longitude of the observer is the arc of the equator between the primary meridian (usually that of Greenwich) and the meridian of the observer. On the celestial sphere the longitude would be the arc of the celestial equator contained between two hour circles whose planes are the planes of the two terrestrial meridians. It would also be the angle at the pole between these two planes. In other words, *the difference in longitude between two points is the difference in time.* Longi-

tude is measured east and west from Greenwich* from 0° to 180° (or from 0^h to 12^h) and must be marked E or W just as latitude must be marked N or S to indicate the proper hemisphere.

17 Relation between the Systems of Coordinates

In studying the relation between different points and circles on the sphere it may be convenient to imagine that the celestial sphere consists of two spherical shells, one within the other. The outer one carries upon its surface the ecliptic, equinoxes, poles, equator, hour circles and all of the stars, the sun, the moon, and the planets. On the inner sphere are the zenith, horizon, vertical circles, poles, equator, hour circles, and the meridian. The earth's daily rotation causes the inner sphere to revolve while the outer sphere is motionless; or, regarding only the apparent motion, the outer sphere revolves once per day on its axis while the inner sphere appears to be motionless. It is evident that the coordinates of a fixed star in the independent equatorial system (declination and right ascension) are practically always the same whereas the coordinates in the horizon system are continually changing.

This concept of two spherical shells forms the basis for the construction of the *celestial coordinator*, a patented diagram for the graphical solution of problems involving the relations between coordinates and for the ready visualization of these relationships. It consists of an under sphere that is projected on a plane, showing in black, by means of parallels of altitude and vertical circles, the hemisphere of sky above the horizon. Centered over this is a movable transparent celluloid sheet on which a projected sphere is

* The Royal Observatory, established at Greenwich in 1675 by Charles II, is currently being moved to Herstmonceux Castle in Sussex where conditions for astronomical observations, away from the lights and smoke of modern London, will be more satisfactory. Longitude and universal time will continue to be measured with reference to the meridian of the old observatory at Greenwich.

printed in red showing parallels of declination and hour circles. By plotting the known coordinates of a celestial

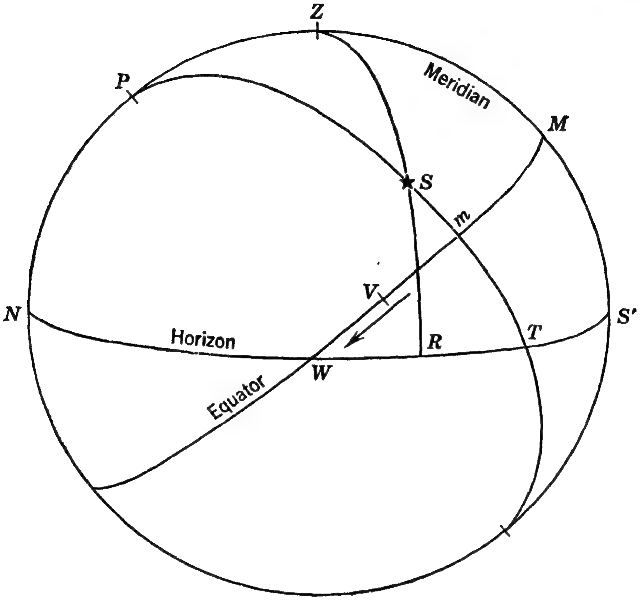


Fig. 20. The Sphere Seen from the Outside

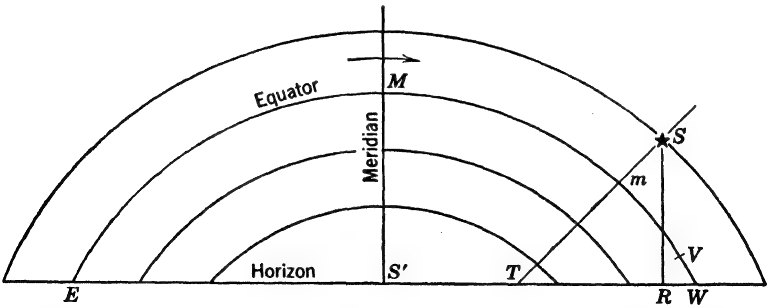


Fig. 21. Portion of the Sphere Seen from the Earth (Looking South)

body and rotating the red projection until the declination of the zenith corresponds to the latitude, the values of unknown coordinates may be readily read from the diagram.

The relationships are also subject to graphical analysis by orthographic projection, but the time required approximates that of mathematical solution, and the results are less precise. The mathematical relations between the

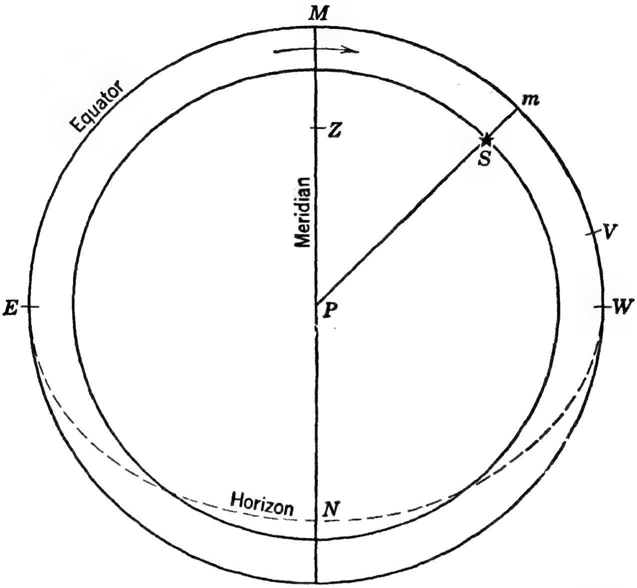


FIG. 22. The Sphere Projected onto the Plane of the Equator

spherical coordinates are discussed in Chapter 4 and in Appendix A.

Figures 20, 21, and 22 show three different views of the celestial sphere with which the student should be familiar. Figure 20 is the sphere as seen from the outside and is the view best adapted to showing problems in spherical trigonometry. The star S has the altitude RS, zenith distance ZS, azimuth $S'R$, declination mS, polar distance PS, hour angle Mm, and right ascension Vm. The meridian is $PZMS'$, the latitude of the observer being represented by the arc ZM. Figure 21 shows a portion of the sphere as seen by an observer looking southward; the points are in-

dicated by the same letters as in Fig. 20. Figure 22 shows the same points projected on the plane of the equator. In this view of the sphere the angles at the pole (i.e., the angles between hour circles) are shown in their true size, and it is therefore a convenient diagram to use when dealing with right ascension and hour angles.

Questions and Problems

1. What coordinates on the sphere correspond to latitude and longitude on the earth's surface?

2. Make a sketch of the sphere, and plot the position of a star having an altitude of 20° and an azimuth of 250° (from the south point clockwise). Locate a star whose H.A. is 16^h; decl. $-10°$. Locate a star whose R.A. is 9^h; decl. N 30°.

3. If a star is on the equator and also on the horizon, what is its azimuth? its altitude? its hour angle? its declination?

4. What changes take place in the azimuth and altitude of a star during 24 hours?

5. What changes take place in the right ascension and declination of the observer's zenith during a day?

6. A person in lat. 40° 43′ N observes *Rigel*, decl. $-8° 17'$, soon after it has passed his meridian. In what order will the star pass the following three circles: (1) the prime vertical, (2) the horizon, and (3) the 6-hour circle? If the star observed had been *Schedir*, decl. $+56° 10'$, instead of *Rigel*, what would have been the order of passing the circles given above?

7. In lat. 40° S does the sun set north or south of the west point of the horizon on December 22?

8. Determine the right ascension, declination, hour angle, azimuth, and zenith distance of the west point of the horizon at the instant when the vernal equinox is at the east point of the horizon.

9. What is the declination of a star which sets due west of an observer in lat. 40° N?

10. A certain star passes through the zenith of an observer in lat. 23° S. What is its declination?

11. Determine the right ascension and declination of the autumnal equinox and the summer and winter solstices.

12. Determine the right ascension, declination, altitude, and hour angle of the north point of the horizon at the instant when the autumnal equinox is at the east point.

13. Make a careful sketch of the celestial sphere for an observer in lat. 45° N. On it show the position of a star having M.A. 30° E; S.H.A. 15°; decl. $-10°$. Estimate from the figure the azimuth and altitude of the star.

14. Determine the azimuth, altitude, meridian angle, and declination of

the east point of the horizon, the zenith, and the north pole of the celestial sphere. The observer is in lat. 45° N.

15. Make a complete sketch of the celestial sphere for lat. 40° S, with the zenith overhead. Place the vernal equinox at the west point of the horizon. Place an arrow on the ecliptic to show the direction of the apparent yearly motion of the sun. To correctly position the ecliptic, the student should remember that near the vernal equinox the declination of the sun is increasing northward with an increasing right ascension.

4

Relation between Coordinates

18 Relation between Altitude of Pole and Latitude of Observer

In Fig. 23, *SZN* represents the observer's meridian; let *P* be the celestial pole, *Z* the zenith, *E* the point of intersection of the meridian and the equator, and *N* and *S* the north and south points of the horizon. By the definitions, *CZ* (vertical) is perpendicular to *SN* (horizon), and *CP* (axis) is perpendicular to *EC* (Equator). Therefore the arc *PN* = arc *EZ*. By the definitions, *EZ* is the declination of the zenith, or the latitude, and *PN* is the altitude of the celestial pole. Hence *the altitude of the pole is always equal to the latitude of the observer.* The same relation may be seen from Fig. 24, in which *OH* is the plane of the horizon, the observer being at *O*; *EQ* is the equator; and *OP′* is a line parallel to that joining *C* and the north pole and, consequently, points to the celestial pole. It may readily be shown that *ECO*, the observer's latitude, equals *HOP′*, the altitude of the celestial pole. A person at the equator would see the north celestial pole in the north point

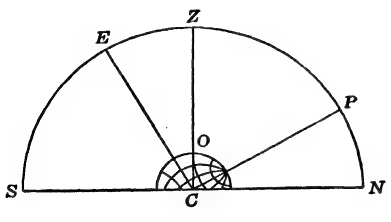


FIG. 23

of his horizon and the south celestial pole in the south point of his horizon. If he traveled northward the north pole would appear to rise, its altitude being always equal to his latitude, while the south pole would immediately go below his horizon. When the traveler reached the north pole of the earth, the north celestial pole would be vertically over his head.

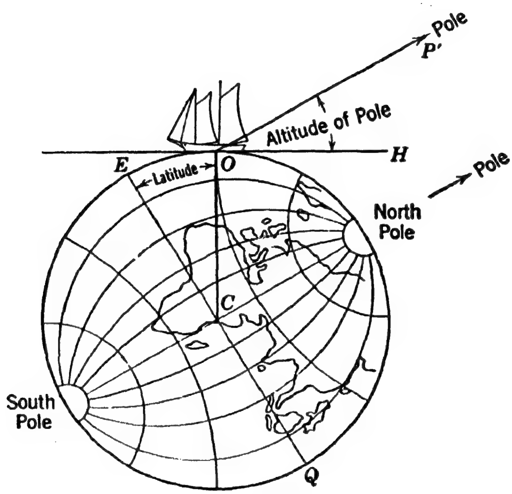


FIG. 24

To a person at the equator all stars would appear to move vertically at the times of rising and setting, and all stars would be above the horizon 12^h and below 12^h during one revolution of the sphere. All stars in both hemispheres would be above the horizon at some time every day (Fig. 25).

If a person were at the earth's north pole, the celestial equator would coincide with his horizon, and all stars in the northern hemisphere would appear to travel around in circles parallel to the horizon; they would be above the horizon for 24^h a day, and their altitudes would not change. The stars in the southern hemisphere would never be visible. The word *north* would cease to have its usual meaning, and

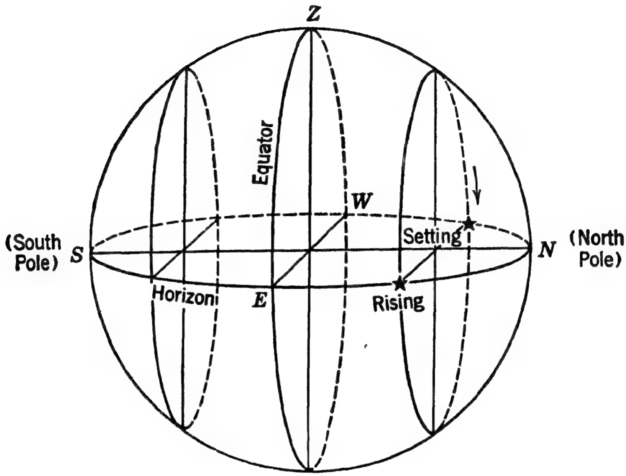


Fig. 25. The Right Sphere (Appearance of sphere to observer at earth's equator)

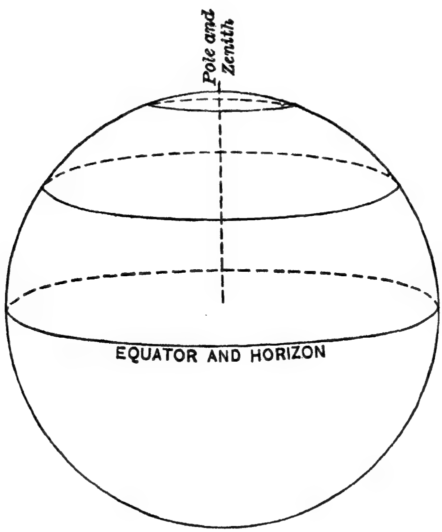


Fig. 26. The Parallel Sphere (Appearance of sphere to observer at earth's pole)

south might mean any horizontal direction. The longitude of a point on the earth and its azimuth from the Greenwich meridian would then be the same (Fig. 26).

At all points between these two extreme latitudes the equator cuts the horizon obliquely. A star on the equator will be above the horizon half the time and below half the time. A star north of the equator will (to a person in the

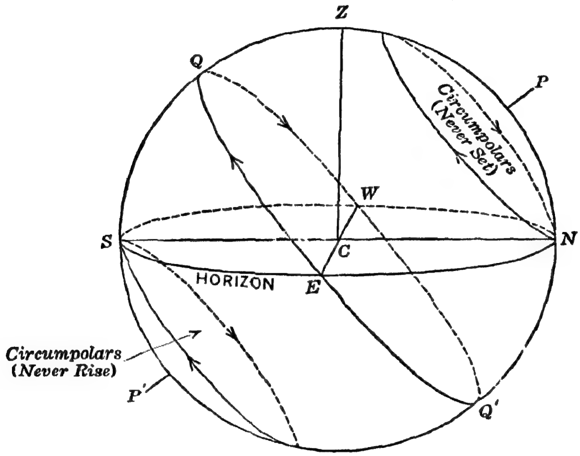


FIG. 27. Circumpolar Stars

northern hemisphere) be above the horizon more than half of the day; a star south of the equator will be above the horizon less than half of the day. If the north polar distance of a star is less than the observer's north latitude, the whole of the star's *diurnal* circle is above the horizon, and the star will therefore remain above the horizon all of the time. It is called in this case a *circumpolar star* (Fig. 27). The south circumpolar stars are those whose south polar distances are less than the observer's south latitude. If the observer travels north until he is beyond the arctic circle, latitude 66° 33′ north, then the sun becomes a circumpolar at the time of the summer solstice. At noon the sun would be at its maximum altitude; at midnight it would

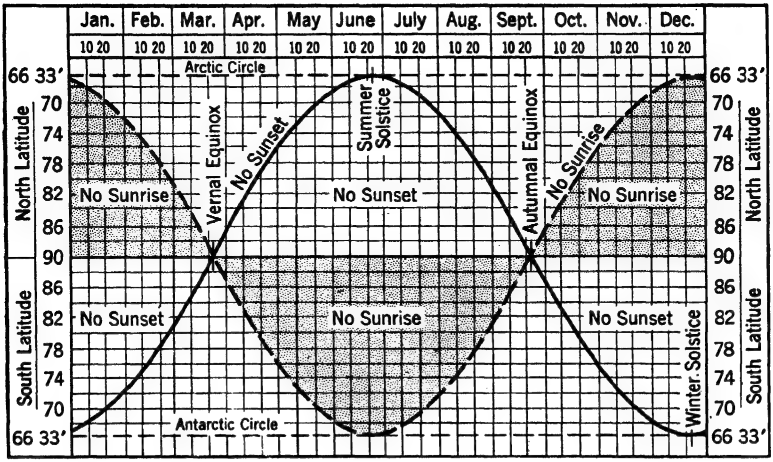


FIG. 28. Calendar of the Midnight Sun *By permission of Prof. C. O. Roth, Jr.*

be at its minimum altitude but would still be above the horizon. This is called the "midnight sun." The calendar of the midnight sun is shown in Fig. 28.

19 Relation between Latitude of Observer and the Declination and Altitude of a Point on the Meridian

The relation between the latitude of the observer and the declination and altitude of a point on the observer's meridian may be seen by referring to Fig. 29. Let A be any point on the meridian, such as a star or the center of the

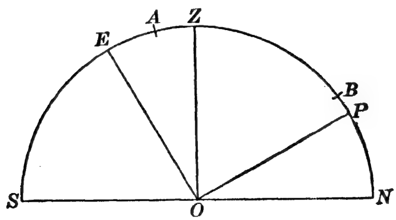


FIG. 29. Star on the Meridian

sun, moon, or a planet, located south of the zenith but north of the equator; then

$EZ = \phi$, the latitude

$EA = \delta$, the declination

$SA = h$, the meridian altitude

$ZA = \zeta$, the meridian zenith distance

From the figure it is evident that

$$\phi = \zeta + \delta \tag{1}$$

If A is south of the equator δ becomes negative, but the same equation applies in this case provided that the quantities are given their proper signs. If A is north of the zenith we should have

$$\phi = \delta - \zeta \tag{2}$$

but if we regard ζ as negative when north of the zenith and positive when south of the zenith, Eq. 1 covers all cases. When the point is below the pole the same formula might be employed by counting the declination beyond 90°. In such cases it is usually simpler to employ the polar distance p instead of the declination.

If the star is north of the zenith but above the pole, as at B, then, since $p = 90° - \delta$,

$$\phi = h - p \tag{3}$$

If B were below the pole we should have

$$\phi = h + p \tag{4}$$

These relations form the basis for the usual observation for latitude. The meridian altitude of the heavenly body is measured with transit or sextant, necessary corrections as described in Chapter 7 are applied, the declination is taken from a list of tabulated positions in the ephemeris or almanac, and the latitude is readily determined from the

above equations. In practice fewer errors will be made if a sketch, similar to Fig. 29, is drawn and the latitude determined from the relationship there indicated.

20 The Astronomical Triangle

By joining the pole, zenith, and any star S on the sphere by arcs of great circles we obtain a triangle from which the

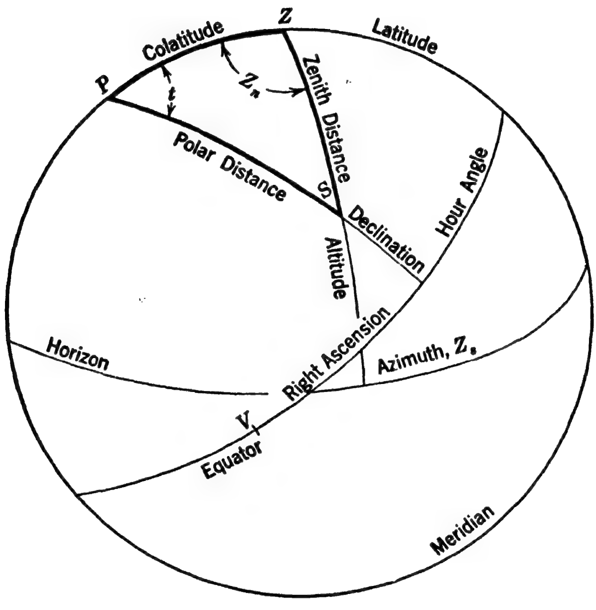


FIG. 30. The Astronomical Triangle

relation existing among the spherical coordinates may be obtained. This triangle is so frequently employed in practical astronomy that it is called the " astronomical triangle " or the " PZS triangle." In Fig. 30 the arc PZ is the complement of the latitude, or colatitude; arc ZS is the zenith distance or complement of the altitude; arc PS is the polar distance or complement of the declination; the angle at P is the hour angle of the star if S is west of the meridian,

or 360° minus the hour angle if S is east of the meridian; and Z is the azimuth of S (from the north point clockwise), or 360° minus the azimuth, according as S is east or west of the meridian. The angle at S is called the *parallactic angle.* If any three parts of this triangle are known the other three may be calculated. The fundamental formulae of spherical trigonometry are (see Appendix):

$$\cos a = \cos b \cos c + \sin b \sin c \cos A \tag{5}$$

$$\sin a \cos B = \cos b \sin c - \sin b \cos c \cos A \tag{6}$$

$$\sin a \sin B = \sin b \sin A \tag{7}$$

If we put $A = t$, $B = S$, $C = Z$, $a = 90° - h$, $b = 90° - \phi$, and $c = 90° - \delta$, these three equations become

$$\sin h = \sin \phi \sin \delta + \cos \phi \cos \delta \cos t \tag{8}$$

$$\cos h \cos S = \sin \phi \cos \delta - \cos \phi \sin \delta \cos t \tag{9}$$

$$\cos h \sin S = \cos \phi \sin t \tag{10}$$

If $A = t$, $B = Z$, $C = S$, $a = 90° - h$, $b = 90° - \delta$, and $c = 90° - \phi$, Eqs. 6 and 7 become

$$\cos h \cos Z = \sin \delta \cos \phi - \cos \delta \sin \phi \cos t \tag{11}$$

$$\cos h \sin Z = \cos \delta \sin t \tag{12}$$

If $A = Z$, $B = S$, $C = t$, $a = 90° - \delta$, $b = 90° - \phi$, and $c = 90° - h$,

$$\sin \delta = \sin \phi \sin h + \cos \phi \cos h \cos Z \tag{13}$$

$$\cos \delta \cos S = \sin \phi \cos h - \cos \phi \sin h \cos Z \tag{14}$$

$$\cos \delta \sin S = \cos \phi \sin Z \tag{15}$$

If $A = Z$, $B = t$, $C = S$, $a = 90° - \delta$, $b = 90° - h$, and $c = 90° - \phi$,

$$\cos \delta \cos t = \sin h \cos \phi - \cos h \sin \phi \cos Z \tag{16}$$

Other forms may be derived, but those already given will suffice for all cases occurring in the following chapters.

The problems arising most commonly in the practice of surveying are:

1. Given the declination, latitude, and altitude to find the azimuth and hour angle.

2. Given the declination, latitude, and hour angle to find the azimuth and altitude.

In following formulae let

$$t = \text{hour angle}$$

$$Z = \text{azimuth*}$$

$$h = \text{altitude}$$

$$\zeta = \text{zenith distance}$$

$$\delta = \text{declination}$$

$$p = \text{polar distance}$$

$$\phi = \text{latitude}$$

$$s = \tfrac{1}{2}(\phi + h + p)$$

For computing t, any of the following formulae may be used.

$$\sin \tfrac{1}{2}t = \sqrt{\frac{\cos s \sin (s - h)}{\cos \phi \sin p}} \tag{17}$$

$$\cos \tfrac{1}{2}t = \sqrt{\frac{\cos (s - p) \sin (s - \phi)}{\cos \phi \sin p}} \tag{18}$$

$$\tan \tfrac{1}{2}t = \sqrt{\frac{\cos s \sin (s - h)}{\cos (s - p) \sin (s - \phi)}} \tag{19}$$

$$\cos t = \frac{\sin h - \sin \phi \sin \delta}{\cos \phi \cos \delta} \tag{20}$$

$$\cos t = \frac{\sin h}{\cos \phi \cos \delta} - \tan \phi \tan \delta \tag{20a}$$

* The trigonometric formulae give the interior angle of the triangle, and consequently the azimuth from the north point, unless the form of the equation is changed so as to give the exterior angle.

$$\text{vers } t = \frac{\cos(\phi - \delta) - \sin h}{\cos \phi \cos \delta} \tag{21}$$

For computing the azimuth Z from the north point either toward the east or the west, we have

$$\sin \tfrac{1}{2}Z = \sqrt{\frac{\sin(s - h)\sin(s - \phi)}{\cos \phi \cos h}} \tag{22}$$

$$\cos \tfrac{1}{2}Z = \sqrt{\frac{\cos s \cos(s - p)}{\cos \phi \cos h}} \tag{23}$$

$$\tan \tfrac{1}{2}Z = \sqrt{\frac{\sin(s - \phi)\sin(s - h)}{\cos s \cos(s - p)}} \tag{24}$$

$$\cos Z = \frac{\sin \delta - \sin \phi \sin h}{\cos \phi \cos h} \tag{25}$$

$$\cos Z = \frac{\sin \delta}{\cos \phi \cos h} - \tan \phi \tan h \tag{25a}$$

$$\text{vers } Z = \frac{\cos(\phi - h) - \sin \delta}{\cos \phi \cos h} \tag{26}$$

Only slight changes are necessary to adapt these to the direct computation of Z_s from the south point of the horizon. For example, Eqs. 24, 25, 25a, and 26 would take the forms

$$\cot \tfrac{1}{2}Z_s = \sqrt{\frac{\sin(s - \phi)\sin(s - h)}{\cos s \cos(s - p)}} \tag{27}$$

$$\cos Z_s = \frac{\sin \phi \sin h - \sin \delta}{\cos \phi \cos h} \tag{28}$$

$$\cos Z_s = \tan \phi \tan h - \frac{\sin \delta}{\cos \phi \cos h} \tag{28a}$$

$$\text{vers } Z_s = \frac{\cos(\phi + h) + \sin \delta}{\cos \phi \cos h} \tag{29}$$

While any of these formulae may be used to determine the angle sought, the choice of formulae should depend somewhat upon the precision with which the angle is defined by the function. If the angle is quite small it is more accurately found through its sine than through its cosine; for an angle near 90° the reverse is true. The tangent, however, on account of its rapid variation, always gives the angle more precisely than either the sine or the cosine. It will be observed that some of the formulae require the use of both logarithmic and natural functions. This causes no particular inconvenience in ordinary five-place computations because engineer's field and office tables almost invariably contain both logarithmic and natural functions. If seven-place logarithmic tables are being used the other formulae will be preferred. All the formulae are adapted to the use of natural functions in conjunction with a calculating machine.

The altitude of an object may be found from the formulae

$$\sin h = \cos(\phi - \delta) - 2 \cos \phi \cos \delta \sin^2 \tfrac{1}{2}t \tag{30}$$

or

$$\sin h = \cos(\phi - \delta) - \cos \phi \cos \delta \operatorname{vers} t \tag{30a}$$

which may be derived from Eq. 8.

If the declination, hour angle, and altitude are given, the azimuth is found by

$$\sin Z = \sin t \cos \delta \sec h \tag{31}$$

For computing the azimuth of a star near the pole when the hour angle is known the following formula is frequently used:

$$\tan Z = \frac{\sin t}{\cos \phi \tan \delta - \sin \phi \cos t} \tag{32}$$

This equation may be derived by dividing Eq. 12 by Eq. 11 and then dividing by $\cos \delta$.

Body on the Horizon. Given the latitude and declination to find the hour angle and azimuth when the object is on the horizon. If in Eqs. 8 and 13 we put $h = 0$, we have

$$\cos t = -\tan \delta \tan \phi \tag{33}$$

and

$$\cos Z = \sin \delta \sec \phi \tag{34}$$

These formulae may be used to compute the approximate time of sunrise or sunset, and the sun's bearing at these times.

Greatest Elongation. A special case of the *PZS* triangle which is of great practical importance occurs when a star which crosses the meridian north of the zenith is at its *greatest elongation.* When in this position the azimuth of the star is a maximum, and its diurnal circle is tangent to the vertical circle through the star; the triangle is therefore right-angled at the point S (Fig. 31). The formulae for the hour angle and azimuth are

$$\cos t = \tan \phi \cot \delta \tag{35}$$

and

$$\sin Z = \sin p \sec \phi \tag{36}$$

from which the time of elongation and the bearing of the star may be found (see Art. 112).

21 Relation between Right Ascension and Hour Angle

In order to understand the relation between the right ascension and the hour angle of a point, we may think of the equator on the outer sphere as graduated into hours, minutes, and seconds of right ascension, zero being at the equinox and the numbers increasing toward the east. The equator on the inner sphere is graduated for hour angles, the zero being at the observer's meridian and the numbers increasing toward the west (see Fig. 32). As the outer

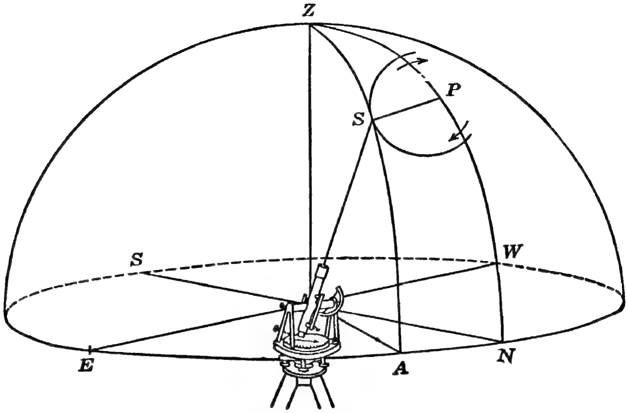


FIG. 31. Star at Greatest Elongation (East)

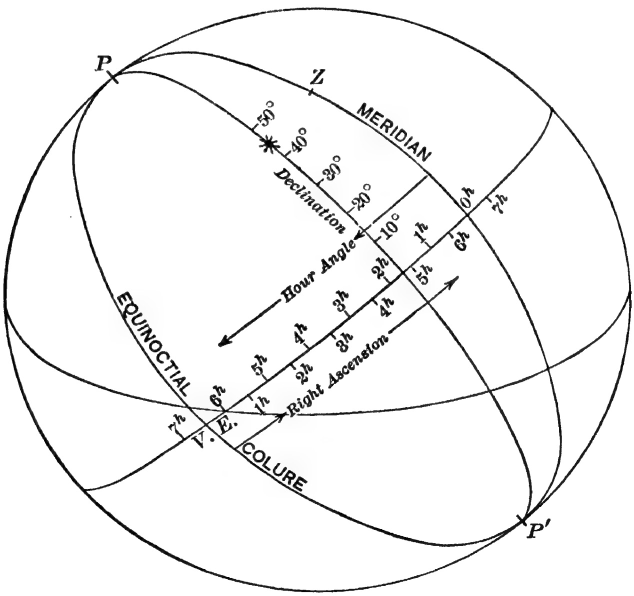


FIG. 32. Right Ascension and Hour Angle

sphere turns, the hour marks on the right ascension scale will pass the meridian in the order of the numbers. The number opposite the meridian at any instant shows how far the sphere has turned since the equinox was on the meridian. If we read the hour-angle scale opposite the

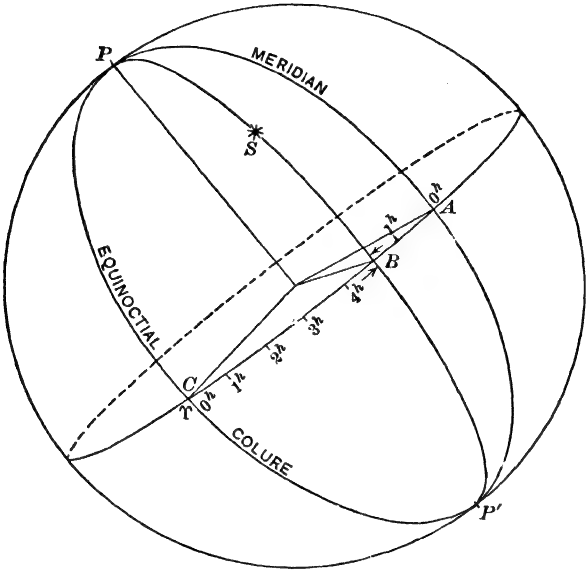


FIG. 33

equinox, we obtain exactly the same number of hours. This number of hours (or angle) may be considered as either the right ascension of the meridian or the hour angle of the equinox. In Fig. 33 the star S has an hour angle equal to AB and a right ascension CB. The sum of these two angles is AC, or the hour angle of the equinox. The same relation will be found to hold true for all positions of S. The general relation existing between these coordinates is, then,

Hour angle of equinox = *Hour angle of star*
+ *Right ascension of star*

Questions and Problems

1. What is the greatest north declination that a star may have and pass the meridian to the south of the zenith?
2. What angle does the plane of the equator make with the horizon?
3. In what latitudes can the sun be overhead?
4. What is the altitude of the sun at noon in Boston (42° 21′ N) on December 22?
5. What are the greatest and smallest angles made by the ecliptic with the horizon at Boston?
6. In what latitudes is *Vega* (decl. = 38° 42′ N) a circumpolar star?
7. Make a sketch of the celestial sphere, like Fig. 14, which corresponds to lat. 20° S and the instant when the vernal equinox is on the eastern horizon. Show position of sun on December 22.
8. Derive Eqs. 33, 34, 35, and 36.
9. Compute the hour angle of *Vega* when it is rising in lat. 40° N.
10. Compute the time of sunrise on June 22 in lat. 40° N. Compute also the bearing of the sun's center at this instant.
11. In lat. 40° 44′ 30″ N, what will be the limiting declinations of circumpolar stars?
12. The meridian altitude of the sun on March 21 was 42° 20′, the sun bearing north of the observer's zenith. Determine the approximate latitude of the observer.
13. Determine the hour angle and declination of a star, if its azimuth is 60° and its zenith distance 40° at a place in lat. 50° N.
14. In lat. 25° S, the sun was observed east of the meridian, and its altitude was found to be 45°; decl. +15°. Determine the azimuth and the hour angle.
15. Compute the hour angle of *Antares* (decl. −26° 17′) when it is setting, in lat. 30° N.
16. Compute the declination of the sun on May 8 (R.A. 3^h). Assume the angle between the planes of the ecliptic and equator as 23° 27′.

5

Measurement of Time

22 The Earth's Rotation

The measurement of intervals of time is made to depend upon the period of the earth's rotation on its axis. Although the period of rotation is not absolutely invariable, the variations are exceedingly small, and the rotation is assumed to be uniform. The most natural unit of time for ordinary purposes is the *solar day*, or the time corresponding to one rotation of the earth with respect to the sun's direction. Because of the motion of the earth around the sun once a year the direction of this reference line is continually changing with respect to the directions of fixed stars, and the length of the solar day is not the true time of one rotation of the earth. In some kinds of astronomical work it is more convenient to employ a unit of time based upon this true time of one rotation, namely, *sidereal time* (or star time).

23 Transit or Culmination

Every point on the celestial sphere crosses the plane of the meridian of an observer twice during one revolution of the sphere. The instant when any point on the celestial sphere is on the meridian of an observer is called the time of *transit*, or *culmination*, of that point over that meridian. When it is on the upper branch of the meridian, it is called

the upper transit; when it is on the lower branch it is called the lower transit. Except for stars near the elevated pole, the upper transit is the only one visible to the observer; hence when the transit of a star is mentioned the upper transit will be understood unless the contrary is stated.

24 Sidereal Day

The *sidereal day* is the interval of time between two successive *upper* transits of the vernal equinox over the same meridian. If the equinox were fixed in position the sidereal day as thus defined would be the true rotation period with reference to the fixed stars, but, since the equinox has a slow (and variable) westward motion caused by the precessional movement of the axis (see Art. 9), the actual interval between two transits of the equinox differs about $0^s.01$ of time from the true time of one rotation. The sidereal day actually used in practice, however, is the one previously defined and not the true rotation period. This causes no inconvenience because sidereal days are not used for reckoning long periods of time (dates always being given in solar days) so that this error never becomes large. The sidereal day is divided into 24 hours, and each hour is subdivided into 60 minutes and each minute into 60 seconds. When the vernal equinox is at upper transit it is 0^h, or the beginning of the sidereal day. This may be called "sidereal noon."

25 Sidereal Time

The sidereal time at a given meridian at any specified instant is equal to the hour angle of the vernal equinox measured from the upper branch of that meridian. It is therefore a measure of the angle through which the earth has rotated since the equinox was on the meridian and shows at once the position of the sphere at this instant with respect to the observer's meridian.

26 Solar Day

A solar day is the interval of time between two successive *lower* transits of the sun's center over the same meridian. The lower transit is chosen in order that the date may change at midnight. The solar day is divided into 24 hours, and each hour is divided into 60 minutes and each minute into 60 seconds. When the center of the sun is on the upper branch of the meridian (upper transit) it is *noon;* when it is on the lower branch it is *midnight.* The instant of midnight is taken as 0^h, or the beginning of the civil day.

27 Solar Time

The solar time at any instant is equal to the hour angle of the sun's center plus 180° or 12^h; in other words, it is the hour angle counted from the lower transit. It is the angle through which the earth has rotated, with respect to the sun's direction, since midnight and measures the time interval that has elapsed.

Since the earth revolves around the sun in an elliptical orbit in accordance with the law of gravitation, the apparent angular motion of the sun is not uniform, and the days are therefore of different length at different seasons. In former times, when sun dials were considered sufficiently accurate for measuring time, this lack of uniformity was unimportant. Under modern conditions, which demand accurate measurement of time by the use of clocks and chronometers, an invariable unit of time is essential. The time ordinarily employed is that kept by a fictitious point called the *mean sun,* which is imagined to move at a uniform rate along the equator,* its rate of motion being such that it makes one apparent revolution around the earth in the same time as the actual sun, that is, in one year. The fictitious sun is

* This statement is true in a general way, but the motion is not strictly uniform because the motion of the equinox itself is variable. The angle from the equinox to the " mean sun " at any instant is the sun's " mean longitude " (along the ecliptic) plus small periodic terms.

so placed that on the whole it precedes the true sun as much as it follows it. The time indicated by the position of the mean sun is called *mean solar time.* The time indicated by the position of the real sun is called *apparent solar time* and is the time shown by a sun dial, or the time obtained by direct instrumental observation of the sun's position. Mean time cannot, of course, be observed directly but must be derived by computation.

28 Equation of Time

The difference between mean time and apparent time at any instant is called the *equation of time* and depends upon how much the real sun is ahead of or behind the position of the mean sun. The amount of this difference varies from about -14^m (mean sun fast) to $+16^m$ (mean sun slow). The exact interval is given in the *American Ephemeris* and also in the *Nautical Almanac* for specified times each day.

This difference between the two kinds of time results from several causes, the chief of which are (1) the inequality of the earth's angular motion in its orbit (see Kepler's second law, Art. 5) and (2) the fact that the real sun moves in the plane of the ecliptic and the mean sun in the plane of the equator, and equal arcs on the ecliptic do not correspond to equal arcs on the equator, or equal angles at the pole.

In the winter, when the earth is nearest the sun, the rate of angular motion about the sun is greater than in the summer (see Art. 7). The sun will then appear to move eastward in the sky at a faster rate than in summer, and its daily revolution about the earth will therefore be slower. This delays the instant of apparent noon, making the solar day longer than the average, and therefore a sun dial will "lose time." About April 1 the sun is moving at its average rate and the sun dial ceases to lose time; from this date until about July 1 the sun dial gains on mean time, making

up what it lost between January 1 and April 1. During the other half of the year the process is reversed; the sun dial gains from July 1 to October 1 and loses from October 1 to January 1. The maximum difference resulting from this cause alone is about 7 minutes, the apparent sun being either ahead of or behind the mean sun by this maximum amount twice a year.

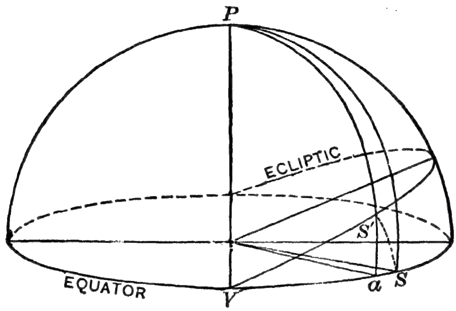


Fig. 34

The second cause of the equation of time is illustrated in Fig. 34. Assume that point S' (sometimes called the "first mean sun") moves uniformly along the ecliptic at the average rate of the actual sun; the time as indicated by this point will evidently not be affected by the eccentricity of the orbit. If the mean sun, S (also called the "second mean sun"), starts at V, the vernal equinox, at the same instant that S' starts, then the arcs VS and VS' are equal since both points are moving at the same rate. By drawing hour circles through these two points it will be seen that these hour circles do not coincide unless the points S and S' happen to be at the equinoxes or at the solstices. Since S and S' are not, in general, on the same hour circle they will not cross the meridian at the same instant, the difference in time being represented by the arc aS. The maximum length of aS is about 10 minutes of time and may be either positive or negative; that is, the

apparent sun may be ahead of or behind the mean sun by this amount. The combined effect of these two causes, or the *equation of time*, is shown in Table C and (graphically) in Fig. 35.

TABLE C

EQUATION OF TIME FOR 1947
(For instant of Greenwich 0^h)

Month	First	Tenth	Twentieth	Thirtieth
January	$-\ 3^m\ 08^s$	$-\ 7^m\ 10^s$	$-10^m\ 48^s$	$-13^m\ 14^s$
February	−13 33	−14 20	−13 58	....
March	−12 42	−10 41	− 7 55	− 4 54
April	− 4 18	− 1 40	+ 0 50	+ 2 39
May	+ 2 47	+ 3 38	+ 3 39	+ 2 45
June	+ 2 29	+ 0 58	− 1 08	− 3 16
July	− 3 28	− 5 02	− 6 09	− 6 21
August	− 6 17	− 5 26	− 3 36	− 0 56
September	− 0 20	+ 2 38	+ 6 09	+ 9 37
October	+ 9 57	+12 40	+14 57	+16 14
November	+16 20	+16 09	+14 36	+11 43
December	+11 21	+ 7 41	+ 2 55	− 2 02

29 Conversion of Mean Time into Apparent Time, and Vice Versa

Mean time may be converted into apparent time by adding algebraically the equation of time for the instant. Thus the equation of time may be regarded as a correction to mean time to obtain apparent time. The value of the equation of time as given by the following relation

$$\text{Equation of time} = \text{Apparent time} - \text{Mean time} \tag{37}$$

is tabulated in the *American Ephemeris* for 0^h civil (mean) time (midnight) at Greenwich each day, together with the proper algebraic sign. For any other time it must be found by adding or subtracting the amount by which the equation has increased or diminished since midnight. In editions of the *Ephemeris* prior to 1934 the variation per hour (the slope of the equation curve at the point for which the

value was tabulated) was given, and it was customary to make a tangent interpolation from the nearer tabular value (see Chapter 6 for interpolation methods). At present the *Ephemeris* shows the difference between tabulated values only, and a chord interpolation is made as shown in the example below. The *Nautical Almanac* shows the

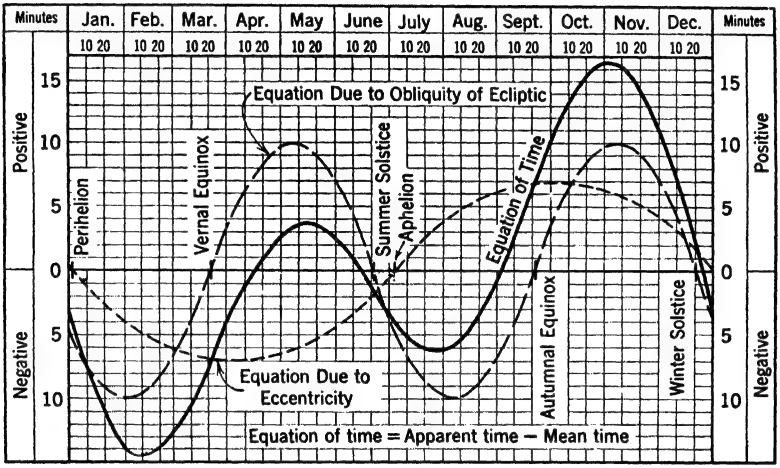


FIG. 35. Equation of Time (Correction to be added to mean time to obtain apparent time) *By permission of Prof. C. O. Roth, Jr.*

equation of time for every 2 hours of Greenwich civil time and tabulates the hourly difference which may be used for that day.

EXAMPLE. Find the apparent time at Greenwich when the mean time (civil) is $14^h\,30^m\,00^s.00$ on March 31, 1947. The *Ephemeris* shows the equation of time at 0^h G.C.T. on this date to be $-4^m\,35^s.93$. At 0^h on April 1 the Eq. T. is $-4^m\,17^s.74$. The tabular difference is $+18^s.19$. The corrected equation of time at $14^h\,30^m$ is therefore $-4^m\,35^s.93 + 18^s.19 \times 14^h.5/24^h = -4^m\,35^s.93 + 10^s.99 = -4^m\,24^s.94$. The G.A.T. is $14^h\,30^m\,00^s.00 - 4^m\,24^s.94 = 14^h\,25^m\,35^s.06$.

When converting apparent time into mean time we may proceed in either of two ways, both of which will be illustrated in the following example. Since the apparent time

is given and the equation of time is tabulated in the *Ephemeris* for definite instants of Greenwich civil (mean) time it is first necessary, in this case, to find the mean time with sufficient accuracy to enable us to take out the correct equation of time. If preferred, the ephemeris of the sun for the meridian of Washington (Part VII of the *Ephemeris*) may be used. Here the equation of time is tabulated for definite instants of apparent time (Washington apparent noon of each day), and its value at the desired instant of apparent time may be interpolated directly.

Example. The G.A.T. is $12^h\ 00^m\ 00^s.00$ on October 20, 1947. Find the G.C.T. The *Ephemeris* shows the equation of time at 0^h G.C.T. on this date to be $+14^m\ 57^s.35$. At Greenwich 0^h on October 21 the value is $+15^m\ 07^s.98$. The difference is $+10^s.63$. Since the equation of time is applied to civil time in accordance with the sign as given, in obtaining apparent time, it is obvious that when the procedure is reversed the sign of the equation of time must also be reversed. In this instance the civil time will be approximately $11^h\ 45^m$, and the correction to the equation of time will be a little less than half of the $10^s.63$ change which takes place during 24^h. This rough mental calculation gives $+15^m\ 02^s$ as the approximate equation of time. Applying this to the apparent time with sign reversed we obtain $11^h\ 44^m\ 58^s$ (or $11^h.749$) for the approximate civil time. The corrected equation of time is therefore $+14^m\ 57^s.35 + 10^s.63 \times 11.749/24 = +15^m\ 02^s.55$, and the G.C.T. is $11^h\ 44^m\ 57^s.45$.

We shall now solve the same problem by using the ephemeris of the sun for the meridian of Washington. Here the equation of time is tabulated for Washington apparent noon and is to be applied to the apparent time in accordance with the sign as given in order to obtain civil time. Since the longitude of Washington is $5^h\ 08^m\ 15^s.78$ west of Greenwich, the Washington apparent time corresponding to Greenwich apparent noon is $6^h\ 51^m\ 44^s.22$ ($6^h.86$). (See Art. 31 for the relationship between longitude and time.) This is $5^h.14$ before Washington apparent noon on October 20, or $18^h.86$ after Washington apparent noon on October 19. From the *Ephemeris* we find the equation of time at Washington apparent noon October 19 to be $-14^m\ 54^s.08$. On October 20 the tabular value is $-15^m\ 04^s.89$. The difference for 24^h is $-10^s.81$. The corrected equation of time is therefore $-14^m\ 54^s.08 + (-10.81 \times 18.86/24) = -15^m\ 02^s.57$. The G.C.T. would therefore be $11^h\ 44^m\ 57^s.43$. This fails to check perfectly with the value previously obtained because the method of interpolation used in the two solutions is imperfect. If a more accurate interpolation formula is used a closer check can be obtained.

30 Astronomical Time — Civil Time — Universal Time

Previous to 1925 the time used in the *Ephemeris* was *astronomical time,* in which 0^h occurred at the instant of noon, the hours being counted continuously up to 24^h. In this system the date changed at noon so that in the afternoon the astronomical and civil dates agreed, but in the forenoon they differed one day. For example, 7^h P.M. of January 3 would be 7^h January 3 in astronomical time; but 3^h A.M. of May 11 would be 15^h May 10 when expressed in astronomical time.

Beginning with the issue for 1925 the time used in the *Ephemeris* is designated as *civil time,* the hours being counted from midnight to midnight. The dates therefore change at midnight, as in ordinary civil time, the only difference being that in the 24-hour system the afternoon hours are greater than 12. This 24-hour system is widely used in the military and naval establishments since it eliminates the confusion which might arise from mistakenly used A.M. and P.M. designations.

For ordinary purposes we prefer to divide the day into halves and to count from two zero points; from midnight to noon is called A.M. (*ante meridiem*) and from noon to midnight is called P.M. (*post meridiem*). When consulting the *Ephemeris* or the *Nautical Almanac* it is necessary to add 12^h to the P.M. hours before looking up corresponding quantities. The data found opposite 4^h are for 4^h A.M.; those opposite 16^h are for 4^h P.M.

Civil time for the meridian of Greenwich, reckoned from midnight and counted from 0^h to 24^h, is known as *universal time.* Substantially all tabular positions for heavenly bodies are given in ephemerides for definite instants of universal time. In many other fields of scientific endeavor, notably in the reporting of weather data, observations are taken at a stated instant of universal time.

31 Relation between Longitude and Time

The hour angle of the sun, counted from the lower meridian of any place, is the solar time at that meridian and will be apparent or mean according to which sun is being considered. The hour angle of the sun from the (lower) meridian of Greenwich is the corresponding Greenwich solar time. The difference between the two times, or hour angles, is the longitude of the place east or west of Greenwich and

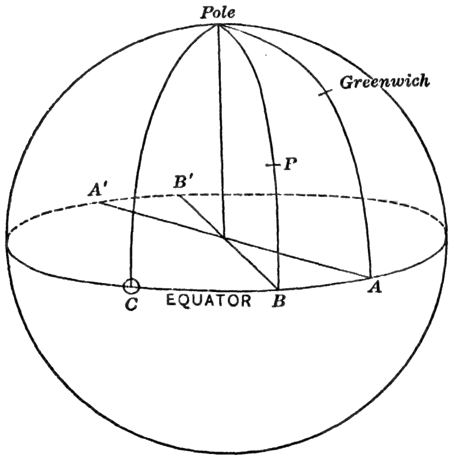


Fig. 36

is expressed either in degrees or in hours according as the hour angles are in degrees or in hours. Similarly, the difference between the local solar times of any two places at a given instant is their difference in longitude in hours, minutes, and seconds. In Fig. 36, $A'AC$ is the Greenwich solar time or the hour angle of the sun from A' and $B'BC$ is the time at P or the hour angle of the sun from B'. The difference $A'B'$, or AB, is the longitude of P west of Greenwich.

It should be observed that the reasoning is exactly the same whether C represents the true sun or the fictitious

sun. The same result would be found if C were to represent the vernal equinox. In this case the arc AC would be the hour angle of the equinox, or the Greenwich sidereal time. BC would be the local sidereal time at P, and AB would be the difference in longitude. *It should be clearly understood that the difference in longitude is equal to the difference in time provided that the same kind of local time is used at the two places.*

The truth of the preceding may be more readily seen by noticing that the difference in the two sidereal times, at meridian A and meridian B, is the interval of sidereal time during which a star would appear to travel from A to B. Since the star requires 24 sidereal hours to travel from A to A again, the time interval AB bears the same relation to 24 sidereal hours that the longitude difference bears to 360°. The difference in the mean solar times at A and B is the number of solar hours that the mean sun would require to travel from A to B; but, since the mean sun requires 24 solar hours to go from A to A again, the time interval from A to B bears the same ratio to 24 solar hours that the longitude difference bears to 360°. Thus the difference in longitude is correctly given when either time is used, provided that the same kind of time is used for both places.

To Change from Greenwich Time to Local Time or from Local Time to Greenwich Time. The method of changing from Greenwich time to local time (and the reverse) is illustrated by the following examples. When proceeding to the east the longitude is added; when going to the west it is subtracted. This will be evident when the apparent daily rotation of celestial objects is considered for the hour angles of these objects at any given instant of time will increase when passing to a more easterly meridian and will decrease in the opposite direction. When converting time to or from that at Greenwich *careful attention should be given to the dates at the two places.*

EXAMPLE 1. The G.C.T. is $19^h\,40^m\,10^s$, June 22, 1947. Determine the L.C.T. at a place in longitude $4^h\,50^m\,21^s$ W.

G.C.T.	$19^h\,40^m\,10^s$	June 22, 1947
Subtract long.	4 50 21	
L.C.T.	$14^h\,49^m\,49^s$	June 22, 1947

EXAMPLE 2. The L.C.T. at a place in long. $4^h\,56^m\,43^s$ W is $21^h\,10^m\,05^s$ at an instant on July 13, 1947. Determine the corresponding G.C.T.

L.C.T.	$21^h\,10^m\,05^s$	July 13, 1947
Add long.	4 56 43	
G.C.T.	26 06 48	
Subtract 24^h	24	
G.C.T.	$2^h\,06^m\,48^s$	July 14, 1947

EXAMPLE 3. The G.C.T. is $20^h\,40^m\,10^s$ on August 14, 1947. Determine the L.C.T. at a place in long. $6^h\,30^m\,25^s$ E.

G.C.T.	$20^h\,40^m\,10^s$	August 14, 1947
Add long.	6 30 25	
L.C.T.	27 10 35	
Subtract 24^h	24	
L.C.T.	$3^h\,10^m\,35^s$	August 15, 1947

32 Relation between Hours and Degrees

Since a circle may be divided either into 24^h or into $360°$, the relation between these two units is constant. Since

$$24^h \simeq 360°$$

we have

$$1^h \simeq 15°$$

$$1^m \simeq 15'$$

$$1^s \simeq 15''$$

Dividing the second equivalent by 15 we have

$$4^m \simeq 1°$$

also

$$4^s \simeq 1'$$

By means of these two sets of equivalents, hours may be converted into degrees and degrees into hours without writing down the intermediate steps. If it is desired to state

the process as a rule it may be done as follows: To convert an angle into time units, divide the degrees by 15, and call the result hours; multiply the remainder by 4, and call the result minutes (of time); divide the minutes (of angle) by 15, and call the result minutes (of time); multiply the remainder by 4, and call it seconds (of time); divide the seconds (of angle) by 15, and call the result seconds (of time).

EXAMPLE. Convert 47° 17′ 35″ into hours, minutes, and seconds.

$$\begin{array}{lll} 47^\circ = 45^\circ + 2^\circ & \approx & 3^h\,08^m \\ 17' = 15' + 2' & \approx & 01^m\,08^s \\ 35'' = 30'' + 5'' & \approx & 02^s.33 \\ \hline \text{Result} & & 3^h\,09^m\,10^s.33 \end{array}$$

To convert hours into degrees, reverse this process.

EXAMPLE. Convert $6^h\,35^m\,51^s$ into degrees, minutes, and seconds.

$$\begin{array}{lll} 6^h & \approx & 90^\circ \\ 35^m = 32^m + 3^m & \approx & 8^\circ\,45' \\ 51^s = 48^s + 3^s & \approx & 12'\,45'' \\ \hline \text{Result} & & 98^\circ\,57'\,45'' \end{array}$$

One should be careful to use " m " and " s " for the minutes and seconds corresponding to hours and ′ and ″ for the minutes and seconds corresponding to degrees.

It should be observed that the relation $15^\circ \approx 1^h$ is quite independent of the length of time which has elapsed. A star requires one sidereal hour to increase its hour angle 15°; the sun requires one solar hour to increase its hour angle 15°. In the sense in which the term is used here 1^h means primarily an angle, not an absolute interval of time. It becomes an absolute interval of time only when a particular kind of time is specified.

Table IX gives the conversion from arc to time. While the use of such a table will minimize errors, the careful computer will find the method outlined above quite as rapid as reference to the table.

33 Standard Time in United States

From the definition of mean solar time it will be seen that at any given instant the solar times at two places will differ by an amount equal to their difference in longitude expressed in hours, minutes, and seconds. Before 1883 it was customary in this country for each large city or town to use the mean solar time of a meridian passing through that place and for the smaller towns in that vicinity to adopt the same time. Before railroad travel became extensive this change of time from one place to another caused no great difficulty, but with the increased amount of railroad and telegraph business these frequent and irregular changes of time became so inconvenient and confusing that in 1883 a uniform system of time was adopted. The country is divided into time belts, each one theoretically 15° wide. These are known as the *Eastern, Central, Mountain,* and *Pacific* time belts. All places within these belts use the mean local time of the 75°, 90°, 105°, and 120° meridians, respectively. The time of the 60° meridian is called *Atlantic* time and is used in the easternmost part of Canada. The actual positions of the dividing lines between these time belts depend partly upon the location of the larger cities and of the division points of the railroads. The lines shown in Fig. 37 represent the present divisions between the several time zones. A proposal to place western Virginia and North Carolina and eastern Tennessee and Kentucky in the eastern zone is currently pending before the Interstate Commerce Commission. Wherever the change of time occurs the amount of the change is exactly one hour. The minutes and seconds of all standard clocks are the same as those of the Greenwich clock. When it is noon at Greenwich it is 7^h A.M. Eastern time, 6^h A.M. Central time, 5^h A.M. Mountain time, and 4^h A.M. Pacific time.

Daylight saving time for any time belt is the time of the belt one hour to the east of that in question. If, for ex-

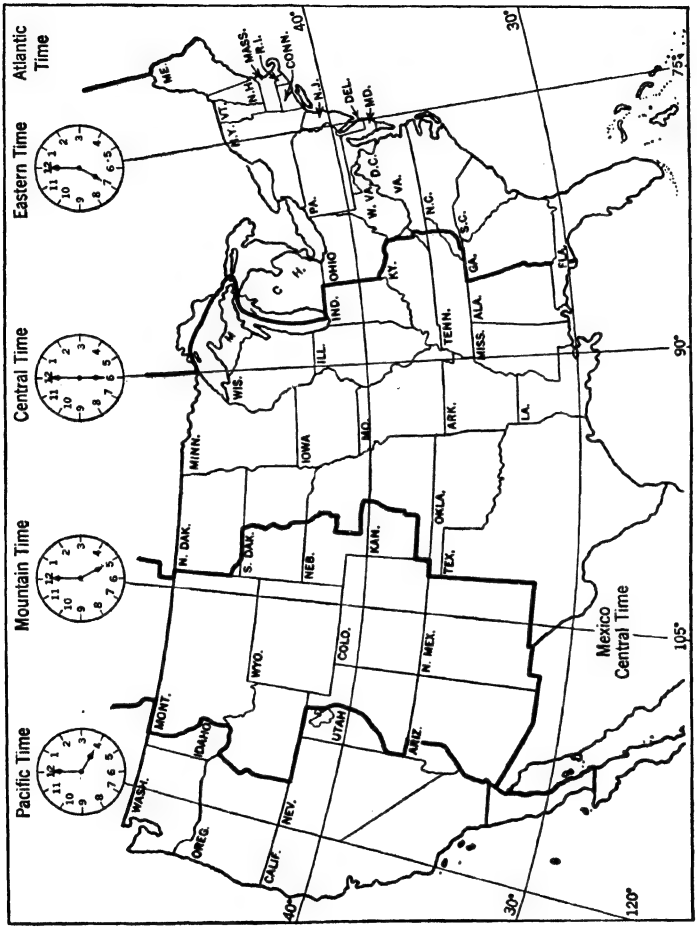


FIG. 37. Map Showing the Standard Time Belts in the United States

(The clocks show the civil time corresponding to the instant of Greenwich mean noon)

ample, in the Eastern states the clocks are set to agree with those of the Atlantic time belt (60° meridian west), this is designated as Eastern daylight saving time. It has been widely used during the spring and summer months, when the hours of daylight exceed those of darkness, to utilize daylight to a greater degree than standard time permits. Normally the clocks are set back to standard time for the fall and winter months. *War time,* in use during the recent war, was no more than daylight saving time used throughout the emergency to effect a vital saving in electric power used for illumination. Some countries advanced the clocks 2 hours ahead of standard time during the spring and summer and 1 hour ahead during the fall and winter. The United States advanced all clocks 1 hour throughout the year, effective during the war.

To Change from Local to Standard Time, or Vice Versa. The change from local to standard time, or vice versa, is made by expressing the difference in longitude between the given meridian and the standard meridian in units of time and adding or subtracting this correction, remembering that the farther west a place is the earlier it is in the day at the given instant of time.

EXAMPLE 1. Find the standard time at a place 71° west of Greenwich when the local time is $4^h\ 20^m\ 00^s$ P.M., September 15, 1947. In long. 71° the standard time would be that of the 75° meridian. The difference in longitude is $4^\circ \approx 16^m$. Since the standard meridian is west of the 71° meridian the time there is 16^m earlier than the local time. The standard time is therefore $4^h\ 04^m\ 00^s$ P.M., September 15, 1947.

EXAMPLE 2. Find the local time at a place 91° west of Greenwich when the Central standard time is $9^h\ 00^m\ 00^s$ A.M., June 12, 1947. The difference in longitude is $1^\circ \approx 4^m$. Since the place is west of the 90° meridian the local time is earlier. The local time is therefore $8^h\ 56^m\ 00^s$ A.M., June 12, 1947.

34 Zone Time

In 1920 *zone time* was adopted for use by the United States Navy. In principle it is based on standard time as used within the continental limits of the United States, but it

has been extended to include all the waters of the globe. The earth is divided into 24 time zones, normally 15° of longitude in width, the central meridians of the zones being 1^h apart. The time used in any zone is that of its central meridian, each zone time differing from Greenwich civil time by plus or minus zero to 12 whole hours, depending on the west or east longitude of the central meridian and being numbered in accordance with that difference. The zero zone extends $7\frac{1}{2}°$ on each side of the Greenwich meridian. Within it, the difference between its zone time and Greenwich civil time is zero. Zones in west longitude are numbered from +1 to +12 since a number of hours equal to the zone number must be added to each zone time to give the Greenwich civil time. In east longitude the zones are numbered from −1 to −12 since the zone number must be subtracted from zone time to give Greenwich time. The twelfth zone is divided by the 180° meridian, the western half being designated the +12 zone and the eastern half the −12 zone. The dates differ on each side of this central meridian (or rather on either side of the international date line which approximates this meridian) by one day. There are certain excepted areas which are indicated on the Hydrographic Office chart of the time zones of the world (H.O. 5192).

35 World Time Zones

Standard time zones for use on land have been adopted by the leading nations of the world. Most of these differ from Greenwich civil time by a whole number of hours in accordance with the longitude of the central meridian used. Figure 38 shows the world time zones on both ocean and land surfaces and indicates those countries and groups of islands which have irregular time belts or which use local time. Detailed information may be obtained from the Hydrographic Office chart referred to previously.

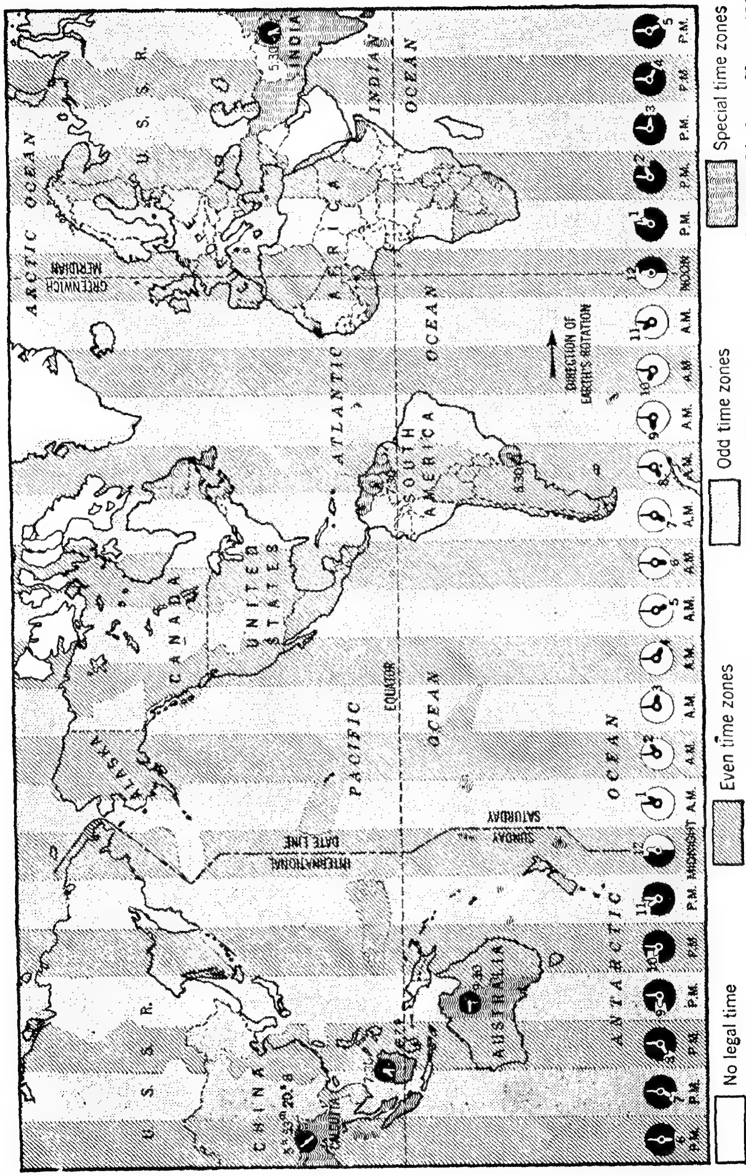


FIG. 38. Map of World Time Zones

(Data from Hydrographic Office Chart 5192)

36 International Date Line

The necessity for a meridian line, on which the date changes, is indicated in Art. 34. To put the matter another way, if a person were to start at Greenwich at the instant of noon and travel westward at the rate of about 600 miles per hour, that is, rapidly enough to keep the sun always on his own meridian, he would arrive at Greenwich 24 hours later, but his own (local) time would not have changed at all; it would have remained *noon* all the time. His date would therefore not agree with that kept at Greenwich but would be a day behind it. When traveling westward at a slower rate the same thing happens except that it takes place in a longer interval of time. The traveler has to set his watch back a little every day in order to keep it regulated to the meridian at which his noon occurs. As a consequence, after he has circumnavigated the globe, his watch has recorded one day less that it has actually run, and his calendar is one day behind that of a person who remained at Greenwich. If the traveler goes east he has to set his watch ahead every day, and, after circumnavigating the globe, his calendar is one day ahead of what it should be. In order to avoid these discrepancies in dates it has been agreed to change the date when crossing the 180° meridian from Greenwich. Whenever a ship crosses the 180° meridian, going westward, a day is omitted from the calendar; when going eastward, a day is repeated. As a matter of practice the change is made at the midnight occurring nearest the 180° meridian. For example, a steamer leaving Sydney, Australia, July 16, 1947, at noon passed the 180° meridian about 8 P.M. of July 21. At midnight, when the date was to be changed, the calendar was set back one day. The ship's log therefore showed two days dated Wednesday, July 21. She arrived in Honolulu July 25 at noon, having taken 10 days for the trip.

The international date line actually used does not follow the 180° meridian in all places but deviates so as to avoid separating the Aleutian Islands, and in the South Pacific Ocean it passes east of several groups of islands so as not to change the date formerly used in these islands.

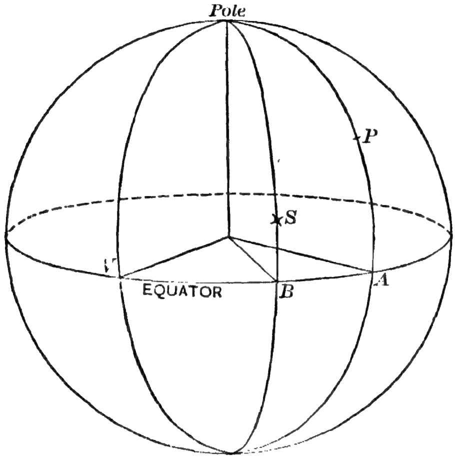


FIG. 39

37 Relation between Sidereal Time, Right Ascension, and Hour Angle of Any Point at a Given Instant

In Fig. 39 the hour angle of the equinox, or local sidereal time, at the meridian of P is the arc AV. The hour angle of the star S at the meridian of P is the arc AB. The right ascension of the star S is the arc VB. It is evident from the figure that

$$AV = VB + AB$$

or

$$S = \alpha + t \tag{38}$$

where S = the sidereal time at P,
α = the right ascension,
t = the hour angle of the star.

This relation is a general one and will be found to hold for all positions, except that it will be necessary to add 24^h to the actual sidereal time when the sum of α and t exceeds 24^h. For instance, if the hour angle is 10^h and the right ascension is 20^h, the sum is 30^h so that the actual sidereal time is 6^h. When the sidereal time and the right ascension are given and the hour angle is required we must first add 24^h (if necessary) to the sidereal time ($24^h + 6^h = 30^h$) before subtracting the 20^h right ascension, to obtain the hour angle 10^h. If, however, it is preferred to compute the hour angle in a direct manner the result is the same. When the right ascension is 20^h the angle from V westward to the point must be $24^h - 20^h = 4^h$. This 4^h added to the 6^h sidereal time gives 10^h for the hour angle as before.

38 Star on the Meridian

When a star is on any meridian the hour angle of the star at that meridian becomes 0^h. The sidereal time at the place then becomes numerically equal to the right ascension of the star. This is of great practical importance because one of the best methods of determining the time is by observing transits of stars over the plane of the meridian. The sidereal time thus becomes known at once when a star of known right ascension is on the meridian.

39 Mean Solar and Sidereal Intervals of Time

It has already been stated that on account of the earth's orbital motion the sun has an apparent eastward motion among the stars of nearly $1°$ per day. This eastward motion of the sun makes the intervals between the sun's transits greater by nearly 4^m than the interval between the transits of the equinox; that is, the solar day is nearly 4^m longer than the sidereal day. In Fig. 40, let C and C' represent the positions of the earth on two consecutive days. When the observer is at O it is noon at his meridian. After the earth makes one complete rotation (with reference to a

fixed star) the observer will be at O', and the sidereal time will be exactly the same as it was the day before when he was at O. But the sun's direction is now $C'O''$ so that the earth must turn through an additional degree (nearly) until the sun is again on this observer's meridian. This will

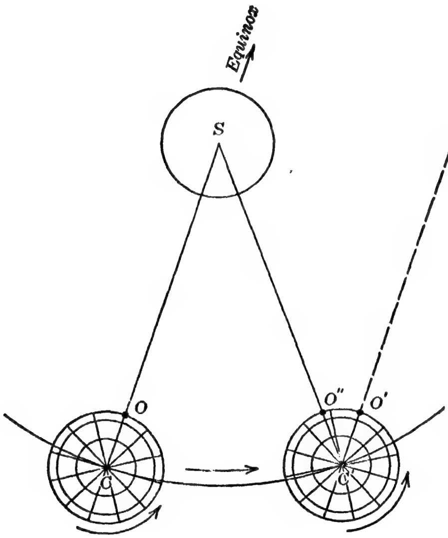


FIG. 40

require nearly 4^m additional time. Since each kind of day is subdivided into hours, minutes, and seconds, all these units in solar time will be proportionally larger than the corresponding units of sidereal time. If two clocks, one regulated to mean solar time and the other to sidereal time, were started at the same instant, both reading 0^h, the sidereal clock would immediately begin to gain on the solar clock, the gain being exactly proportional to the time elapsed, that is, about 10^s per hour, or more nearly $3^m\ 56^s$ per day.

In Fig. 40, C and C' may be taken to represent the earth's position at the date of the equinox and any subsequent date. The angle CSC' will then represent that angle through

which the earth has revolved in the interval since March 21, and the angle $SC'X$ (always equal to CSC') will represent the accumulated difference between solar and sidereal time since March 21. This angle is, of course, equal to the sun's right ascension. The angle $SC'X$ becomes 24^h or $360°$ when the angle CSC' becomes $360°$; in other words, at the end of one year, the sidereal clock has gained exactly one day.

This fact enables us to establish the exact relation between the two time units. It is known that the *tropical year* (equinox to equinox) contains 365.2422 mean solar days. Since the number of sidereal days is one greater we have

$$366.2422 \text{ sidereal days} = 365.2422 \text{ solar days}$$

or

$$1 \text{ sidereal day} = 0.99726957 \text{ solar days} \tag{39}$$

and

$$1 \text{ solar day} = 1.00273791 \text{ sidereal days} \tag{40}$$

Eqs. 39 and 40 may be written

$$24^h \text{ sidereal time} = (24^h - 3^m\ 55^s.909) \text{ mean solar time} \tag{41}$$

$$24^h \text{ mean solar time} = (24^h + 3^m\ 56^s.555) \text{ sidereal time} \tag{42}$$

These equations may be put into more convenient form for computation by expressing the difference in time as a correction to be applied to any interval of time to change it from one unit to the other. If I_m is a mean solar interval and I_s is the corresponding number of sidereal units, then

$$I_s = I_m + 0.00273791 \times I_m \tag{43}$$

and

$$I_m = I_s - 0.00273043 \times I_s \tag{44}$$

These give $+9^s.8565$ and $-9^s.8296$ as the corresponding corrections for one hour of solar and sidereal time respec-

tively. Tables II and III (pp. 302 and 303) were constructed by multiplying different values of I_m and I_s by the constants in Eqs. 43 and 44. More extended tables (II and III) will be found in the *Ephemeris*.

EXAMPLE 1. Assuming that a sidereal chronometer and a solar clock start together at a zero reading, what will be the reading of the solar clock when the sidereal chronometer reads $9^h\ 23^m\ 51^s.0$? From Table II, opposite 9^h comes the correction $-1^m\ 28^s.466$; opposite 23^m and in the fourth column comes $-3^s.768$; and opposite 51^s and in the last column, $0^s.139$. The sum of these three partial corrections is $-1^m\ 32^s.373$; $9^h\ 23^m\ 51^s.0 - 1^m\ 32^s.373 = 9^h\ 22^m\ 18^s.627$, the reading of the solar clock.

EXAMPLE 2. Reduce $7^h\ 10^m$ in solar time units to the corresponding interval in sidereal time units. In Table III the correction for 7^h is $+1^m\ 08^s.995$; for 10^m it is $+1^s.643$. The sum, $1^m\ 10^s.638$, added to $7^h\ 10^m$ gives $7^h\ 11^m\ 10^s.638$ of sidereal time.

It should be remembered that the conversion of time discussed above concerns the change of a short interval of time from one kind of unit to another and is like changing a distance from yards to meters. When changing a long interval of time, such, for example, as finding the local sidereal time on August 1 when the local solar time is 10^h A.M., we make use of the total accumulated difference between the two times since March 21, which is the same thing as the right ascension of the mean sun.

40 Approximate Corrections

Since both corrections are nearly equal to 10^s per hour, or 4^m per day, we may use these as rough approximations. For a still closer correction we may allow 10^s per hour, and then deduct 1^s for each 6^h in the interval. The correction for 6^h would then be $6 \times 10^s - 1^s = 59^s$. The error of this correction is only $0^s.023$ per hour for solar time and $0^s.004$ per hour for sidereal time.

41 Relation between Sidereal Time and Mean Solar Time at Any Instant

If in Fig. 39, Art. 37, the point B is taken to represent

the position of the mean sun, Eq. 38 becomes

$$S = \alpha_s + t_s \tag{45}$$

where α_s and t_s are the right ascension and hour angle respectively of the *mean sun* at the instant that is being considered. If the civil time is represented by T, then $t_s = T + 12^h$, and

$$S = \alpha_s + T + 12^h \tag{46}$$

which equation enables us to find the sidereal time when the civil time is given, and vice versa.

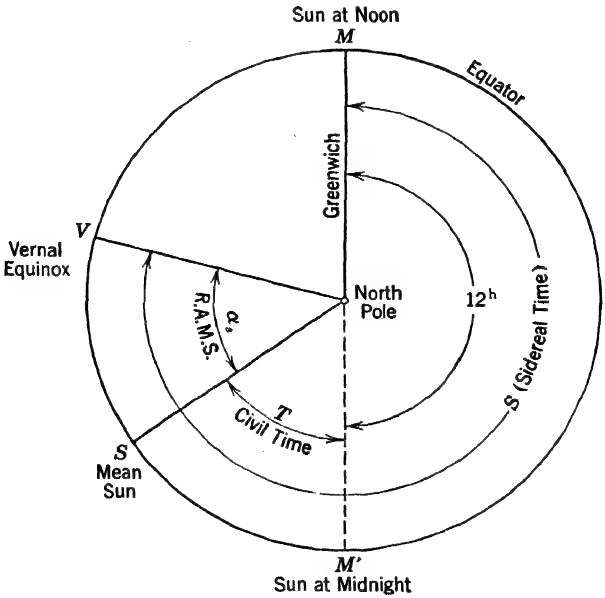


Fig. 41

This will be easily understood by referring to Fig. 41. The sidereal time, or hour angle of the equinox, is the arc $MM'V$ and is made up of three parts; $MM' = 12^h$, $M'S = T$, and $VS = \alpha_s$, the right ascension of the mean sun. This equation holds true for any meridian whatsoever.

If Eq. 46 is written

$$S - T = \alpha_s + 12^h \tag{47}$$

then, since the value of α_s does not depend upon the local time at any place but only upon the absolute instant of time considered, it is clear that the difference between the sidereal time and the civil time at any given instant is the same for all places on the earth. The actual values of S and T will be different for different meridians, but the difference, $S - T$, will be the same for all places at the given instant.

In order that Eq. 46 shall hold true it is essential that the values of α_s and T shall refer to the same position of the sun; that is, their values must correspond to the same absolute instant of time. As ordinarily expressed, this means that they must correspond to the same instant of Greenwich civil time.

The value of the "right ascension of the sun + 12^h" obtained from the *Ephemeris* is that corresponding to the instant of 0^h (midnight) of Greenwich civil time and should apply to that 0^h which immediately precedes the given time. To correct the value of the right ascension at 0^h in order to obtain the value at the given instant, it is necessary to increase this right ascension by a correction equal to the product of the hourly change in the right ascension times the number of hours in the civil time T. The hourly increase in the right ascension of the mean sun is constant and is equal to $+9^s.8565$ per solar hour. This is the same quantity that was tabulated for converting mean solar into sidereal time and is given in Table III. The difference between solar and sidereal time is caused by the very fact that the sun's right ascension increases so that the two corrections are numerically the same. It is not necessary in practice to multiply this constant by the hours of civil time, but the correction may be taken directly from Table III. Similarly Table II furnishes at once the correction

to the right ascension for any number of sidereal hours. Equation 46 will not hold true, therefore, until this correction to α_s (for 0^h) has been applied; and this correction may be regarded either as the change from solar to sidereal time (or from sidereal to solar) or as the increase in the sun's right ascension during the time T, whichever is more convenient.

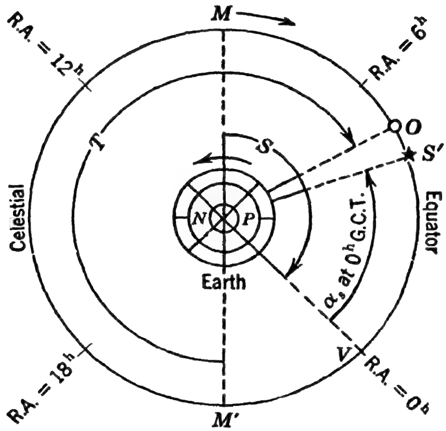


FIG. 42

Suppose that the sun O (Fig. 42) and a star S' passed the lower branch of the Greenwich meridian MM' at the same instant of time and that at civil time T it is desired to compute the corresponding sidereal time S. Since the sun is apparently moving at a slower rate than the star, it will describe the arc $M'MO$ ($= T$) while the star describes the arc $M'MS'$. The arc OS' represents the gain of sidereal time on mean time during the mean time interval T because the star is farther around to the west. But S' is the position in the sky, *relative to the vernal equinox*, where the sun was at Greenwich 0^h (midnight), and VS' is the right ascension (α_s) of the sun at 0^h. The required right ascension is VO (or the actual value of α_s at the time T)

so that the value of α_s at 0^h must be increased by the amount $S'O$, or the correction from Table III corresponding to T hours.

If α_s in the equation represents the value at the instant of 0^h Greenwich civil time the equation may be used to compute time at the meridian of Greenwich. If α_s in the equation is taken to represent the value at 0^h of civil time at any other meridian, then the equation may be used to compute the time at that meridian. The meridian M (Fig. 41) then represents the given meridian instead of the Greenwich meridian.

The right ascension of the mean sun (increased by 12^h) is given in the *Ephemeris* under the heading "sidereal time of 0^h G.C.T." or "right ascension of the mean sun + 12^h." For convenience we may write Eq. 46 in the form

$$S = (\alpha_s + 12^h) + T + \text{Table III correction} \qquad (46a)$$

where a correction from Table III is to be added to convert the interval T to sidereal units.

When changing Greenwich solar time to Greenwich sidereal time, or vice versa, we apply Eq. 46*a* directly, the value of $(\alpha_s + 12^h)$ being given in the *Ephemeris*. When making this change at any other meridian than that of Greenwich there are two general methods of procedure. First, we may find the actual Greenwich time corresponding to the given instant (by adding the west longitude or subtracting the east longitude), then make the change in time at the Greenwich meridian, and finally change back either to local time by applying the longitude with sign reversed from that in the initial step or to standard time by applying the appropriate number of hours. Second, we may correct the tabulated value of $(\alpha_s + 12^h)$ so as to obtain the value for 0^h of local civil time at the given meridian. This is done by adding to the tabular $(\alpha_s + 12^h)$ a correction equal to the hourly variation times the number of hours in the west longitude (Table III). The computation is then exactly

the same as though changing at Greenwich. Both of these methods will be illustrated by examples.

If the date changes in passing from local time to Greenwich time this will result in different values of ($\alpha_s + 12^h$) being used in the two methods. The value used should always be for the 0^h immediately preceding the given time. For example, if the local civil time is 22^h June 15 at a place 5^h west of Greenwich and we are to find sidereal time by the first method, the Greenwich civil time is 3^h June 16. We must therefore use the value of ($\alpha_s + 12^h$) for 0^h of June 16. By the second method we should use the value of ($\alpha_s + 12^h$) for June 15, correcting it for 5^h of longitude from Table III before applying Eq. 46*a*.

It should be remembered that dates (days of month) are in mean solar time regardless of whether we are using solar or sidereal time.

If the student has difficulty in understanding the process indicated by Eq. 46*a* it may be helpful to remember that all the quantities represented are really angles and may be expressed in degrees, minutes, and seconds. If all three parts, the sun's right ascension + 12^h, the hour angle of the mean sun from the lower meridian (T), and the increase in the sun's right ascension since midnight (Table III), are expressed as angles, then it is not difficult to see that the hour angle of the equinox is the sum of these three parts.

Another view of it is that the actual sidereal time interval from the transit of the equinox over the upper meridian to the transit of the "mean sun" over the lower meridian (midnight) is $\alpha_s + 12^h$; to obtain the sidereal time (since the upper transit of the equinox) we must add to this the sidereal time interval since midnight, which is the mean time interval since midnight plus the correction in Table III.

Example 1. To find the G.S.T. corresponding to the G.C.T. $8^h\ 00^m\ 00^s$ on April 10, 1947. The "right ascension of the mean sun + 12^h" at Greenwich 0^h on this date is tabulated in the *Ephemeris* as $13^h\ 09^m\ 31^s.440$. The increase in the sun's right ascension for 8^h (as given in Table III) is $01^m\ 18^s.852$. The sidereal time is then found as follows:

G.C.T.	$8^h\ 00^m\ 00^s$	April 10, 1947
R.A.M.S. + 12^h at Greenwich 0^h	13 09 31 .440	April 10, 1947
Table III	01 18 .852	
G.S.T.	$21^h\ 10^m\ 50^s.292$	

If it is desired to find the civil time T when the sidereal time S is given the equation is

$$T = S - (\alpha_s + 12^h). \tag{46b}$$

In this instance it is not possible to correct the right ascension at once for the change since 0^h because that is not yet known. It is possible, however, to find the number of sidereal hours since midnight for this results directly from the subtraction of the tabulated value of $(\alpha_s + 12^h)$ from S. This is called the *sidereal interval since midnight.* T is therefore found by subtracting from this last result the corresponding correction in Table II, that is, by a direct conversion from sidereal to solar time.

Example 2. If the G.S.T. $21^h\ 10^m\ 50^s.292$ had been given, same date as before, to find the civil time we should first subtract from S the tabulated value of R.A.M.S. + 12^h, obtaining the sidereal interval since midnight. This interval less the correction in Table II is the civil time, T.

G.S.T.	$21^h\ 10^m\ 50^s.292$	Civil date, April 10, 1947
R.A.M.S. + 12^h at 0^h	13 09 31 .440	April 10, 1947
Sid. int.	$8^h\ 01^m\ 18^s.852$	

From Table II we find

for 08^h	$01^m\ 18^s.636$
for 01^m	00 .164
for $18^s.852$	00 .052
Total correction	$01^m\ 18^s.852$

Subtracting this from the above sidereal interval we have

G.C.T. $8^h\ 00^m\ 00^s.000$ April 10, 1947

Example 3. If the time given is that for a meridian other than that of Greenwich the corresponding Greenwich time may be found at once (Art. 31) and the problem solved as before. Suppose that the L.C.T. is $20^h\ 00^m\ 00^s$ at a place in longitude $4^h\ 56^m\ 40^s$ W and the date is April 21, 1947. Find the L.S.T.

L.C.T.	$20^h\ 00^m\ 00^s$	April 21, 1947
Add long. W	4 56 40	
G.C.T.	24 56 40	After 0^h April 21
G.C.T.	$0^h\ 56^m\ 40^s$	April 22, 1947
R.A.M.S. + 12^h at Greenwich 0^h	13 56 50 .080	From *Ephemeris* for April 22, 1947
Table III for 56^m	09 .199	
for 40^s	00 .110	
G.S.T.	14 53 39 .389	
Subtract long. W	4 56 40	
L.S.T.	$9^h\ 56^m\ 59^s.389$	

Example 4. If the L.S.T. on the same civil date as in Example 3 (April 21, 1947) had been given as $9^h\ 56^m\ 59^s.389$, the computation to determine the L.C.T. would be as follows:

L.S.T.	$9^h\ 56^m\ 59^s.389$	
Add long. W	4 56 40	
G.S.T.	14 53 39 .389	
R.A.M.S. + 12^h at Greenwich 0^h	13 56 50 .080	From *Ephemeris* for April 22, 1947
Sid. int.	0 56 49 .309	
Table II	09 .309	
G.C.T.	0 56 40 .000	April 22, 1947
Add 24^h	24 56 40 .000	After 0^h, April 21, 1947
Subtract long. W	4 56 40	
L.C.T.	$20^h\ 00^m\ 00^s$	April 21, 1947

Example 5. Alternative method. The same result deduced in Example 3 may be obtained by applying to the tabulated R.A.M.S. + 12^h at Greenwich 0^h on the local date a correction to reduce it to its value at 0^h L.C.T. The time interval between 0^h at Greenwich and 0^h at the given place is equal to the difference in longitude, in this case, $4^h\ 56^m\ 40^s$. We therefore proceed as follows:

R.A.M.S. + 12^h at Greenwich 0^h	$13^h\ 52^m\ 53^s.528$	From *Ephemeris* for April 21, 1947
Table III correction for		
longitude 4^h	39.426	
56^m	9.199	
40^s	0.110	
R.A.M.S. + 12^h at local 0^h	13 53 42.263	For civil date April 21, 1947

R.A.M.S. + 12^h at local 0^h	13 53 42 .263	April 21, 1947
L.C.T.	20 00 00	
Table III (for 20^h)	03 17 .129	
L.S.T.	33 56 59 .392	
Subtract 24^h L.S.T.	$9^h\ 56^m\ 59^s.392$	

It should be noted that if the longitude had been east instead of west the Table III correction for longitude would have been subtractive.

Example 6. Alternative method. We may, in a similar manner, work through the local meridian when converting from L.S.T. to L.C.T. The computations for the data of Example 4, using this alternative method, are as follows:

L.S.T.	$9^h\ 56^m\ 59^s.389$	
Add 24^h	33 56 59 .389	
R.A.M.S. + 12^h at local 0^h	13 53 42 .263	See Example 5
Sid. int.	20 03 17 .126	
Table II	03 17 .128	
L.C.T.	$19^h\ 59^m\ 59^s.998$	
L.C.T. (very closely)	$20^h\ 00^m\ 00^s$	April 21, 1947

It should be noted that the two methods used in Examples 3 and 5, and in Examples 4 and 6, do not yield results which check precisely. The differences are in the order of 2 or 3 thousandths of a second. While such discrepancies are of no significance in the solution of engineering problems the reason for them merits explanation.

An examination of Example 5 will indicate that we have taken out a total correction from Table III for $24^h\ 56^m\ 40^s$ ($4^h\ 56^m\ 40^s$ in one place and 20^h in another). In Example 3, on the other hand, a Table III correction has been taken out for only $0^h\ 56^m\ 40^s$, the balance of the increase in the sun's right ascension being accounted for by the difference in the tabular values of $\alpha_s + 12^h$ as given in the *Ephemeris* for 0^h on April 21 and April 22. This tabular difference, representing the actual increase in the sun's right ascension, is $03^m\ 56^s.552$ while the mean 24^h increase, as given in Table III, is $03^m\ 56^s.555$. This accounts for the difference in results obtained.

If the position of the vernal equinox remained fixed and the mean sun revolved around the celestial equator at a constant speed, the right ascension of the mean sun would

increase at a constant rate of $03^m\ 56^s.555$ in each 24^h period of solar time. This forms the basis for the construction of Table III. But we have seen, in Chapter 1, that the position of the vernal equinox changes very slowly, being subject to the effects of precession and nutation. Since both the equinox and the mean sun are moving, one at a very slow, variable rate and the other at a more rapid, constant rate, it is clear that the sun's right ascension will not show a constant rate of increase. The tabular values in the *Ephemeris* indicate the actual increase from day to day while Table III shows the mean increase. The difference involved is very small, and our engineering problems can be solved with sufficient precision by either of the methods indicated.

In converting from civil to sidereal time it is shorter and more convenient to work through the meridian of Greenwich as in Example 3. Then there is never any question as to the Greenwich date and therefore no doubt as to which tabulated value of $(\alpha_s + 12^h)$ to use. In converting from sidereal time to civil time, on the other hand, there may at times be some doubt as to the Greenwich date. Under this condition, confusion will be avoided by working through the local meridian. It is obvious that in Example 4 we had sufficient knowledge of the local civil time to know that the corresponding Greenwich civil time would fall on April 22 instead of on April 21, and we accordingly used $(\alpha_s + 12^h)$ for April 22. Sometimes we do not have this knowledge. One of the most satisfactory means of determining the error of a watch is to observe the time of transit of a star across the local meridian. From Art. 38 we know that the right ascension of the star at that instant is equal to the local sidereal time. If such an observation had been made about 7^h P.M. Eastern standard time and the watch had an uncertain error we would not at once know whether the corresponding Greenwich time fell on the same date or on the next day. In this case, that is, when the observation has

been made at about Greenwich midnight, the reduction to civil time can be made best through the local meridian. Similarly, when observations are made near local midnight, less confusion will result by working through Greenwich. At other times either method may be used.

42 Greenwich Hour Angle of the Sun

In making solar observations the surveyor frequently has occasion to use the equation of time in converting from apparent time to civil time, or vice versa, by the methods of Art. 29. Similarly, when he observes the stars, he follows the procedures of Art. 41 in changing from sidereal time to civil time, or vice versa. Conversions by these methods are required where a high degree of precision in the results is necessary. In other instances, however, closely approximate results will suffice, and the surveyor can simplify his computations by adopting methods which have recently been developed in marine and air navigation practice.

The navigator of a ship or aircraft, the position of which is constantly changing, is more concerned with the rapidity with which his observations may be reduced to a determination of his position at the instant of observation than he is with a high degree of precision in the results. The introduction of tabular values of the Greenwich hour angle of the sun, moon, planets, and stars in the *Nautical Almanac*, beginning with the issue for 1934, provides a means of speeding up the navigator's calculations while maintaining the required limited precision of results. The *Air Almanac* follows a similar procedure but omits the Greenwich hour angles of the stars and includes that of the vernal equinox for reasons explained in Art. 43.

The Greenwich hour angle of a body is simply its hour angle measured westward from the meridian of Greenwich.

The local hour angle at the meridian of the observer may be obtained from the Greenwich hour angle by adding the local longitude when east of Greenwich and subtracting it when in west longitude. Thus

$$\text{L.H.A.} = \text{G.H.A.} + \lambda_e \tag{48}$$

or

$$\text{L.H.A.} = \text{G.H.A.} - \lambda_w \tag{49}$$

Since the Greenwich hour angle is tabulated in units of arc it is unnecessary to convert the longitude to time units as is required when dealing with apparent or sidereal time.

The Greenwich hour angle of the sun is tabulated in the *Nautical Almanac* for every hour of Greenwich civil time. To determine its value at any intermediate instant of Greenwich civil time we take from the *Almanac* the tabular value for the preceding instant of Greenwich civil time and correct it for the remaining minutes and seconds by use of the correction table on the yellow pages. A slight error is introduced here since the correction table is based on the constant rate of movement of the mean sun rather than upon the variable rate of the true sun. The tabular values are close enough together so that the error never becomes large and the results are sufficiently precise for many purposes.

The Greenwich hour angle is particularly useful to the surveyor in determining the standard time of local apparent noon, that is, the instant at which the sun may be observed on the meridian. Other examples of its use will be given in succeeding chapters of this book.

EXAMPLE 1. Determine the E.S.T. of L.A.N. on May 26, 1947, in long. 74° 11′.2 W by use of the G.H.A. Compare this with the solution using the equation of time.

When the sun transits at L.A.N. the L.H.A. is zero. The G.H.A. is therefore equal to the west longitude.

G.H.A. at L.A.N., May 26, 1947	74° 11′.2
At 16^h G.C.T. *Almanac* shows G.H.A.	60 47 .2
Remainder	13 24 .0
From correction table in *Almanac* for $0^h\ 53^m$	13 15 .0
Remainder	9 .0
From correction table in *Almanac* for 36^s	9 .0
Remainder	0 .0
G.C.T. of L.A.N., May 26, 1947	$16^h\ 53^m\ 36^s$
Long. W	5
E.S.T. of L.A.N., May 26, 1947	$11^h\ 53^m\ 36^s$ A.M.

The solution using the equation of time is

L.A.T., May 26, 1947	$12^h\ 00^m\ 00^s$	
Long. 74° 11′.2 W	4 56 44.8	
G.A.T.	16 56 44.8	
Eq. T.	+03 08.4	(Subtract)
G.C.T., May 26, 1947	16 53 36.4	
Long. W	5	
E.S.T., May 26, 1947	$11^h\ 53^m\ 36^s.4$ A.M.	

While the second solution may appear to be shorter it actually requires more time because of the necessity of changing the local longitude from arc to time and of interpolating for the equation of time.

EXAMPLE 2. Determine the G.H.A., L.H.A., and M.A. of the sun at $15^h\ 20^m\ 35^s$ G.C.T., May 26, 1947, in long. 74° 40′.7 W.

G.C.T.	$15^h\ 20^m\ 35^s$	May 26, 1947
G.H.A. at 14^h G.C.T.	30° 47′.3	
Correction for $1^h\ 20^m$	20 00 .0	
Correction for 35^s	08 .8	
G.H.A.	50° 56′.1	
+360°	360 00 .0	
G.H.A.	410 56 .1	
Long. W	74 40 .7	
L.H.A.	336° 15′.4	
Subtract from 360°		
M.A.	23° 44′.6 E	

43 Greenwich Hour Angle and Sidereal Hour Angle of the Stars

The position of a star may be fixed by giving its right ascension and declination. It may be fixed equally well by giving its Greenwich hour angle and declination. The latter coordinates are convenient in that they are both in

units of arc, and the need for converting from arc to time is eliminated in many problems. The *Nautical Almanac*

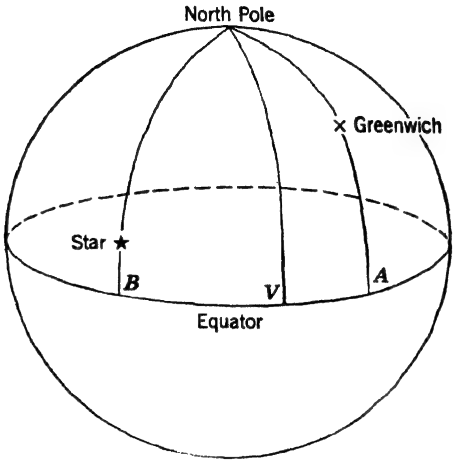


FIG. 43*a*

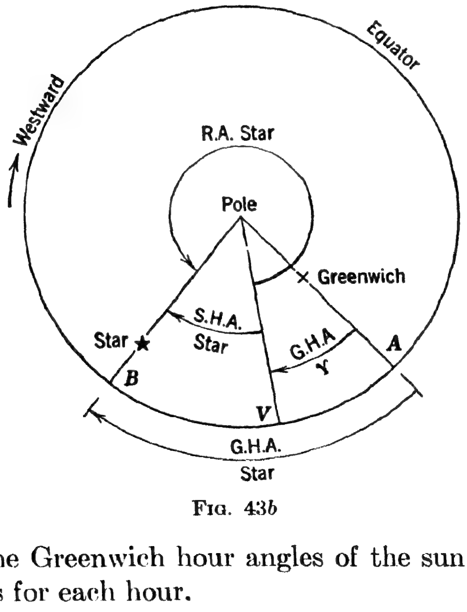


FIG. 43*b*

tabulates the Greenwich hour angles of the sun, moon, and four planets for each hour.

The air navigator obtains the Greenwich hour angles of the stars in a little different way. He must work rapidly and requires all data for a given date tabulated on a single sheet of the *Air Almanac*, with general tables on the covers of the volume. Accordingly, the daily sheet shows the Greenwich hour angle of the vernal equinox while a table on the cover shows the nearly constant sidereal hour angles of the navigational stars. Recent editions of the *Nautical Almanac* show a similar tabulation. The sum of the Greenwich hour angle of the equinox and the sidereal hour angle of a star equals the Greenwich hour angle of the star. In Fig. 43 this relationship is made evident: *AV* is the Greenwich hour angle of the vernal equinox, *VB* is the sidereal hour angle of the star, and *AB* is the Greenwich hour angle of the star.

44 The Calendar

Before the time of Julius Caesar the calendar was based upon the *lunar month*, and, as this resulted in a continual change in the dates at which the seasons occurred, the calendar was frequently changed in an arbitrary manner in order to keep the seasons in their places. This resulted in extreme confusion in the dates. In the year 45 B.C., Julius Caesar reformed the calendar and introduced one based on a year of $365\frac{1}{4}$ days, since called the Julian Calendar. The one-fourth day was provided for by making the ordinary year contain 365 days, but every fourth year, called *leap year*, was given 366 days. The extra day was added to February in such years as were divisible by 4.

Since the year actually contains 365.2422 days, or 365^d 05^h 48^m 46^s, while the Julian calendar assumed the year to be 365^d 06^h, this calendar made the year too long by 11^m 14^s. This error amounts to one day in 128 years. After many centuries the difference had accumulated to about 10 days. In order to rectify this error Pope Gregory XIII, in 1582, ordered the calendar to be corrected by dropping 10 days and future dates to be computed by

omitting leap year in those century years not divisible by 400; that is, such years as 1700, 1800, and 1900 would not be counted as leap years. The effect of this omission of three days in every four hundred years is to reduce the length of the mean calendar year to 365.2425 days or to $365^d\ 05^h\ 49^m\ 12^s$. This is in excess of the true solar year by 26^s. The error amounts to one day in 3323 years. It has been proposed to omit leap year in the year 4000 and all even multiples thereof. This would preserve the uniformity of the use of the number 4 in the intercalations and would result in an error of no more than a day in 20,000 years.

The Gregorian change was at once adopted by the Catholic nations. In England it was not adopted until 1752. The error then amounted to 11 days since the year 1700 was a leap year in accordance with the Julian calendar. The British Calendar (new style) Act of 1750 provided that the day following September 2, 1752, should be designated the 14th of that month. At the same time the beginning of the legal year was changed from the 25th of March to the 1st of January. This latter change had been made in Scotland in 1600. When consulting English or British Colonial records referring to dates previous to 1752 it is necessary to determine whether they are dated according to "old style" or "new style." The date March 5, 1740, would now be written March 16, 1741. "Double dating," such as 1740–1741, is frequently used to avoid ambiguity. George Washington was born on February 11, 1731 (old style or Julian calendar.) This day is now referred to as February 22, 1732 (new style or Gregorian calendar.)

Questions and Problems

1. If a sun dial shows the time to be 9^h A.M. on May 1, 1947, at a place in long. 71° W, what is the corresponding E.S.T.? The corrected equation of time is $+2^m\ 52^s$.

2. When it is apparent noon on October 1, 1947, at a place in long. 76° W, what is the E.S.T.? The corrected equation of time is $+10^m\ 10^s$.

3. Make a design for a horizontal sun dial for a place whose latitude is 42° 21′ N. The gnomon *ad* (Fig. 44), or line which casts the shadow on the horizontal plane, must be parallel to the earth's rotation axis; the angle which

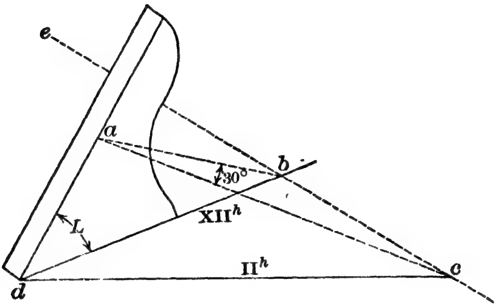


Fig. 44

the gnomon makes with the horizontal plane therefore equals the latitude. The shadow lines for the hours (X, XI, XII, I, II, etc.) are found by passing planes through the gnomon and finding where they cut the horizontal plane of the dial. The vertical plane *adb* coincides with the meridian and therefore is the noon (XII^h) line. The other planes make, with the vertical plane, angles equal to some multiple of 15°. In finding the trace *dc* of one of these planes on the dial it should be observed that the foot of the gnomon, *d*, is a point common to all such traces. In order to find another point *c* on any trace, or shadow line, pass a plane *abc* through some point *a* on the gnomon and perpendicular to it. This plane (the plane of the equator) will cut an east and west line *ce* on the dial. If a line is drawn in this plane making an angle of $n \times 15°$ with the meridian plane, it will cut *ce* at a point *c* which is on the shadow line. Joining *c* with the foot of the gnomon gives the required line.

In making a design for a sun dial it must be remembered that the west edge of the gnomon casts the shadow in the forenoon and the east edge in the afternoon; there will be, of course, two noon lines, and the two halves of the diagram will be symmetrical and separated from each other by the thickness of the gnomon. The dial may be placed in position by leveling the horizontal surface and then computing the watch time of apparent noon and turning the dial so that the shadow is on the XII^h line at the calculated time.

Prove that the horizontal angle *bdc* is given by the relation

$$\tan bdc = \tan t \sin \phi$$

where t = the sun's hour angle,
ϕ = the latitude.

4. By selecting two points at which the solar time differs by say 3^h and then converting the solar time at each place into sidereal time, prove that the difference in longitude of two points is independent of the kind of time used.

5. On January 20, 1947, the E.S.T. at a certain instant is $7^h\ 30^m$ P.M. (civil time $19^h\ 30^m$). What is the L.S.T. at this instant at a place in long. 72° 10′ W? R.A.M.S. + 12^h at 0^h G.C.T. January 20 = $7^h\ 54^m\ 07^s.1$; and R.A.M.S. + 12^h for January 21 = $7^h\ 58^m\ 03^s.7$.

6. At a place in long. 87° 30′ W the L.S.T. is found to be $19^h\ 13^m\ 10^s.5$ on September 30, 1947. What is the Central standard time at this instant? R.A.M.S. + 12^h at 0^h G.C.T. September 30 = $0^h\ 31^m\ 35^s.623$; and at 0^h October 1 = $0^h\ 35^m\ 32^s.171$.

7. If a vessel leaves San Francisco on July 16 and makes the trip in 18 days, on what date will she arrive at Vladivostok?

8. At a certain instant on July 13, 1947, the L.C.T. in Newark, N. J., long. 74° 10′ 43″ W, is $22^h\ 07^m\ 45^s$. Determine for this same instant (*a*) G.C.T.; (*b*) E.S.T.; (*c*) L.C.T. in Bombay, long. $4^h\ 51^m\ 16^s$ E; (*d*) zone time at a point in the Indian Ocean due south of Bombay; (*e*) Central daylight saving time; (*f*) Mountain war time; and (*g*) L.C.T. in San Francisco, long. $8^h\ 09^m\ 43^s$ W. Indicate the date in each of the above cases.

9. A vessel sailed from a Siberian port every Monday morning on a weekly round trip to an Alaskan port. She left Siberia Monday, February 1, in a leap year. How many Mondays were recorded in the ship's log during the month of February?

10. Design a horizontal sun dial for a place on the equator. The top of the gnomon is to be 4 inches above the horizontal base. Determine the distance from the base of the gnomon to the shadow line for each hour.

In each of the following problems the student is to refer to the *Ephemeris* or to the *Nautical Almanac* for the current year for data required in the solution.

11. Determine the E.S.T. of L.A.N. in Millburn, N. J., long. 74° 17′ 22″ W, on October 10.

12. The E.S.T. at a certain instant on April 15 is $3^h\ 52^m\ 32^s$ P.M. Determine the L.A.T. at that instant in Montreal, long. $4^h\ 54^m\ 18^s.65$ W.

13. At a certain instant on August 19 the Mountain standard time is $4^h\ 18^m\ 20^s$ A.M. Determine the L.S.T. in Denver, long. $6^h\ 59^m\ 48^s$ W.

14. At a certain instant on November 14 the L.S.T. in Maplewood, N. J., long. 74° 15′ 22″ W, is $23^h\ 14^m\ 45^s$. Determine the E.S.T. at this instant.

15. Determine the G.H.A., L.H.A., and M.A. of the sun at a place in long. 67° 32′.0 W., at the instant on October 28 when the E.S.T. is $3^h\ 10^m\ 30^s$ P.M.

6

Ephemerides—Star Catalogues—Interpolation

45 Ephemerides

In discussing the problems of the previous chapters it has been assumed that the right ascensions and declinations of the celestial objects and the various other required data not obtained by direct observation are known to the computer. These data consist of results calculated from observations made with large instruments at the astronomical observatories and published in ephemerides by the major governments of the world. The United States governmental publications in this category which tabulate data for the use of astronomers, engineers, navigators, and aviators include (1) the *American Ephemeris and Nautical Almanac,** (2) the *American Nautical Almanac,* (3) the *American Air Almanac,* and (4) *Ephemeris of the Sun and Polaris, and Tables of Azimuths and Altitudes of Polaris.* The first three are prepared by the United States Naval Observatory personnel; the last is published by the General Land Office, United States Department of the Interior.

* Similar publications by other governments are the *Nautical Almanac* (Great Britain), *Berliner Astronomisches Jahrbuch* (Germany), *Connaissance des Temps* (France), *Almanaque Nautico* (Spain), and *Apparent Places of Fundamental Stars* (published by the British Admiralty with international cooperation).

All may be secured from the Superintendent of Documents. In addition, many instrument manufacturers put out pocket-sized ephemerides containing data required for observations on the sun and *Polaris*. It should be noted that the quantities given in these ephemerides vary with the time and are therefore computed for equidistant intervals of solar time at some assumed meridian, usually that of the Greenwich (England) Observatory. To keep the volumes of convenient size they are issued annually and, in the case of the *Air Almanac*, thrice yearly. The major publications are described in the subsequent paragraphs.

46 The American Ephemeris and Nautical Almanac

The *American Ephemeris and Nautical Almanac*, hereinafter referred to as the *Ephemeris*, is an annual publication containing data of value to astronomers and to engineers using precision instruments for astronomical work. It may be obtained, a year or two in advance, from the Superintendent of Documents, Government Printing Office, Washington. The *Ephemeris* for 1947 is divided into seven parts. Part I contains data for the sun, moon, and planets, at stated hours of Greenwich civil time, usually 0^h (midnight), or the beginning of the civil day. (Previous to 1925 such data were given for Greenwich mean noon.) Part II contains the lists of star places, the data being referred to the meridian of Greenwich, the instant being that of transit. Previous to 1941 the *Ephemeris* listed the places (right ascensions and declinations) of some 887 stars. In the issues for 1941 and following years this list has been reduced to some 213 stars. Since most of these will not be suitable for observations at a particular place, date, and hour, data for an extended series of observations may now be secured from an annual volume entitled *Apparent Places of Fundamental Stars*, produced by international cooperation and published in Great Britain, which contains the apparent places of 1535 stars. Part III of the *Ephemeris*

contains data needed for the prediction of eclipses and occultations. Part IV is an ephemeris for physical observations of the sun, moon, and certain planets. Part V contains data on the satellites. Part VI covers miscellaneous tables. Part VII is an ephemeris for the meridian of the United States Naval Observatory at Washington ($5^h\ 08^m\ 15^s.78$ west of Greenwich). Parts I, II, VI, and VII are of particular importance to the surveyor.

In Part I we find for the sun, tabulated for Greenwich 0^h on each day, the apparent right ascension, apparent declination, semidiameter, horizontal parallax, equation of time, and the sidereal time or right ascension of the mean sun plus 12^h. For the moon we find the right ascension and declination tabulated for each hour of Greenwich civil time; the phases; the semidiameter, horizontal parallax, and Greenwich time of upper and lower culmination for each half day. For the major planets the right ascension, declination, semidiameter, horizontal parallax, and time of transit are tabulated daily.

In Part II we find a table of the mean places of stars. This shows the catalogue number, the star name, its special name, if any, its magnitude, right ascension, and declination. This is followed by a table showing the apparent right ascensions and declinations of these stars for every ten days throughout the year. Data are furnished at the bottom of each page for the accurate determination of these values at any instant for use in connection with observations made with precision instruments.

The miscellaneous tables in Part VI contain data required for the reduction of many observations. The tabulations showing time of sunrise, sunset, moonrise, and moonset are often useful.* The apparent place of *Polaris* is

* The time of sunrise is defined as the instant of appearance of the upper limb of the sun above a level horizon. At this instant the apparent zenith distance of the upper limb will be 90°. Applying a mean refraction of 34′, a semidiameter correction of 16′, and disregarding the small parallax correc-

shown daily for the Greenwich civil time of upper transit. Numbered tables from I to VII inclusive are of particular importance to the surveyor, and the student should become thoroughly familiar with the information contained therein.

Part VII contains for the sun, for the instant of Washington apparent noon, the apparent place, the equation of time, the semidiameter, and the sidereal time of the semidiameter passing the meridian, also the sidereal time of 0^h Washington civil time. This is followed by a table of "moon culminations" which gives data required in determining longitude by observing meridian transits of the moon. We also find data for the major planets for the instant of their Washington transit. This section is followed by several pages describing the use of the tables, an index to the apparent places of stars, and a general index.

47 The American Nautical Almanac

The *American Nautical Almanac* is an annual publication intended primarily for navigators. Since the position of a ship at sea needs to be known with somewhat less precision than that of an astronomical station, the data pub-

---

tion, we obtain 90° 50′ for the true zenith distance of the sun's center at sunrise (see Chapter 7 for corrections to altitudes). The tables in the *Ephemeris* and *Almanac* are based on this value for the zenith distance. Sunset is similarly taken as the instant when the upper limb disappears below the horizon.

Moonrise and moonset occur with the moon's upper limb on the horizon. The tables are based on a true geocentric zenith distance of the center of 90° 34′ + semidiameter − parallax, the refraction correction being taken as 34′ as for the sun, and the magnitude of the other corrections varying with the distance of the moon from the earth.

The phenomena of evening and morning twilight (dawn) result from the reflection of sunlight from particles in the upper atmosphere. *Astronomical twilight* lasts until the faintest stars appear. It is taken to end (or begin in the morning) when the true zenith distance of the sun's center is 108°. *Civil twilight* includes that period before sunrise and after sunset during which outdoor activities can be carried on in clear weather without artificial light. In latitude 40° its duration is about 30^m. Civil twilight tables in the *Air Almanac* are based on a true zenith distance of the sun's center of 96°.

lished in the *Almanac* are shown, in general, with a lower degree of refinement than those given in the *Ephemeris.* While the precision is adequate for observations made with the engineer's transit and the arrangement of data is very convenient, certain useful tables published in the *Ephemeris* are not printed in the current *Almanac.* The surveyor will find the *Ephemeris* more generally useful in the reduction of his astronomic observations.

Beginning with the 1950 issue the format and contents of the *Almanac* were changed considerably as compared with previous editions. Many of the examples and references to the *Almanac* in this text were based on the 1947 issue. In the second printing it has been possible to make certain changes conformable to the present *Almanac* but the reader will not always find the methods of solution identical nor the references always applicable when using the current *Almanac.*

Daily pages give the Greenwich hour angle of the vernal equinox and the Greenwich hour angles and declinations of the sun, moon, and four planets, tabulated for each hour of Greenwich civil time. A section of yellow pages gives tables which are used to reduce the hourly values printed on the daily pages to the values at the instant of observation.

The daily pages also show the sidereal hour angles and declinations of the navigational stars, values of the equation of time, times of Greenwich transit for the moon and four planets, altitude corrections, and times of sunrise, sunset, moonrise, moonset, and beginning and ending of twilight.

Additional tables are available to determine the azimuth of *Polaris* and the observer's latitude from an observed altitude of *Polaris.*

48 The American Air Almanac

Development of celestial and radio navigational aids has accompanied modern development of long-range air transport. A *Lunar Ephemeris for Aviators* was published in the last quarter of 1929. The first *Air Almanac* appeared in 1933. Demand for the volume proved small, and its publication was discontinued. The *British Air Almanac* was first published in 1937, and in 1941 the Naval Observatory staff renewed publication of the *American Air Almanac,* greatly modified in form to permit the more rapid reduction of observations.

The *Air Almanac* is published in three volumes, one for each four months of the year. The binding is such that the book lies flat when opened to any page. It is so designed that the two sides of a leaf furnish all data for a single day. A flap in the back shows the navigational star chart. The principal auxiliary tables are printed on the reverse side of this chart and on the covers so that the turning of pages in the computation of observations is reduced to a minimum.

The daily sheets show the Greenwich hour angles at 10-minute intervals for the sun, vernal equinox, the three planets most suitable for observation at the time, and the moon. Declinations are shown at 10-minute intervals for the moon and hourly for the sun and planets. These sheets also show the semidiameters of the sun and moon; corrections to the hour angle of the moon; parallax corrections for the moon; data on time of sunrise, sunset, moonrise, moonset, beginning of morning twilight, and end of evening twilight; and a diagram of the region along the ecliptic showing the positions of the sun, moon, planets, vernal equinox, and bright stars in this region.

The magnitudes, sidereal hour angles, and declinations of the 55 navigational stars are shown on the inside of the back cover. Data for the brightest stars appear on the flap and on the inside of the front cover. A *Polaris* table on the flap shows, for various values of the local hour angle

of the equinox, the correction to be applied to an observed altitude of *Polaris* to obtain the latitude. Tables for interpolation of Greenwich hour angle, refraction, and dip are given on either the flap or covers.

Data on astrograph settings; correction tables for the effect of altitude on the time of sunrise, etc.; Coriolis acceleration correction tables for the bubble sextant; arc-time conversion tables; sunlight and moonlight graphs; sky diagrams; and suitable explanations of the use of the tables are included.

This almanac, designed for the air navigator, is perhaps less well adapted to the astronomical work of the surveyor than are the two volumes previously described. Nevertheless, the arrangement of the data is extremely convenient, and the volume can be utilized advantageously on many occasions.

49 Smaller Ephemerides

The General Land Office, Department of the Interior, publishes the *Ephemeris of the Sun and Polaris and Tables of Azimuths and Altitudes of Polaris.* It contains the data indicated and is particularly adapted to solar observations with the solar attachment. Many of the manufacturers of surveying instruments publish small ephemerides containing data for the reduction of solar and *Polaris* observations as well as numerous useful tables. In describing the above publications many quantities have been referred to with which the student is as yet unfamiliar. He will find need for them in practical work and will find them described in detail in later chapters of this text. For the present he should be concerned with familiarizing himself with the sources of the data which he will need. The problems at the end of this chapter have been designed with that end in view.

50 Star Catalogues

Whenever it becomes necessary to observe stars which are not included in the list given in the *Ephemeris*, their positions must be taken from one of the star catalogues. These catalogues give the mean place of each star at some epoch, such as the beginning of the year 1890 or 1900, together with the necessary data for reducing it to the mean place for any other year. The mean place of a star is that obtained by referring it to the mean equinox at the beginning of the year; that is, the position it would occupy if its place were not affected by the small periodic terms of the precession.

The year employed in such reductions is that known as the Besselian fictitious year. It begins when the sun's mean longitude (arc of the ecliptic) is 280°, that is, when the right ascension of the mean sun is $18^h\ 40^m$, which occurs about January 1. After the catalogued position of the star has been brought up to the mean place at the beginning of the given year it must still be reduced to its "apparent place," for the exact date of the observation, by employing formulae and tables given for the purpose in Part II of the *Ephemeris*.

There are many star catalogues, some containing the positions of a very large number of stars but which are determined with rather inferior accuracy; others contain a relatively small number of stars, but the places are determined with the greatest accuracy. Among the best of these latter may be mentioned the Greenwich 10-year (and other) catalogues and Boss' *Catalogue of 6188 Stars for the Epoch 1900* (Washington 1910).

The British annual publication, *Apparent Places of Fundamental Stars*, previously referred to, is also of considerable value for precision observations where the limited star list given in the present *Ephemeris* is inadequate.

51 Interpolation

When taking data from any of the ephemerides corresponding to any given instant of Greenwich civil time it will generally be necessary to interpolate between the tabulated values of the function. The usual method of interpolating, such as in tables of logarithms or of trigonometric functions, consists in assuming that the function varies uniformly between two successive values in the table, and, if applied to the *Ephemeris* or *Almanac*, consists in giving the preceding tabulated value an increase (or decrease) directly proportional to the time elapsed since the tabulated Greenwich time.

EXAMPLE. To illustrate this method of interpolating, let it be assumed that the sun's declination is required when the G.C.T. is 21^h February 5, 1947. The tabulated values for 0^h (midnight) on February 5 and February 6 are as follows:

	Sun's Declination	*Difference*
February 5, 0^h	$-16° \, 14' \, 24''.0$	
February 6, 0^h	$-15 \;\; 56 \;\; 20\;.1$	$+1083''.9$

The given time is 21^h, and the difference in declination for 24^h is $+1083''.9$. Therefore the required declination is

$$-16° \, 14' \, 24''.0 + \frac{1083.9}{24} \times 21 = -15° \, 58' \, 35''.6.$$

If the function is represented graphically it will be seen that this process places the computed point on a *chord* of the function curve. For observations made with the ordinary engineer's transit this method of interpolation is sufficiently accurate.

Since the differences in the tabulated values vary from day to day, the above method may not give a sufficiently close result for precise observations. To obtain the value of the function with a higher degree of precision we may use Bessel's interpolation formula; that is,

$$F_{(n)} = F + na' + \frac{n(n-1)b_o}{1 \cdot 2} + \frac{n(n-1)\,(n-\frac{1}{2})c'}{1 \cdot 2 \cdot 3} + \cdots \qquad (50)$$

F is the tabular value from which we start, and n is the time interval expressed as a fraction of a day. a' and c' are odd differences, and b_o (and d_o) are even differences as will be made clear in the following numerical example.

EXAMPLE. Suppose, as in the preceding example, it is desired to compute the declination of the sun at 21^h G.C.T., February 5, 1947. From the *Ephemeris* we take the following:

Date	*Declination*	*Difference*
February 3	−16° 49′ 41″.1	
		+1049″.9
February 4	−16 32 11 .2	
		+1067 .2
February 5	−16 14 24 .0	
		+1083 .9
February 6	−15 56 20 .1	
		+1100 .3
February 7	−15 37 59 .8	
		+1116 .3
February 8	−15 19 23 .5	

Taking differences and completing the table we have

Date	*Declination*	*1st Difference*	*2nd Difference*	*3rd Difference*	*4th Difference*
February 3	−16° 49′ 41″.1				
		+1049″.9			
February 4	−16 32 11 .2		+17″.3		
		+1067 .2		−0″.6	
February 5	−16 14 24 .0		+16 .7		+0″.3
		+1083 .9	[+16 .55]	−0 .3	[+0 .1]
February 6	−15 56 20 .1		+16 .4		−0 .1
		+1100 .3		−0 .4	
February 7	−15 37 59 .8		+16 .0		
		+1116 .3			
February 8	−15 19 23 .5				

F is the value of the declination opposite the date, February 5. Between February 5 and February 6 we draw horizontal lines as shown. Between these lines are found the odd differences a', c', etc. We next fill in between these lines the means of the even differences lying above and below the horizontal lines. $b_o = \frac{1}{2}[+16.7 + 16.4] = [+16.55]$ and $d_o = \frac{1}{2}[+0.3 - 0.1] =$

[+0.1]. n, the fractional part of a day, equals $\frac{21}{24}$. Making the substitutions

$$F_{(n)} = -16° \, 14' \, 24''.0 + \tfrac{21}{24}[+1083.9] + \frac{\frac{21}{24}[-\frac{3}{24}][+16.55]}{2} + \frac{\frac{21}{24}[-\frac{3}{24}][\frac{9}{24}][-0.3]}{6} + \cdots$$

$$= -16° \, 14' \, 24''.0 + 948''.4 - 0.9'' + 0''.0 = -15° \, 58' \, 36''.5$$

The foregoing method of interpolation need only be used when observations are made with precision instruments, and it is included here merely to exemplify how values may be closely interpolated when conditions warrant. It is well to keep in mind that, when it is necessary to interpolate,

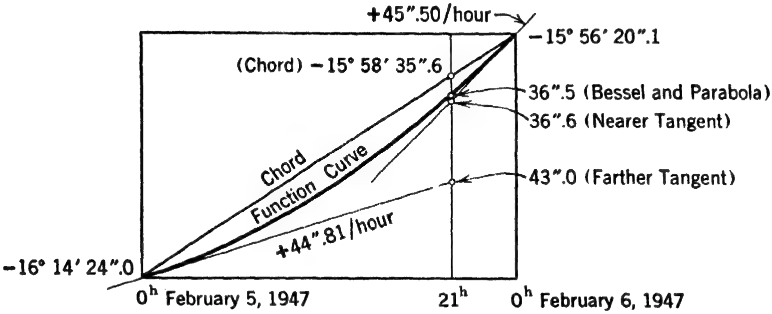


FIG. 45. Interpolation of Apparent Declination of Sun at 21^h G.C.T., February 5, 1947

the value sought should be determined with a precision comparable to the precision of the observations. The methods of interpolation explained in this chapter may be used for interpolating in tables whose arrangement is similar to those in the *Ephemeris*.

Before 1934, the "variation per hour," or differential coefficient of the function, was given in the *Ephemeris* tables opposite each value of the function, instead of the total difference between the tabulated values as given now. Certain ephemerides still follow the former practice, and it is therefore necessary to explain the method of interpolation

to be used. Where the " variation per hour " is given this value is simply multiplied by the time difference from the nearer instant for which a value is tabulated. An inspection of the diagram (Fig. 45) will show that this is also a more accurate method than where straight-line interpolation is used, provided that we always work from the nearer tabulated value; when the differential coefficient is used the computed point lies on the *tangent* line, and the curve is nearer to the tangent than to the chord for any distance that is less than half the interval between tabulated values.

EXAMPLE. To illustrate these methods of interpolating we shall apply them to the same problem which has already been worked by chord interpolation and by Bessel's formula. While the present *Ephemeris* does not give the variations per hour for February 5 and 6, we may compute these values, very closely, by taking $\frac{1}{24}$ of the means of the tabular differences given. We then have

Date	*Declination*	*Difference*	*Mean*	*Variation per Hour* ($\frac{1}{24}$ *of Mean*)
February 4	$-16° 32' 11''.2$			
		$+1067''.2$		
February 5	$-16\ 14\ 24\ .0$		$+1075''.55$	$+44''.81$
		$+1083\ .9$		
February 6	$-15\ 56\ 20\ .1$		$+1092\ .10$	$+45\ .50$
		$+1100\ .3$		
February 7	$-15\ 37\ 59\ .8$			

The given time 21^h is nearer to 0^h February 6 than it is to 0^h February 5 so that we must correct the value $-15° 56' 20''.1$ by subtracting (algebraically) a correction equal to $+45''.50$ multiplied by 3^h, giving $-15° 58' 36''.6$ for the declination. Had we worked from the value for 0^h, February 5, we would have obtained the less precise value of $-16° 14' 24''.0 + 44''.81 \times 21^h = -15° 58' 43''.0$.

These two values are shown on the tangents to the function curve in Fig. 45. The chord interpolation value is shown on the chord. The more precisely determined value from Bessel's formula is shown on the function curve. It is clear that the latter is the best value, that the value obtained from the variation per hour for the nearer date is next in precision, that the chord value comes next, while

the precision using the variation per hour for February 5 is low.

A value somewhat better than that obtained from the nearer tangent may be obtained more rapidly than Bessel's formula permits by interpolating between the given values of the differential coefficients to obtain a more precise value of the rate of change for the particular interval employed. If we imagine a parabola (Fig. 46) with its axis vertical and

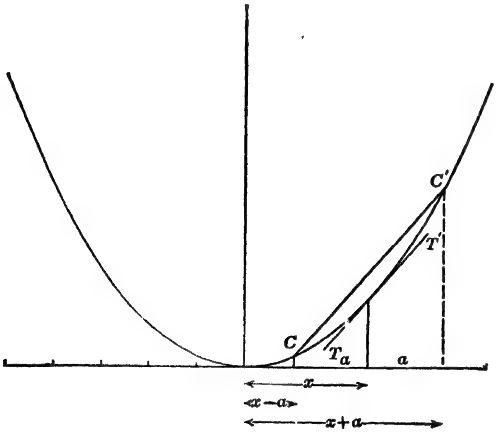


FIG. 46. Parabola $x^2 = ky$

so placed that it passes through the two given points C and C' of the function curve and has the same slope at these points, then it is obvious that this parabola must lie very close to the true function curve at all points between the tabulated values. By the following process we may find a point exactly on the *parabola* and consequently close to the true value. The second differential coefficient of the equation of the parabola is constant, and the slope (dy/dx) may therefore be found for any desired point by simple interpolation between the given values of dy/dx. If we determine the value of dy/dx for a point whose abscissa is *half way* between the tabulated time and the required

time we obtain the slope of a tangent line TT' which is also the slope of a chord of the parabola extending from the point representing the tabulated value to the point representing the desired value; for it may be proved that for this particular parabola such a chord is exactly parallel to the tangent (slope) so found. By finding the value of the variation per hour corresponding to the *middle* of the time interval over which we are interpolating and employing this in place of the given variation per hour for the tabular value, we place our point exactly on the parabola and therefore close to the true point on the function curve. In the preceding example this interpolation would be carried out in the following manner.

EXAMPLE. From 0^h February 6 back to 21^h February 5 is 3^h, and the time at the middle of the interval is $22^h\ 30^m$. Interpolating between the two values of the variation per hour which we found for the tangent interpolation (and which are tabulated in many ephemerides) we find their difference to be $+45''.50 - 44''.81 = +0''.69$, and the rate of change at $22^h\ 30^m$ is $+45''.50 - \frac{1\frac{1}{2}}{24} \times 0''.69 = +45''.46$.

The declination is therefore

$$-15°\ 56'\ 20''.1 - 45''.46 \times 3^h = -15°\ 58'\ 36''.5.$$

This checks the result obtained by Bessel's formula and therefore shows as the same point in Fig. 45.

To work this same parabolic interpolation when the total differences between the tabulated values, instead of the variations per hour, are given, we take the following data from the *Ephemeris*.

Date	*Declination*	*First Difference*	*Second Difference*
February 5	−16° 14′ 24″.0		
		+1083″.9	
February 6	−15 56 20 .1		+16″.4
		+1100 .3	
February 7	−15 37 59 .8		

Proceeding as in the previous example we find the time at the middle of the interval is $22^h\ 30^m$. In accordance with this method we may consider the differences $+1083''.9$ and $+1100''.3$ to be the rates of change at 12^h G.C.T. on February 5 and February 6 respectively. Computing the rate of change

at $22^h\,30^m$ on February 5 we find the value to be

$$+1083''.9 + \frac{10.5}{24} \times 16''.4 = +1091''.1$$

Interpolating back from the declination given at 0^h February 6 to 21^h on February 5 we find the required declination to be, as before,

$$-15°\,56'\,20''.1 - 1091''.1 \times \tfrac{3}{24} = -15°\,58'\,36''.5$$

52 Double Interpolation

When the tabulated quantity is a function of two or more variables the interpolation presents greater difficulties. If the tabular intervals are not large, and they never are in a well-planned table, the interpolation may be carried out in the following manner. Starting from the nearest tabulated value, determine the change in the function produced by each variable separately, and apply these corrections to the tabulated value.

Example. In Table G, p. 268, we find the following:

$p'\sin t$

H.A.	1945	1950
$1^h\,52^m$	28′.1	27′.4
1 56	29 .0	28 .2

Suppose that we require the value $p'\sin t$ for the year 1947 and for the hour angle $1^h\,53^m.5$. We may consider that the value 28′.1 is increased because the hour angle increases and is decreased by the change of 2 years in the date, and we may consider that the two changes are independent. The increase caused by the $1^m.5$ increase in hour angle is $\frac{1.5}{4.0}\,(0'.9) = 0'.34$. The decrease caused by the change in date is $\frac{2}{5}\,(0'.7) = 0'.28$. The corrected value is $28'.1 + 0'.34 - 0'.28 = 28'.16$.

In a similar manner the tabulated quantity may be corrected for three variations.

Example. Suppose that it is desired to take from the tables of the sun's azimuth (H.O. 71) the azimuth corresponding to decl. $+11°\,30'$ and H.A. (apparent time from noon) $3^h\,02^m$, the lat. being $42°\,20'$ N. From the page for lat. 42° we find

Declination

H.A.	11°	12°
$3^h\ 10^m$	112° 39′	111° 45′
3 00	114 56	114 01

and from the page for lat. 43° we find

Declination

H.A.	11°	12°
$3^h\ 10^m$	113° 22′	112° 29′
3 00	115 41	114 48

Selecting the value for lat. 42°, H.A. $3^h\ 00'$, and decl. 11°, namely, 114° 56′, as the basic quantity from which to start, we correct for the three variations as follows:

For lat. 42°, decrease in 10^m time = 2° 17′; therefore decrease for 2^m time = 27′.4. For lat. 42°, decrease for 1° change in decl. = 55′; therefore decrease for 30′ of decl. = 27′.5. For $3^h\ 00^m$ the increase in azimuth for 1° increase in lat. = 45′; therefore the increase for 20′ of latitude = 15.0′. The corrected value is

$$114°\ 56' - 27'.4 - 27'.5 + 15'.0 = 114°\ 16'.1$$

For more general interpolation formulae the student is referred to Chauvenet's *Spherical and Practical Astronomy*, Doolittle's *Practical Astronomy*, Hayford's *Geodetic Astronomy*, and Rice's *Theory and Practice of Interpolation.*

Questions and Problems

1. Indicate the symbol commonly used to designate each of the following heavenly bodies or points of reference: (*a*) the sun, (*b*) the moon, (*c*) Venus, (*d*) the earth, (*e*) Mars, (*f*) Jupiter, (*g*) Saturn, and (*h*) the vernal equinox.

2. Determine from the *Ephemeris* for the current year the values indicated. Use chord interpolation where necessary.

a. Apparent R.A. of sun	at $0^h\ 00^m\ 00^s$	G.C.T.	February	2
b. Apparent R.A. of sun	at 14 01 54	G.C.T.	April	25
c. Apparent decl. of sun	at 08 42 30	G.C.T.	May	9
d. Sun's semidiameter	at 0	G.C.T.	July	13
e. Sun's horizontal parallax	at 0	G.C.T.	September	12

f.	Eq. T.	at $0^h\,00^m\,00^s$	G.C.T.	August	6
g.	Eq. T. at G.A.N.			October	4
h.	Sidereal time	at 0 00 00	G.C.T.	November	3
i.	R.A.M.S. $+12^h$	at 15 10 42	G.C.T.	January	18
j.	Civil time of sidereal 0^h			September	22

3. Determine values from the *Ephemeris* as indicated in Problem 2.

a.	R.A. of moon	at $10^h\,36^m\,00^s$	G.C.T.	February	18
b.	Decl. of moon	at 20 15 00	G.C.T.	March	25
c.	Date and time of new moon in October				
d.	Date and time of first quarter in June				
e.	Date and time of full moon in February				
f.	Date and time of last quarter in December				
g.	Moon's semidiameter	at $0^h\,00^m\,00^s$	G.C.T.	January	3
h.	Moon's horizontal parallax	at 18 00 00	G.C.T.	January	1
i.	Time of U.C. of moon at Greenwich			July	11
j.	Time of L.C. of moon at Greenwich			August	14

4. Determine values from the *Ephemeris* as indicated in Problem 2.

a.	Apparent R.A. of Venus	at $20^h\,10^m\,30^s$	G.C.T.	June	20
b.	Apparent decl. of Venus	at 20 10 30	G.C.T.	June	20
c.	Apparent R.A. of Mars	at 0 00 00	G.C.T.	December	15
d.	Apparent decl. of Mars	at 0 00 00	G.C.T.	December	15
e.	Civil time of transit at Greenwich for Mars			December	3
f.	Apparent R.A. of Jupiter at instant of Greenwich transit			April	1
g.	Apparent decl. of Jupiter at instant of Greenwich transit			April	1
h.	Apparent R.A. of Saturn	at $18^h\,24^m\,00^s$	G.C.T.	October	20
i.	Apparent decl. of Saturn	at 18 24 00	G.C.T.	October	20
j.	Civil time of transit at Greenwich for Saturn			October	20

5. Determine values from the *Ephemeris* as indicated in Problem 2.

a.	Mean R.A. of α *Eridani* and magnitude of this star				
b.	Mean decl. of α *Tauri*				
c.	Apparent R.A. of α *Ceti*	at 0^h	G.C.T.	October	8
d.	Apparent decl. of β *Orionis*	at 20	G.C.T.	August	12
e.	L.C.T. of sunrise, lat. 40° N, long. 70° W			June	12
f.	Zone + 5 time of moonrise, lat. 18° S, long. 68° W			February	24
g.	Apparent place of *Polaris*	at 0^h	G.C.T.	January	2
h.	s.t.s.d.p. at Washington apparent noon			February	2
i.	Eq. T. at Washington apparent noon (Why is the sign reversed from that shown in the *Ephemeris* for Greenwich?)			May	17
j.	Washington civil time of U.C. of moon			July	14

6. Determine from the *Nautical Almanac* for the current year the values indicated. Use chord interpolation where necessary.

a. The correction to an observed altitude of 40° 00′ on *Aldebaran*.
b. The correction to an observed altitude of 36° 30′ on Venus on July 31.

c.	The G.H.A. of the sun	at 12^h	45^m	30^s	G.C.T.	March	18
d.	Sun's decl.	at 12	00	00	G.C.T.	July	11
e.	Eq. T.	at 0	00	00	G.C.T.	July	13
f.	Eq. T.	at 12	00	00	G.A.T.	July	13
g.	G.H.A. of vernal equinox	at 6	20	40	G.C.T.	April	25
h.	S.H.A. of *Arcturus*	at 12	00	00	G.C.T.	August	24
i.	G.H.A. of *Arcturus*	at 12	00	00	G.C.T.	August	24
j.	R.A. of *Rigel*	on				May	30

7. Determine values from the *Nautical Almanac* as indicated in Problem 6.

a. The time of Greenwich transit of *Polaris* on February 2.

b.	The decl. of *Regulus*	on				August	14
c.	The G.H.A. of the moon	at 22^h	14^m	18^s	G.C.T.	September	15
d.	The decl. of the moon	at 22	14	18	G.C.T.	September	15
e.	The R.A. of the apparent sun	at 0	00	00	G.C.T.	December	25

f. The E.S.T. of sunrise in lat. 44° 00′ N, long. 67° 45′ W on May 15
g. The magnitude of Jupiter

h.	The R.A. of the moon	at 20	10	30	G.C.T.	November	11

i. The latitude of the observer if the altitude of *Polaris* was 43° 20′ at 3^h 18^m 40^s G.C.T. on October 10 for an observer in long. 74° 15′ W.
j. The azimuth of *Polaris* in (i) above.

8. From the *Nautical Almanac* for the current year take out the values of the G.H.A. of the vernal equinox, the G.H.A. of the apparent sun, and the equation of time, all for 0^h G.C.T. February 9. Prepare a diagram similar to Fig. 43 b, page 93, showing the Greenwich meridian, the vernal equinox, and the apparent and mean suns in their correct relative positions. Making the appropriate conversions from arc to time, use the values from the *Almanac* to determine R.A.M.S. + 12^h at 0^h G.C.T. February 9. Check with the value given in the *Ephemeris*. Set up an equation expressing the relationship existing between the several terms involved in the computation. Check the equation by using the data for December 9 and comparing with the value for R.A.M.S. + 12^h given in the *Ephemeris*.

9. Determine from the *Air Almanac* for the first third of the current year the values indicated.

a.	G.H.A. of sun	at 5^h	14^m	16^s	G.C.T.	January	6
b.	Decl. of sun	at 5	14	16	G.C.T.	January	6
c.	G.H.A. of vernal equinox	at 6	20	32	G.C.T.	February	7
d.	G.S.T.	at 0	00	00	G.C.T.	January	6
e.	G.H.A. of Mars	at 6	20	32	G.C.T.	February	7
f.	Decl. of Jupiter	at 6	20	32	G.C.T.	February	7
g.	G.H.A. of Venus	at 20	30	10	G.C.T.	March	2

h. Decl. of Saturn	at 18	00	00	G.C.T.	February	22
i. G.H.A. of moon	at 22	15	30	G.C.T.	January	15
j. Decl. of moon	at 22	15	30	G.C.T.	January	15

10. Determine from the *Air Almanac* for the first third of the current year the values indicated.

a. S.H.A. of *Vega*
b. Decl. of *Regulus*
c. Magnitude of *Sirius*
d. G.H.A. of *Achernar* at 0^h G.C.T. January 2
e. On January 5, what planet is rising at sunset?
f. On January 5, what planet is setting at sunrise?
g. Is the moon above the horizon at sunset on January 15?
h. L.C.T. of sunrise at 20,000 feet altitude in latitude 40° N January 2
i. Moon's horizontal parallax January 14
j. Sun's s.d. January 14

11. Determine from the *Ephemeris* for the current year the apparent declination of the sun at 9^h 00^m 00^s G.C.T. on April 22. Use the following methods of interpolation: (*a*) chord interpolation. (*b*) and (*c*) Compute "variation per hour" at Greenwich 0^h on April 22 and April 23, and interpolate using tangent interpolation from (*b*) April 22 and (*c*) April 23. (*d*) Use parabolic interpolation. (*e*) Use Bessel's interpolation formula. On graph paper plot sufficient tabular values to determine the shape of the function curve and show all interpolated points. Arrange answers in descending order of precision.

12. From the *Ephemeris* for the current year, using Table IV, determine the azimuth of *Polaris* for lat. 41° 26 N and H.A. 9^h 34^m.

13. In Part I of the *Ephemeris* for the current year locate in the ephemerides of the planets an example of planetary regression, and prepare a diagram similar to Fig. 5*a*, p. 12.

7

The Earth's Figure—Corrections to Observed Altitudes

53 The Earth's Figure

The form of the earth's surface is approximately that of an ellipsoid of revolution whose shorter axis is the axis of revolution. The actual figure departs slightly from that of the ellipsoid, but this difference is relatively small and may be neglected in astronomical observations of the character considered in this book. Each meridian may therefore be regarded as an ellipse and the equator and the parallels of latitude as perfect circles. In fact, the earth may, without appreciable error, be regarded as a sphere in such problems as arise in navigation and in field astronomy with small instruments. The semimajor axis of the meridian ellipse, or radius of the equator on the Clarke (1866) spheroid, used as the datum for geodetic surveys in the United States, is 3963.27 statute miles, and the semiminor (polar) axis is 3949.83 miles in length. This difference of about 13 miles, or about $\frac{1}{300}$ part, would only be noticeable in precise work. The length of 1° of latitude at the equator is 68.703 miles; at the pole it is 69.407 miles. The length of 1° of the equator is 69.172 miles. The radius

of a sphere having the same volume as the ellipsoid is about 3958.9 miles.* On the Hayford (1909) spheroid the semimajor axis is 3963.34 miles, and the semiminor axis is 3949.99 miles.

In locating points on the earth's surface by means of spherical coordinates there are three kinds of latitude to be considered. The latitude as found by direct astronomical observation is dependent upon the direction of gravity as indicated by the spirit levels of the instrument; this is distinguished as the *astronomical latitude*. It is the angle which the vertical or plumb line makes with the plane of the equator. The *geodetic latitude* is that shown by the direction of the normal to the surface of the spheroid, or ellipsoid. It differs at each place from the astronomical latitude by a small amount which, on the average, is about $3''$, but occasionally it is as great as $30''$. This discrepancy is known as the "local deflection of the plumb line," or the "station error"; it is a direct measure of the departure of the actual surface from that of an ellipsoid. Evidently the geodetic latitude cannot be observed directly but must be derived by calculation. If a line is drawn from any point on the surface to the center of the earth the angle which this line makes with the plane of the equator is called the *geocentric latitude*. In Fig. 47, AD is normal to the surface of the spheroid, and the angle ABE is the geodetic latitude. The plumb line, or line of gravity, at this place would coincide closely with AB, say AB', and the angle it makes ($AB'E$) with the equator is the astronomical latitude of A. The angle ACE is the geocentric latitude. The difference between the geocentric and geodetic latitudes is the angle BAC, called the *angle of the vertical*, or the *reduction of latitude*. The geocentric lati-

* The reference sphere which serves as the basis for the determination of the nautical mile of 6080.27 feet is a sphere having a surface area equal to that of the earth. For discussion see Appendix 12, United States Coast and Geodetic Survey Report for 1881.

tude is always less than the geodetic by an amount which varies from 0° 11′ 30″ in latitude 45° to 0° at the equator and at the poles. Whenever observations are made at any point on the earth's surface it becomes necessary to reduce the measured values to the corresponding values at the earth's center before they can be combined with other data

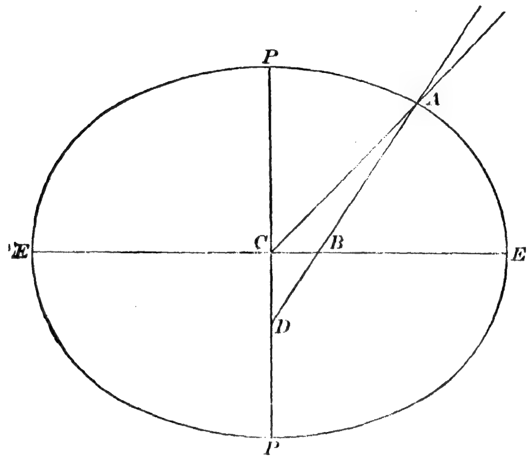


Fig. 47

referred to the center. In making this reduction the geocentric latitude must be employed if great exactness in the results is demanded. For the observations of the character treated in the following chapters it will be sufficiently accurate to regard the earth as a sphere when making such reduction.

54 The Parallax Correction

The coordinates of celestial objects as given in the *Ephemeris* are referred to the center of the earth whereas the coordinates obtained by direct observation are necessarily measured from a point on the surface and hence must be

reduced to the center. The case of most frequent occurrence in practice is that in which the altitude (or the zenith distance) of an object is observed and the geocentric altitude (or zenith distance) is desired. For all objects except the moon the distance from the earth is so great that it is sufficiently accurate to regard the earth as a sphere, and even for the moon the error involved is not large when compared with the errors of measurement with small instruments.

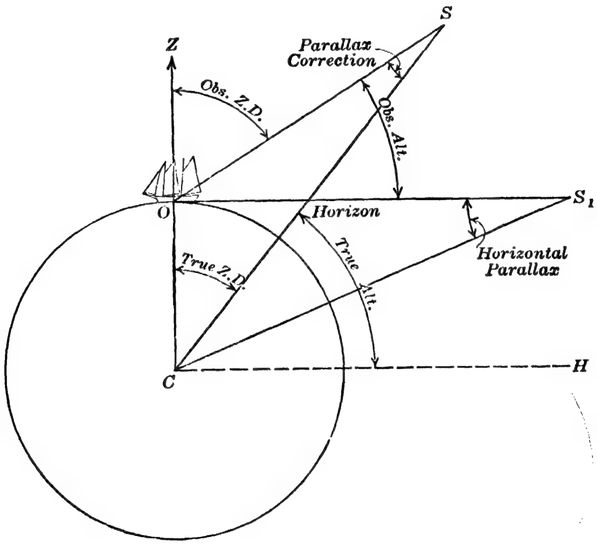


FIG. 48

In Fig. 48 the angle ZOS is the observed zenith distance, and S_1OS is the observed altitude; ZCS is the true (geocentric) zenith distance, and HCS is the true altitude. The object therefore appears to be lower in the sky when seen from O than it would if seen from C. This apparent displacement of the object on the celestial sphere is called *parallax*. The effect of parallax is to decrease the altitude of the object. If the effect of the spheroidal form of the

earth is considered it is seen that the azimuth of the body is also affected, but this small error will not be considered here. In the figure it is seen that the difference in the directions of the lines OS and CS is equal to the small angle OSC, the *parallax correction.* When the object is vertically overhead points C, O, and S are in a straight line, and the angle is zero; when S is on the horizon (at S_1) the angle OS_1C has its maximum value and is known as the *horizontal parallax.*

In the triangle OCS, the angle at O may be considered as known since either the altitude or the zenith distance has been observed. The distance OC is the semidiameter of the earth (about 3959 statute miles), and CS is the distance from the center of the earth to the center of the object and is known for bodies in the solar system. To obtain S we solve this triangle by the law of sines, obtaining

$$\sin S = \sin ZOS \times \frac{OC}{CS} \tag{51}$$

From the right triangle OS_1C we see that

$$\sin S_1 = \frac{OC}{CS_1} \tag{52}$$

The angle S_1, or *horizontal parallax,* is given in the *Ephemeris* for each object; we may therefore write

$$\sin S = \sin S_1 \sin ZOS \tag{53}$$

or

$$\sin S = \sin S_1 \cos h \tag{54}$$

At this point it is to be observed that S and S_1 are very small angles, about $9''$ for the sun and only $1°$ for the moon. We may therefore make a substitution of the angles themselves (in radians) for their sines since these are very nearly

the same.* This gives

$$S \text{ (radians)} = S_1 \text{ (radians)} \times \cos h \tag{55}$$

To convert these angles expressed in radians into angles expressed in seconds† we substitute

$$S \text{ (radians)} = S'' \times 0.000004848 \cdots$$

and

$$S_1 \text{ (radians)} = S_1'' \times 0.000004848 \cdots$$

the result being

$$S'' = S_1'' \cos h \tag{56}$$

that is,

$$\text{Parallax correction} = \text{Horizontal parallax} \times \cos h \tag{57}$$

As an example of the application of Eq. 57 let us compute

* To show the error involved in this assumption express the sine as a series.

$$\sin x = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \cdots$$

Since we have assumed that $\sin x = x$ the error is approximately equal to the next term $\frac{x^3}{6}\cdot$ For $x = 1°$ the series is

$$\sin 1° = 0.0174533 - 0.0000009 + 0.0000000.$$

The error is therefore 9 in the seventh place of decimals and corresponds to about 0″.18. For angles less than 1° the error would be much smaller than this since the term varies as the cube of the angle. If, as is frequently done, the cosine of a small angle is replaced by 1, the error is that of the small terms of the series

$$\cos x = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \cdots$$

For 1° this becomes

$$\cos 1° = 1 - 0.00001523 + \cdots$$

The error therefore corresponds to an angle of 31″.42, much larger than for the first series.

† To reduce radians to seconds we may divide by arc 1″ (= 0.000004848137) or multiply by 206264.8.

the parallax correction of the sun on May 1, 1947, when at an apparent altitude of 50°. From the *Ephemeris* the horizontal parallax is found to be 8″.73. The correction is therefore

$$8''.73 \times \cos 50° = 5''.61$$

and the altitude corrected for parallax is 50° 00′ 05″.61.

Table IV (*A*) gives approximate values of this correction for the sun.

55 The Refraction Correction

Astronomical refraction is the apparent displacement of a celestial object caused by the bending of the rays of light from the object as they pass through the atmosphere. The angular amount of this displacement is the *refraction cor-*

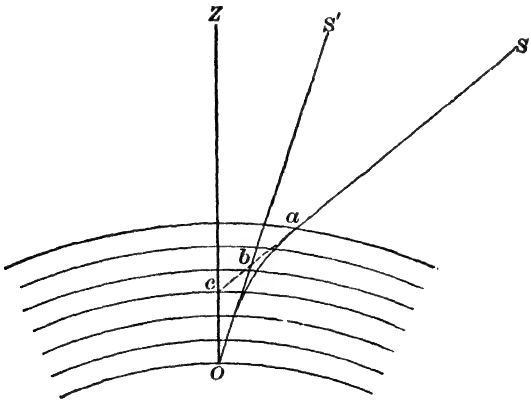


Fig. 49

rection. On account of the greater density of the atmosphere in the lower portion the ray is bent into a curve, which is convex upward and more sharply curved in the lower portion. In Fig. 49 the light from the star S is curved from a down to O, and the observer at O sees the light apparently coming from S', along the line bO. The star seems

to him to be higher in the sky than it really is. The difference between the direction of S and the direction of S' is the correction which must be applied either to the apparent zenith distance or the apparent altitude to obtain the true zenith distance or the true altitude. A complete formula for the refraction correction for any altitude, any temperature, and any pressure, is rather complicated. For observations with a small transit a simple formula will answer provided its limitations are understood. The simplest method of deriving such a formula is to consider that the refraction takes place at the upper limit of the atmosphere just as it would at the upper surface of a plate of glass. This is not the fact, but its use may be justified on the ground that the total amount of refraction is the same as though it did happen this way. In Fig. 50 light from the star S is bent at O' so that it assumes the direction $O'O$, and the observer sees the star apparently at S'. $ZO'S$ $(= \zeta')$ is the true zenith distance, $ZO'S'$ $(= \zeta)$ is the apparent zenith distance, and $SO'S'$ $(= r)$ is the refraction correction; then, from the figure,

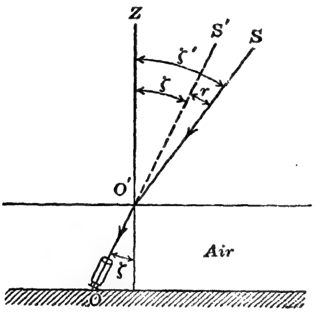


FIG. 50

$$\zeta' = \zeta + r. \tag{58}$$

Whenever a ray of light passes from a rare to a dense medium (in this case from vacuum into air) the bending takes place according to the law

$$\sin \zeta' = n \sin \zeta \tag{59}$$

where n is the index of refraction. For air, this may be taken as 1.00029. Substituting Eq. 58 in Eq. 59 we have

$$\sin (\zeta + r) = n \sin \zeta \tag{60}$$

Expanding the first member,

$$\sin \zeta \cos r + \cos \zeta \sin r = n \sin \zeta \tag{61}$$

Since r is a small angle, never greater than about 0° 34′, we may write with small error (see note, p. 122).

$$\sin r = r$$

and

$$\cos r = 1$$

whence

$$\sin \zeta + r \cos \zeta = n \sin \zeta \tag{62}$$

from which

$$r = (n - 1) \tan \zeta \tag{63}$$

r being in radians.

To reduce r to minutes we divide by arc 1′ (= 0.0002909 · · ·).

The final value of r is therefore approximately

$$r \text{ (minutes)} = \frac{0.00029}{0.00029} \tan \zeta \tag{64}$$

$$= \tan \zeta \tag{65}$$

$$= \cot h \tag{66}$$

This formula is simple and convenient but must not be regarded as showing the true law of refraction. The correction varies nearly as the tangent of ζ from the zenith down to about $\zeta = 80°$ ($h = 10°$), beyond which the formula is quite inaccurate. The extent to which the formula departs from the true refraction may be judged by a comparison with Table I, which gives the values as calculated by a more accurate formula for a temperature of 50° F. and pressure of 29.5 inches.

As an example of the use of Eq. 66 and Table I suppose that the lower edge of the sun has a (measured) altitude

of 31° 30′. By Eq. 66 the value of r is 1′.63, or 1′ 38″. The corrected altitude is therefore 31° 28′ 22″. By Table I the correction is 1′ 33″, and the corrected altitude is 31° 28′ 27″. This difference of 5″ is not very important in observations made with an engineer's transit. Table I, or any good refraction table, should be used when possible; the formula may be used when a table of tangents is available and no refraction table is at hand. For altitudes lower than 10° the formula should not be considered reliable. More accurate refraction tables may be found in any of the textbooks on astronomy to which reference has been made (p. 113). Table VIII, p. 309, gives the refraction and parallax corrections for the sun.

As an aid in remembering the approximate amount of the refraction it may be noted that at the zenith the refraction is 0; at 45° it is 1′; and at the horizon it is about 0° 34′, or a little larger than the sun's angular diameter. Because the horizontal refraction is 34′ while the sun's diameter is 32′, the entire disc of the sun is still visible (apparently above the horizon) after it has actually set.

56 Semidiameters

The discs of the sun and the moon are sensibly circular, and their *angular semidiameters* are given for each day in the *Ephemeris*. Since a measurement may be taken more accurately to the edge, or limb, of the disc than to the center, the altitude of the center is usually obtained by measuring the altitude of the upper or lower edge and applying a correction equal to the angular semidiameter. The angular semidiameter as seen by the observer may differ from the tabulated value for two reasons. When the object is above the horizon it is nearer to the observer than it is to the center of the earth, and the angular semidiameter is therefore larger than that stated in the *Ephemeris*. When the object is in the zenith it is about 4000 miles nearer the observer than when it is in the horizon. The moon is about

240,000 miles distant from the earth so that its apparent semidiameter is increased by about one sixtieth part or about 16″.

Refraction is greater for a lower altitude than for a higher altitude; the lower edge of the sun (or the moon) is always apparently lifted more than the upper edge. This causes an apparent contraction of the vertical diameter. This is most noticeable when the sun or the moon is on the horizon, at which time it appears elliptical in form. This contraction of the vertical diameter has no effect on an observed altitude, however, because the refraction correction applied is that corresponding to the altitude of the edge observed; but the contraction must be allowed for when the angular distance is measured (with the sextant) between the moon's limb and the sun, a star, or a planet. The approximate angular semidiameter of the sun on the first day of each month is given in Table IV (B).

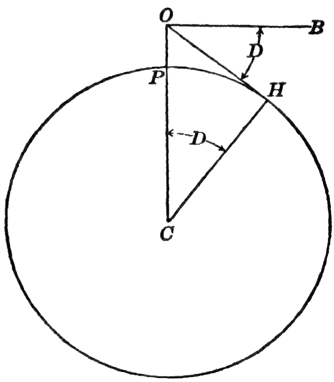


FIG. 51.

57 Dip of the Sea Horizon

If altitudes are measured above the sea horizon, as when observing on board ship with a sextant, the measured altitude must be diminished by the angular *dip* of the sea horizon below the true horizon. In Fig. 51 suppose the observer to be at O; the true horizon is OB, and the sea horizon is OH. Let $OP = h$, the height of the observer's eye above the water surface, expressed in feet; $PC = R$, the radius of the earth, regarded as a sphere; and D, the angle of dip. Then from the triangle OCH

$$\cos D = \frac{R}{R + h} \tag{67}$$

Replacing cos D by its series $1 - \frac{D^2}{2} + \cdots -$ and neglecting terms in powers higher than the second, we have

$$\frac{D^2}{2} = \frac{h}{R + h}$$

Since h is small compared with R this may be written

$$\frac{D^2}{2} = \frac{h}{R}$$

or

$$D \text{ (radians)} = \sqrt{\frac{2h}{R}}$$

Replacing R by its value in feet (20,884,000) and dividing by arc 1′ (= 0.0002909) to reduce D to minutes gives

$$D \text{ (minutes)} = \frac{1}{\sqrt{\frac{R}{2}} \times \text{arc } 1'} \times \sqrt{h}$$

$$= 1.064\sqrt{h} \tag{68}$$

This shows the amount of the dip with no allowance for refraction. But the horizon itself is apparently lifted by refraction, and the dip which affects an observed altitude is therefore less than that given by Eq. 68. If the coefficient 1.064 is arbitrarily taken as unity the formula is much nearer the truth and is very simple, although the dip is still somewhat too large. It then becomes

$$D \text{ (minutes)} = \sqrt{h} \text{ feet} \tag{69}$$

that is, the dip in minutes approximately equals the square root of the height in feet. A more precise relation for the dip correction is given by Eq. 70

$$D'' = 58''.8\sqrt{h} \text{ feet} \tag{70}$$

TABLE D

CORRECTIONS TO OBSERVED ALTITUDES

Correction	*When Made*	*Sign of Correction*	*Remarks*
Instrumental (Index correction)	When instrument does not read zero when telescope or sights show horizontal	Positive or negative	See Chapter 8. Correction negative when measured angle is greater than arc passed over
Dip	Observations at sea with sextant (ship or airplane)	Always negative	Depends upon height of eye above sea (see Table IV C)
Refraction	All observations	Always negative	For accurate observations temperature and barometric pressure must be noted (see Table I)
Semidiameter	Observations on sun, moon, and some planets when only one limb (upper or lower) has been observed	Positive or negative	Either limb of sun may usually be observed. One limb of moon. Correction is positive if lower limb is observed. Table IV(B) for sun. *Ephemeris* or *Almanac* for other bodies
Parallax	Observations on sun, moon, and planets	Always positive	Correction very large for lunar observations (see Table IV(A) for parallax of sun and Table VIII for combined refraction and parallax of sun; see *Ephemeris* or *Almanac* for other bodies)

Table IV (C), based upon this more accurate formula, will be seen to give smaller values than Eq. 69. When sextant observations are made from an airplane, using the sea horizon, this correction may be quite large (see table on back cover of *Air Almanac*).

58 Sequence of Corrections

Strictly speaking, the corrections to the observed altitude must be made in the following order: (1) instrumental corrections, (2) dip (if made at sea), (3) refraction, (4) semidiameter, and (5) parallax. In practice, however, it is seldom necessary to follow this order exactly. The parallax correction for the sun will not be appreciably different for the altitude of the center than it will for the altitude of the upper or lower edge; if the altitude is low, however, it is important to employ the refraction correction corresponding to the edge observed because this may be sensibly different from that for the center. In navigation it is customary to combine all the corrections, except the first, into a single correction given in a table whose arguments are the "height of eye," and "observed altitude." (See Bowditch, *American Practical Navigator*, Table 46.) Table D shows a summary of corrections to observed altitudes.

Problems

1. Compute the sun's mean horizontal parallax. The sun's mean distance is 92,900,000 miles; for the earth's radius see Art. 53. Compute the sun's parallax at an altitude of 60°.

2. Compute the moon's mean equatorial horizontal parallax. The moon's mean distance is 238,800 miles; for the earth's radius use 3963.27 miles. Compute the moon's parallax at an altitude of 45°.

3. If the altitude of the sun's center is 21° 10′, what is the parallax correction? the corrected attitude?

4. If the observed altitude of a star is 15° 30′, what is the refraction correction? the corrected altitude?

5. If the observed altitude of the lower edge of the sun is 27° 41′ on May 1, what is the true central altitude, corrected for refraction, parallax, and semidiameter?

6. The altitude of the sun's lower limb is observed at sea, December 1,

and is found to be $18° 24' 20''$. I.C. of the sextant $+1' 20''$; height of eye 30 feet. Compute the true altitude of the center.

7. Observed altitude of the sun's upper limb on July 11, as measured with an engineer's transit, $29° 38' 30''$; temperature 95° F; I.C. $-1' 00''$. Determine the true central altitude.

8. Plot to scale a diagram showing the apparent shape of the sun's disc at the instant on October 1 when the apparent altitude of the sun's lower limb is $0° 00'$.

9. The apparent altitude of the sun's lower limb, with reference to the visible sea horizon, was taken from an airplane at an altitude of 4000 feet on July 1 and was found to be $55° 38' 20''$; I.C. of the sextant $+40''$. Determine the true central altitude, making corrections in the proper order.

10. Observed altitude of the moon's upper limb, as measured with a transit, $47° 38'$ at 8^h P.M. E.S.T. on January 20, I.C. $-1'$. Using data from the *Nautical Almanac* for the current year determine the true central altitude.

8

Description of Instruments

59 The Engineer's Transit

The engineer's transit is an instrument for measuring horizontal and vertical angles (see Fig. 56, p. 145). For the purpose of discussing the theory of the instrument it may be regarded as a telescopic line of sight having motion about two axes at right angles to each other, one vertical, the other horizontal. The line of sight is determined by the optical center of the object glass and the intersection of two cross hairs* placed in its principal focus. The vertical axis of the instrument coincides with the axes of two spindles, one inside the other, each of which is attached to a horizontal circular plate. The lower plate carries a graduated circle for measuring horizontal angles; the upper plate has two verniers, on opposite sides, for reading angles on the circle. On the top of the upper plate are two uprights, or standards, supporting the horizontal axis to which the telescope is attached and about which it rotates. At one end of the horizontal axis is a vertical arc, or a circle, and on the standard is a vernier, in contact with the circle, for reading the angles. The plates and the horizontal axis are provided with clamps and slow-motion screws to control the motion. On the upper plate are two spirit levels for

* Also called wires or threads; they are made of spider threads, platinum wires, lines ruled upon glass, or spun-glass threads.

leveling the instrument, or, in other words, for making the vertical axis coincide with the direction of gravity.

The whole instrument may be made to turn in a horizontal plane by a motion about the vertical axis, and the telescope may be made to move in a vertical plane by a motion about the horizontal axis. By means of a combination of these two motions, vertical and horizontal, the line of sight may be made to point in any desired direction. The motion of the line of sight in a horizontal plane is measured by the angle passed over by the index of the vernier along the graduated horizontal circle. The angular motion in a vertical plane is measured by the angle on the vertical arc indicated by the vernier attached to the standard. The direction of the horizon is defined by means of a long spirit level attached to the telescope. When the bubble is central, the line of sight should lie in the plane of the horizon.

For the instrument to be in perfect adjustment the following conditions must be met: (1) the axis of each of the spirit levels* attached to the horizontal plate must be in a plane at right angles to the vertical axis of the spindles; (2) the horizontal axis of the telescope must be at right angles to the vertical axis; (3) the line of sight should be at right angles to the horizontal axis of the telescope; (4) the axis of the telescope level should be parallel to the line of sight; and (5) the vernier of the vertical arc should read zero when the bubble is in the center of the level tube attached to the telescope.

When a transit is completely set up, the various parts bear a distinct relationship to certain points and circles of reference on the celestial sphere associated with the horizon system of coordinates (Art. 13). It will be seen that the extension of the vertical spindle (or the plumb

* The axis of a level may be defined as a line tangent to the curve of the glass tube at the point on the scale taken as the zero point, or at the center of the tube.

line) upwards will pierce the celestial sphere at the zenith. If the plane of the horizontal plate is extended until it intersects the celestial sphere, this great circle will then be identical with the horizon circle. The horizon circle may also be defined optically by setting the vernier of the vertical circle or arc to read 0° and rotating the instrument about the vertical axis, the horizon being traced by the horizontal wire. Furthermore, if the telescope is clamped at any altitude and the instrument turned about the vertical axis, the line of sight describes a cone and traces a parallel of altitude (or almucantar) on the celestial sphere. Again, if the horizontal circles are clamped in any position and the telescope is rotated about its horizontal axis, the line of sight defined by the vertical wire describes a vertical circle on the celestial sphere. Indeed, when the lower plate is so turned that the vernier reads 0° when the telescope points towards true south and the lower plate remains clamped in this position, the telescope may then be pointed at any celestial body, and the vernier readings of the horizontal and vertical circles will give at once the azimuth and altitude of the body, thus instrumentally reproducing the celestial coordinates of the body in the horizon system.

60 Elimination of Errors

It is usually more difficult to measure an altitude precisely with the ordinary transit than to measure a horizontal angle. While the precision of horizontal transit angles may be increased by means of repetitions, in measuring altitudes the precision cannot be so increased because of the construction of the instrument. The vertical arc or circle usually has only one vernier so that the eccentricity cannot be eliminated, and this vernier often does not read as closely as the horizontal vernier.

One error, which is likely to be large but which may be eliminated readily, is that known as the *index error*. The

measured altitude of an object may differ from the true reading for two reasons: first, the zero of the vernier may not coincide with the zero of the circle when the telescope bubble is in the center of its tube; second, the line of sight may not be horizontal when the bubble is in the center of the tube. The first part of this error can be corrected by simply noting the vernier reading when the bubble is central and applying this as a correction to the measured altitude. This correction, of course, must be determined in the same vertical circle wherein the altitude was measured. To eliminate the second part of the error the altitude may be measured twice, once from the point on the horizon directly beneath the object observed and again from the opposite point of the horizon. In other words, the instrument may be reversed (180°) about its vertical axis and the vertical circle read in each position while the horizontal cross hair of the telescope is sighting the object. The mean of the two readings is free from the error in the sight line. Evidently this method is practicable only with an instrument having a complete vertical circle. If the reversal is made in this manner the error due to non-adjustment of the vernier is eliminated at the same time so that it is unnecessary to make a special determination of it as described above. If the circle is graduated in one direction, it will be necessary to subtract the second reading from 180° and then take the mean between this result and the first altitude.

In the preceding description it is assumed that the plate levels remain central during the reversal of the instrument, indicating that the vertical axis is truly vertical. If this is not the case, the instrument should be releveled before the second altitude is measured, the difference in the two altitude readings in this case including all three errors. If it is not desirable to relevel, the error of inclination of the vertical axis may still be eliminated by reading the vernier of the vertical circle in each of the two positions when the

telescope bubble is central and applying these corrections separately. With an instrument provided with a vertical arc only, it is essential that the axis of the telescope bubble be made parallel to the line of sight and that the vertical axis be made truly vertical. To make the axis vertical without adjusting the levels themselves, bring both bubbles to the centers of their tubes, turn the instrument 180° in azimuth, and then bring each bubble *halfway* back to the center by means of the leveling screws. When the axis is truly vertical, each bubble should remain in the same part of its tube in all azimuths.

Since the plate bubbles of the ordinary transit are not particularly sensitive and since the close determination of altitudes is necessary in most astronomical observations, it is best to make the vertical axis more truly vertical by use of the long, sensitive bubble attached to the telescope. The instrument is leveled as carefully as possible by means of the plate bubbles. The telescope bubble is then placed over one pair of leveling screws and centered by means of the tangent screw on the standard; the telescope is then turned 180° about the vertical axis, and if the bubble moves from the center of its tube it is brought halfway back by means of the tangent screw and then centered by means of the leveling screws. This process should be repeated to test the accuracy of the leveling; the telescope is then turned at right angles to the first position and the bubble centered by means of the leveling screws only. With due care the bubble should remain centered for all azimuths. The transit is thus much more accurately leveled than is possible with the plate bubbles alone.

If the line of sight is not at right angles to the horizontal axis, or if the horizontal axis is not perpendicular to the vertical axis, the errors resulting from these two causes may be eliminated by combining two sets of measurements, one in each position of the instrument. If a horizontal angle is measured with the vertical circle on the observer's

right and if the same angle is again observed with the circle on his left, the mean of these two angles is free from both these errors because the two positions of the horizontal axis are placed symmetrically about a true horizontal line,* and the two directions of the sight line are situated symmetrically about a true perpendicular to the rotation axis of the telescope. If the horizontal axis is not perpendicular to the vertical axis the line of sight describes a plane which is inclined to the true vertical plane. In this case the sight line will not pass through the zenith, and both horizontal and vertical angles will be in error.

In instruments intended for precise work a striding level is provided, which may be set on the pivots of the horizontal axis. This enables the observer to level the axis or to measure its inclination without reference to the plate bubbles. The striding level should be used in both the direct and the reversed position and the mean of the two results used in order to eliminate the errors of adjustment of the striding level itself. If the line of sight is not perpendicular to the horizontal axis it will describe a cone whose axis is the horizontal axis of the instrument. The line of sight will, in general, not pass through the zenith, even though the horizontal axis be in perfect adjustment. The instrument must either be used in two positions, or else the cross hairs must be adjusted. Except in large transits it is not usually practicable to determine the amount of the error and allow for it.

The next few articles cover various attachments to the engineer's transit which are necessary or useful in astronomical work.

61 Reflector and Illuminating Devices

When making star observations with the transit it is necessary to make some arrangement for illuminating the

* Strictly speaking, they are placed symmetrically about a perpendicular to the vertical axis.

field of view. Some transits are provided with a special shade tube into which is fitted an annular mirror set at an angle of 45°. By means of a lantern or a flashlight, held at one side of the telescope, light is reflected down the tube. The cross hairs appear as dark lines against the bright field. The stars can be seen through the opening in the center of the mirror. If no special shade tube is provided, the flashlight may easily be held in such a position that enough light enters the telescope to illuminate the cross hairs without rendering the stars invisible and without shining in the eye of the observer.

Formerly, only geodetic and special astronomical instruments were equipped with illuminating devices, but there has been an increasing demand in recent years for transits so equipped. They are useful in tunnel work and on construction projects requiring night surveys, and the call for such instruments has been particularly urgent for military surveys. Such transits are equipped with battery-powered lamps for illuminating the cross hairs through a hollow horizontal axis and also for properly illuminating the verniers or micrometers on the horizontal and vertical circles. Contact switches permit turning the light on or off at will. They are most helpful in increasing the precision of reading angles since the light bulbs may be most advantageously placed to minimize parallax errors.

62 Prismatic Eyepiece

When altitudes greater than about 55° are to be measured, it is necessary to attach to the eyepiece a totally reflecting prism which reflects the rays at right angles to the sight line. By means of this attachment altitudes as great as 75° can be measured. In making observations on the sun it must be remembered that the prism inverts the image so that with a transit having an erecting eyepiece with the prism attached the apparent lower limb is the true

upper limb; the positions of the right and left limbs are not affected by the prism.

63 Sun Glass

In making direct observations on the sun it is necessary to cover the eyepiece with a shade of dark-colored glass to protect the eye from the sunlight while observing. The sun glass should not be placed in front of the objective. While an eyepiece provided with a glass of this type is most convenient and is relatively inexpensive, the surveyor frequently has occasion to use a transit which is not so equipped for solar observations and must resort to the use of a card as described in the next article.

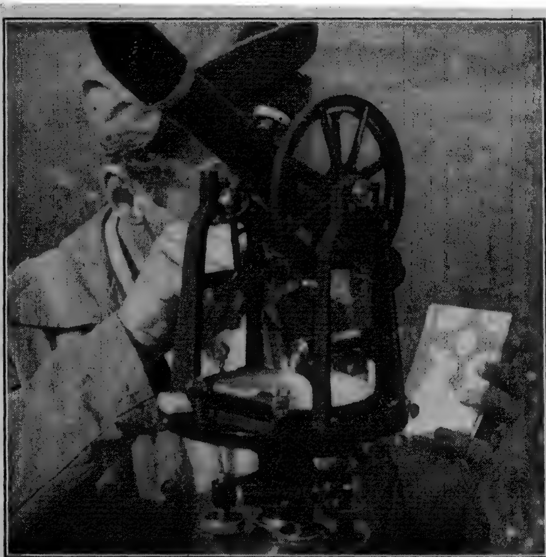
64 Card for Solar Observations

An indirect solar observation may be made by holding a white or light-colored card or sheet of paper behind the eyepiece in such a manner that the image of the sun is thrown upon it. By adjusting the position of the eyepiece tube and varying the distance at which the card is held, the images of the sun and cross hairs may be brought into sharp focus and excellent pointings made as rapidly as with the sun glass. Figure 52 shows the use of a card for this purpose.

65 The Wall Simplex Solar Shield

In determining the azimuths of survey lines it is normally convenient to use solar rather than star observations. Star observations frequently involve much overtime and special trips to the survey area. On the other hand, in the past, more precise results were generally secured from star observations, and this often justified the additional time spent in obtaining the field data. In either case it was possible to obtain the declination, latitude, longitude, and time with comparable precision, but the altitude and the horizontal

angle from the reference mark to the celestial body could be read more surely when sighting on a star because of the relatively greater difficulty in making accurate pointings on the sun. The *Simplex Solar Shield* is a device designed to improve the accuracy of the solar pointings and the

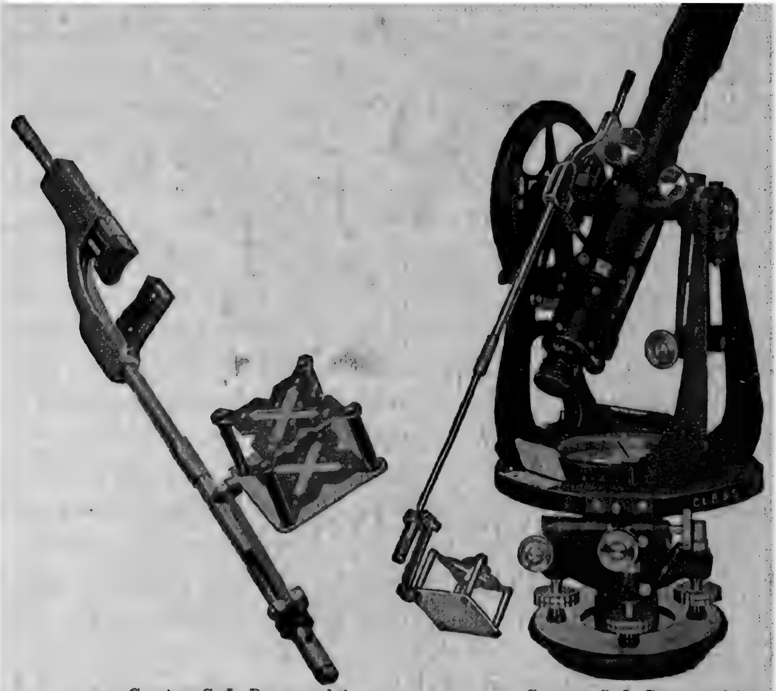


Courtesy Professor Lawrence Perez

FIG. 52. Use of Card for Solar Observation

determination of the times at which the pointings are made. It consists of a shield with symmetrically designed openings which is placed between the eyepiece and a plate upon which the images of the sun and cross hairs are focused. The rod which carries the shield is attached to the horizontal axis of the transit by means of leather grips. The device is shown in Fig. 53 with the shadow of the shield projected on the plate. Figure 54 shows the shield in position on the instrument. The shield is adjusted on the rod until the image of the sun is of such size as to show points of light in the upper and lower V-notches. It is also adjusted so

that the images of the cross hairs appear at the points and notches. It is then possible for the observer to keep the center of the image of the sun on the horizontal cross hair



Courtesy C. L. Berger and Sons

FIG. 53. Wall Solar Shield

Courtesy C. L. Berger and Sons

FIG. 54. Wall Solar Shield

during a series of pointings taken at such times as the image of the edge of the disc appears on the cross hairs or in the several small holes in the shield. Observations made with this device indicate that results compare much more favorably with stellar observations than do the results of direct solar observations.

66 The Solar Attachment

The *solar attachment* is a device which provides for a rapid, mechanical solution of the astronomical triangle

which joins the sun, the pole, and the zenith. Properly positioned, the polar axis of the attachment will point to the celestial pole, the attachment telescope itself or a reflecting device will be directed toward the sun while the vertical axis of the transit points to the zenith. In this position the construction is such that the transit telescope rotates in the plane of the meridian, and the azimuth of a traverse line may be read directly on the horizontal circle. The only data required for positioning the attachment are the latitude of the station; the time of observation; and the sun's declination, corrected for atmospheric refraction, for the date and hour of observation. If desired, all data may be determined in advance, leaving nothing to compute in the field. The attachment thus permits azimuth checks to be applied concurrently with the progress of the field surveys whereas the results of direct solar observations must be obtained normally from subsequent office computations.

While the device will not yield results as close as may be obtained from star observations, azimuths may be maintained within $1\frac{1}{2}$ minutes of true values when the observations are made with the sun in a favorable position, that is, with the sun well above the horizon but not within $1\frac{1}{2}$ hours of the meridian. The attachment has the great advantage of rapidly providing close determinations of the true azimuth, permitting the elimination of accumulative errors incident to running long lines over difficult terrain. It is also useful for latitude determinations and for running the true latitudinal curve in public land surveys.

Three types of attachment, the Smith, the Burt, and the Saegmuller, all depending upon the same principle but differing in construction, are in common use.

The Saegmuller pattern has a polar axis mounted on top of the transit telescope. This axis can be adjusted so that it is perpendicular to the line of sight of the transit and to the horizontal axis of the instrument. The axis carries a solar telescope provided with the usual cross hairs

and four additional hairs forming a square and so set that the sun's image may be just encompassed within them. A bubble tube, adjusted parallel to the line of sight of the solar telescope, is attached to this telescope to be used in

Courtesy C. L. Berger and Sons

FIG. 55. Saegmuller Solar Attachment

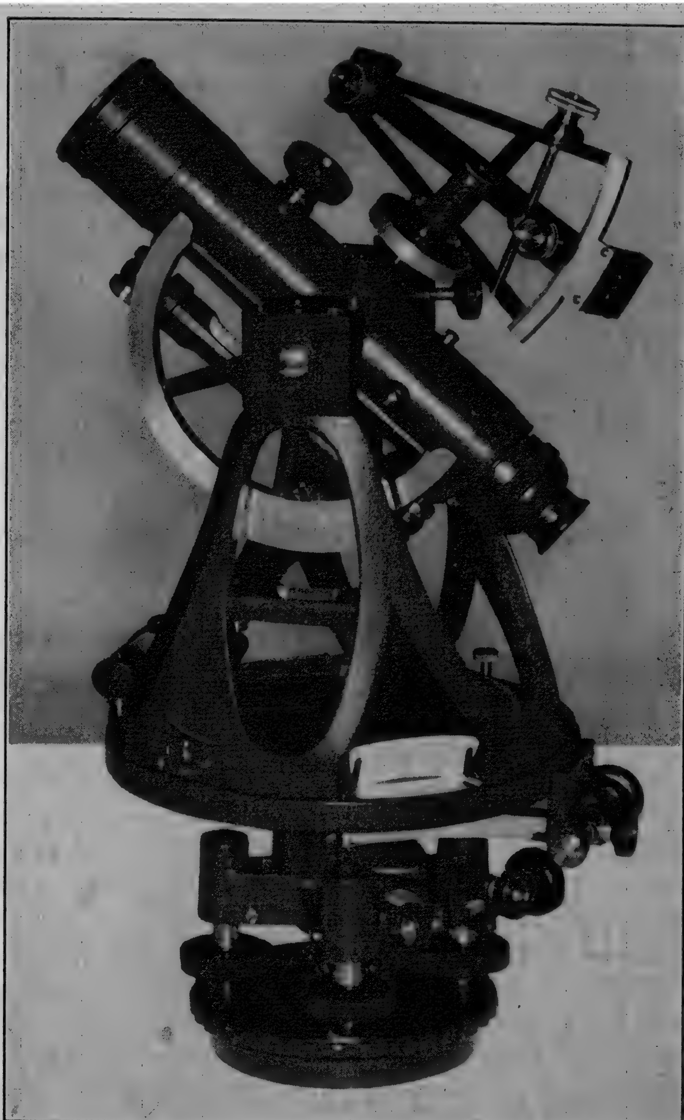
connection with the vertical circle of the transit in laying off the sun's declination. The solar telescope is provided with clamps and tangent screws so that it may be rotated in hour angle about the polar axis and also rotated in declination. Figure 55 shows the Saegmuller attachment.

The Burt attachment also has a polar axis mounted on top of the transit telescope, but the solar telescope is replaced by a bar carrying a small lens and a silver screen

provided with lines within which the sun's image may be centered. Control of movement of this arm in declination is provided for by a vernier on a declination arc, the arm rotating about the center of this arc. Double sets of lenses and screens are placed at opposite ends of the bar. One pair is used for north declinations, the other for south declinations. This makes it possible to have the declination arc read in one direction only. Movement of the arm in hour angle is controlled by an hour circle rotating about the polar axis. The vertical circle of the transit is used to lay off the colatitude so that the polar axis will be parallel to the earth's axis of rotation. The Burt attachment is illustrated in Fig. 56.

The Smith attachment differs from the other two in being fastened to the side of the transit. A horizontal axis is attached to one standard of the transit (east standard with telescope in direct position pointed north); a counterweight is attached to the other. This axis carries a vertical limb provided with a latitude arc and collar bearings supporting the solar telescope. The solar telescope serves as the polar axis of the instrument. At the objective end of this telescope there is pivoted a plane mirror with its axis normal to the line of collimation. This mirror is attached to a graduated declination arc. Motion along the latitude and declination arcs is controlled by clamps and tangent screws. The auxiliary telescope may be rotated in hour angle about its longitudinal axis, an index on one of the collars permitting the hour circle to be set to correspond to the local apparent time.

To obtain the direction of the true meridian, the horizontal circle of the transit is set at zero, the latitude of the station is laid off on the latitude arc, the sun's declination (corrected for refraction) is set on the declination arc, and the auxiliary telescope is rotated in its collar bearings until the index of the hour circle is set at the local apparent time. With the transit carefully leveled, the lower motion is then



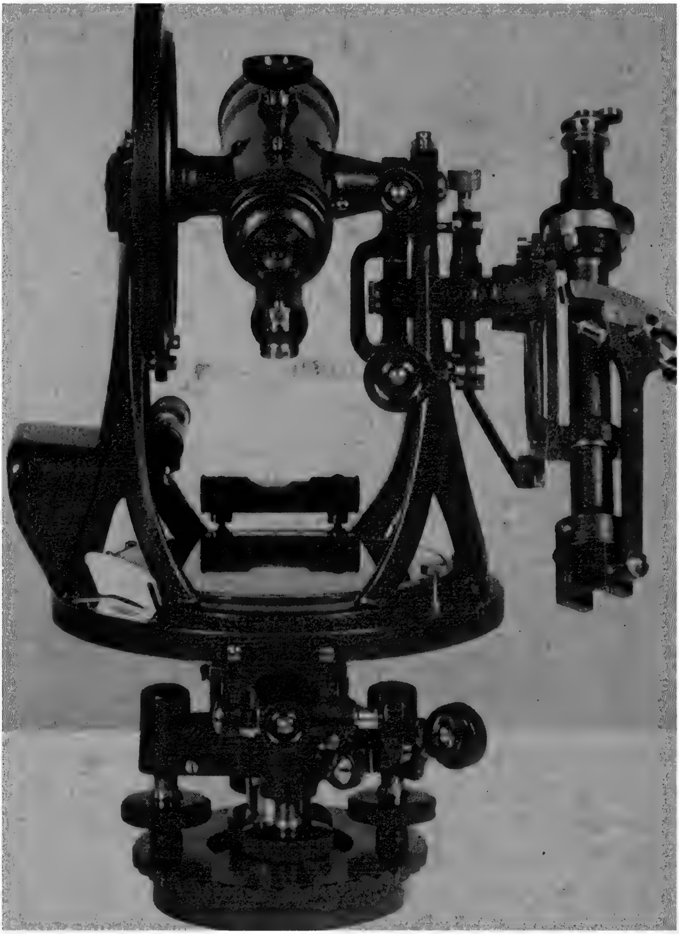
Courtesy W. and L. E. Gurley

FIG. 56. Burt Solar Attachment

released, and the instrument is rotated on its vertical axis until the image of the sun is reflected from the mirror to the objective of the solar telescope and thence brought to focus at the cross hairs. The reticule of this telescope is provided with a set of equatorial hairs placed parallel to the axis of the mirror and so spaced that the sun's image may be just included between them. Only when the transit telescope is in the plane of the meridian can the sun's image be maintained between these equatorial hairs as the solar telescope is slowly rotated to keep pace with the increasing hour angle. The lower motion may then be locked and the horizontal angle turned off to the next traverse station by use of the upper motion, the reading of the horizontal circle being then a measure of the azimuth. The Smith solar attachment is shown in Fig. 57.

The widest user of the solar attachment today is probably the General Land Office in connection with surveys of the public lands of the United States. This office prefers the Smith attachment because of its proven accuracy, the rapidity with which results are obtained, the ease of making field adjustments and the stability of these adjustments, and the compactness and low center of gravity of its design. This attachment has a very great advantage over other types in that its position on the side of the standards permits the unrestricted use of the transit for all normal survey purposes. The declination and latitude arcs may remain clamped and may be readily adjusted by means of their tangent screws for the changes in declination and latitude caused by the passage of time and movement from one station to another. On the other hand, this instrument should be used on transits which have been designed to carry it and which have sufficient rigidity in the standards. Ordinary engineer's transits may be more easily modified by the factory to carry one of the other two types. The Burt and Saegmuller attachments are so constructed that it is impossible to plunge the transit telescope in normal operations unless the attachment is removed.

All these attachments are largely restricted to work in the temperate zones because the sun is not well positioned



Courtesy W. and L. E. Gurley

FIG. 57. Smith Solar Attachment

for observations in the tropics or in arctic regions. For further discussion of the use of the solar attachment in azimuth observations see Art. 118, Chapter 12.

67 The Marine Sextant

The *marine sextant* is an instrument for measuring the angular distance between two objects, the angle always lying in the plane through the two objects and the eye of the observer. It is particularly useful at sea because it

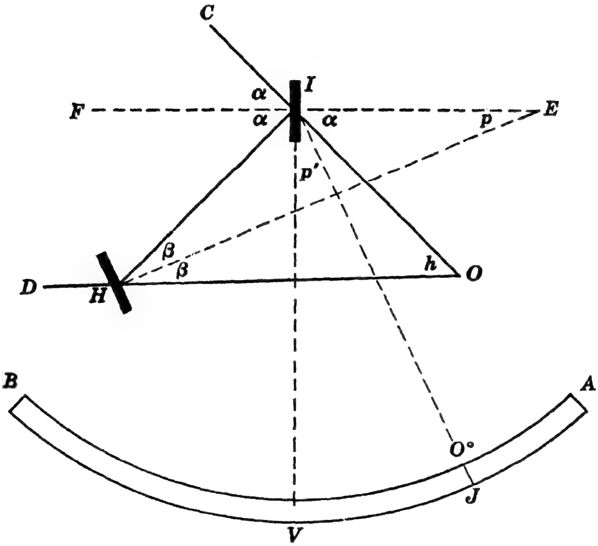


Fig. 58. Principle of the Marine Sextant

does not require a steady support as does the transit. It consists of a frame carrying a graduated arc AB, Fig. 58, about 60° long and two mirrors I and H, the first one movable and the second fixed. At the center of the arc, at I, is a pivot on which swings an arm IV, 6 to 8 inches long. This arm carries a vernier V for reading the angles on the arc AB. Upon this arm is placed the index glass I. At H is the horizon glass. Both of these mirrors are set so that their planes are perpendicular to the plane of the arc AB and so that when the vernier reads zero the mirrors are parallel. The half of the horizon mirror H which is

furthest above the frame is unsilvered so that objects may be viewed directly through the glass. In the silvered portion, objects may be seen by reflection from the mirror I to the mirror H and thence to the point O. At a point near O (on the line HO) is a telescope of low power for viewing the objects. Between the two mirrors and also to the left of H are colored shade glasses to be used when making observations on the sun. The principle of the instrument is as follows: A ray of light coming from an object at C is reflected by the mirror I to H where it is again reflected to O. The observer sees the image of C in apparent coincidence with a second object at D. The arc is so graduated that the reading of the vernier gives directly the angle between OC and OD. Basing the theory on the physical law that when light is reflected from a plane mirror the angles of incidence and reflection are equal, we have for the triangle IOH

$$h = HIC - OHI = 2\alpha - 2\beta$$

In the triangle IEH

$$p = HIF - EHI = \alpha - \beta$$

Therefore $h = 2p$. Since IJ is the position of the index arm when the vernier reads 0°, IJ is parallel to the horizon glass, and the angles p and p' are equal. The angle between the mirrors is therefore half the angle between the objects which are made to appear to coincide. In order that the true angle may be read directly from the arc each half degree is numbered as if it were a degree. Thus the sextant, with an arc of 60°, will measure angles up to 120°. The *quadrant*, formerly used in marine navigation, is similar to the sextant except that its arc covers one *eighth* (not one quarter) of a circle, permitting angles to be measured up to 90°. To be consistent with the term sextant it should have been called an *octant*. Modern octants are used in air navigation. The term sextant is frequently used to

designate all instruments of this type. Figure 59 shows a marine sextant.

Study of Fig. 58 will indicate that, as the angle between two objects changes, the position of point O will vary, the amount depending upon the distance of the reflected object from the observer. Since in astronomical work the dis-

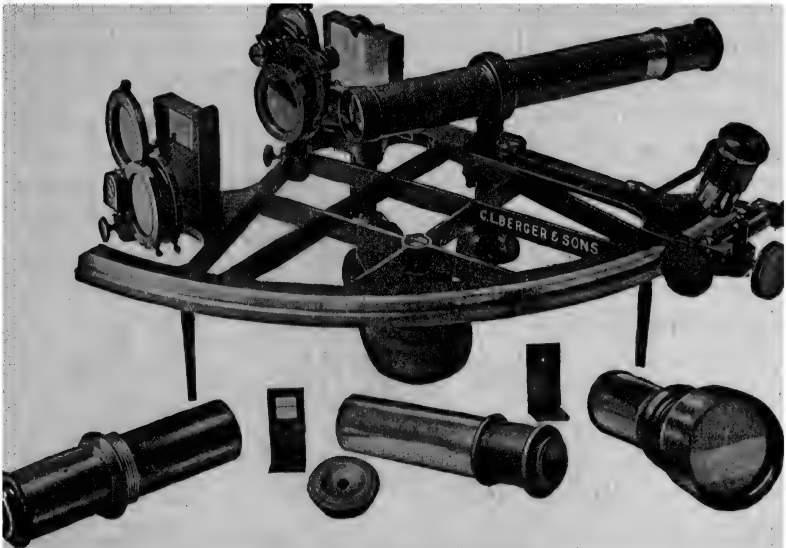


Fig. 59. Marine Sextant

tances to observed objects are always large, the errors caused by changes in the position of O are always negligible.

The sextant is in adjustment when (1) both mirrors are perpendicular to the plane of the arc, (2) the line of sight of the telescope is parallel to the plane of the arc, and (3) the vernier reads zero when the mirrors are parallel to each other. When this last condition is not fulfilled an index correction must be applied to all angles measured with the instrument. The amount of the correction may be found by either of two general methods. If a distant continuous line, such as the visible sea horizon or the flat roof

or vertical edge of a distant building, is available, the error of the sextant may be quickly determined. The line is viewed through the clear portion of the horizon glass, and the reflected image of the same line is brought into coincidence. If the instrument is in adjustment the vernier should read zero. Any lack of coincidence between the vernier and the arc may be noted and applied as the index correction. If the reading is *off* the arc, the correction is given a plus sign and must be *added* to all readings; if the reading is *on* the arc, the correction is minus and must be *subtracted.* Use of the visible sea horizon is valuable where the horizon glass must be adjusted since the vernier may be set to read 0° and the doubly reflected image brought into agreement with the horizon as seen directly. Another method of obtaining the index correction through the use of the sun presents some advantages. The correction is determined as follows. Set the vernier to read about 30′ *on* the arc, and place the shades for both the index mirror and the horizon glass in position. When the sun is sighted, if the index correction is small, the two images will be seen with their edges nearly in contact. This contact should be perfected and the vernier reading recorded. Several repetitions of this observation will increase the accuracy. Then set the vernier about 30′ on the opposite side of the zero point (*off* the arc), and repeat the whole operation, the reflected image now being below the direct image. If the shade glasses are of different colors the contact can be made more accurately. Half the difference of the two (average) readings is the index correction. It will also be apparent that half the sum of the two readings should agree with the tabulated value for the diameter of the sun for the date. This method is preferable for the determination of the index correction because of the presence of a check.

Example. On June 11, 1947, the average reading on the sun with the vernier on the arc was 0° 28′ 40″. The average reading off the arc was

0° 34′ 20″. Determine the index correction and determine the sun's apparent diameter as a check.

On the arc	−0° 28′ 40″	
Off the arc	+0 34 20	
Difference	+0 05 40	
Sum	1 03 00	(without regard to sign)
I.C. = $\frac{1}{2}$ difference	+0° 02′ 50″	
Sun's diameter = $\frac{1}{2}$ sum	0° 31′ 30″	

The sun's semidiameter, June 11, 1943 (from *Ephemeris*) is 15′ 46″.93. The correct apparent diameter is therefore 0° 31′ 34″.

Many sextants, as now manufactured, are fitted with an endless tangent screw carrying a micrometer drum from which the minutes and fractions of a minute (usually 10″ or 0′.1) are read instead of from a vernier. The degrees are obtained from divisions on the arc together with an index mark on the index bar. Large movements of the index bar may be secured by means of a release which disengages the endless tangent screw from the worm teeth on the arc. When measuring an angle the two may therefore be engaged at any convenient position. Obviously this arrangement facilitates the reading of angles since the vernier is eliminated. Some sextants of this type are fitted with a small light near the micrometer together with a battery in the handle for night observations. The index correction may be determined as outlined in the previous discussion. However, before using the sextant, the observer should satisfy himself that there is no backlash between the tangent screw and the worm gear. This may be best determined by making an index correction observation on the visible sea horizon, first bringing the reflected image *down* to the direct image and then bringing the reflected image *up*. If the two index corrections agree, there is probably no backlash. If they disagree, however, the mechanical fault should be corrected. If there is no opportunity to do so, one or the other of the two index corrections should be adopted and angles observed by moving the

tangent screw in the same direction used for obtaining the correction.

In measuring the altitude of the sun above the sea horizon the observer directs the telescope to the point on the horizon vertically under the sun and then moves the index arm until the reflected image of the sun comes into view. The sea horizon can be seen through the plain glass, and the sun is seen in the mirror. The sun's lower limb is then set in contact with the horizon line. In order to be certain that the angle is measured to the point vertically beneath

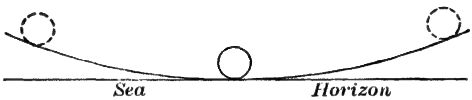


Fig. 60.

the sun, the instrument is tipped slowly right and left, causing the sun's image to describe an arc. This arc should be just tangent to the horizon (see Fig. 60). If at any point the sun's limb goes below the horizon the altitude measured is too great. The tangent screw should be turned until this condition of tangency is realized. The vernier reading corrected for index error and dip is the apparent altitude of the lower limb above the true horizon. To obtain the true altitude of the sun's center corrections for refraction, semidiameter, and parallax must be applied.

In measuring the altitude of a star certain modifications of this procedure are necessary. The usual solar telescope provided with the instrument is erecting. For star observations a second telescope is furnished to provide increased magnification of the stars. In order to preserve the light-gathering power to the maximum degree, this telescope is normally inverting. Some observers find this confusing and prefer to use the solar telescope for all observations.

With solar observations we look through the horizon glass at the sea horizon and bring the image of the sun down

to it with the mirror. In making star observations the large number of stars makes sure identification practically impossible if this procedure is followed. Instead, we set the vernier to read zero and sight the star through the horizon glass. If there is no index correction the star and its reflected image will now coincide. Then, keeping one eye on the reflected image, we move the index arm slowly forward, at the same time moving the instrument so that horizon glass drops slowly downward until the horizon is visible in the clear portion of the horizon glass. Thus the image of the star is brought down to the horizon and identification is definite since we have kept this image in constant view.

Many observers prefer to make a rough altitude setting by inverting the sextant, sighting the star through the horizon glass, and bringing the reflected image of the horizon into coincidence with the star. The instrument is then placed in the normal position and the horizon sighted in the direction of the star. The reflected image of the star will then show in the silvered portion of the horizon glass, and the altitude setting may be perfected with assurance as to the star observed since the navigational stars are few in number and no other bright star will have that particular direction and approximate altitude at that instant.

68 Artificial Horizon

When altitudes are to be measured with the sextant on land the visible horizon cannot be used, and an artificial horizon must be used instead. The surface of any heavy liquid, such as mercury, molasses, or heavy oil, may be used for this purpose. When the liquid is placed in a basin and allowed to come to rest, the surface is perfectly level, and in this surface the reflected image of the sun may be seen, the image appearing as far below the horizon as the sun is above it. Such an artificial horizon should be protected from the wind by placing over it a roof-shaped glass

cover. The two surfaces of each glass pane must be parallel to eliminate refraction. In an emergency a cover of fine mosquito net will answer the purpose.

Another convenient form of horizon consists of a piece of black glass, with plane surfaces, mounted on a frame supported by leveling screws. This horizon is brought into position by placing a spirit level on the glass surface and leveling alternately in two positions at right angles to each other. This form of horizon is not so accurate as the mer-

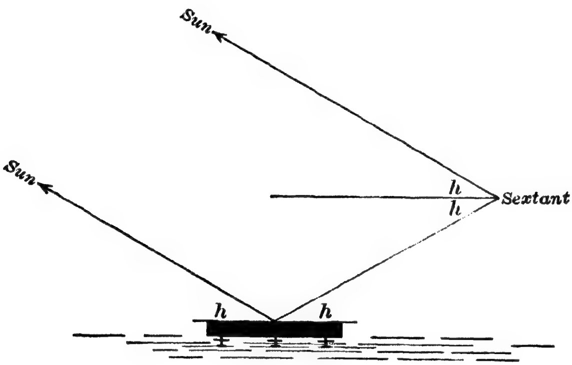


FIG. 61. Artificial Horizon

cury surface but is often more convenient. The principle of the artificial horizon may be seen from Fig. 61. Since the image seen in the horizon is as far below the true horizon as the sun is above it, the angle between the two is $2h$. In measuring this angle the observer points the telescope toward the artificial horizon and then brings the reflected sun down into the field of view by means of the index arm. By placing the apparent lower limb of the reflected sun in contact with the apparent upper limb of the image seen in the mercury surface, the angle measured is twice the altitude of the sun's lower limb. The two points in contact are really images of the same point. If the telescope inverts the image this statement applies to the upper limb.

The index correction must be applied *before* the angle is divided by 2, and corrections for refraction, semidiameter, and parallax must be applied after division to obtain the correct altitude.

69 The Bubble Sextant

A description of instruments for measuring the altitudes of celestial bodies would be incomplete without mention of the bubble sextant. While the instrument is designed primarily for the air navigator, the engineer may find it useful in exploration and reconnaissance surveys.

The air navigator requires a type of sextant or octant embracing a special form of artificial horizon which may be used when the sea horizon is not visible. In flights over land or above an overcast ocean the sea horizon is not available. Many long flights, requiring celestial observations for position checks, are made during the hours of darkness when observations with the marine sextant would not be possible. Furthermore, at altitudes above 1000 feet the sea and sky often tend to merge, and a distinct horizon line is not always seen on over-water flights. Dip corrections are large in aerial navigation, and altimeter readings cannot always be relied upon for the determination of these corrections with precision.

The necessity for an instrument capable of measuring altitudes from a self-contained artificial horizon is at once evident. Several arrangements for such an horizon, not requiring a stable support, are possible. Of these, the circular level bubble is the simplest and is used in the majority of modern airplane sextants. These instruments are used primarily for altitude measurements, that is, for angles up to a maximum of 90° plus the amount of the dip correction if the sea horizon is used. Although frequently termed sextants, they are usually of the octant type, capable of measuring angles slightly in excess of 90°.

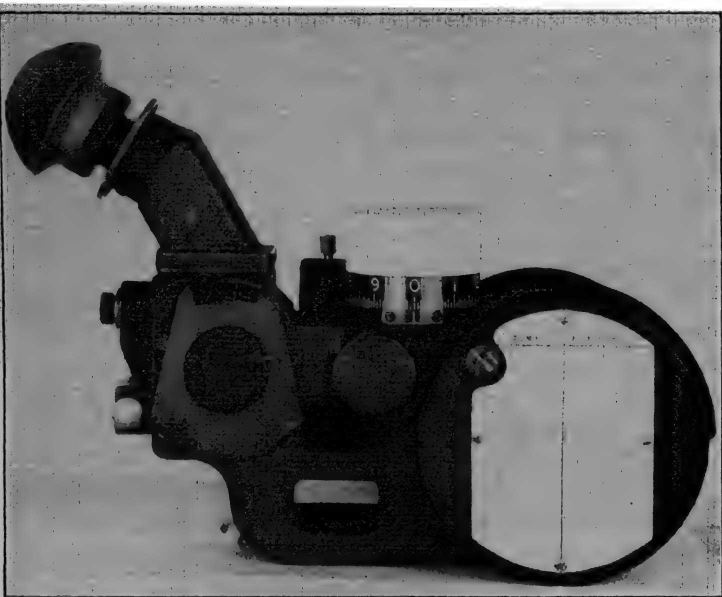
All bubble sextant designs are based on the same funda-

mental principles as the marine sextant but have the added provision of an artificial horizon which may be used in place of the sea horizon when desired. In measuring altitudes, the image of the celestial body is brought into coincidence with the image of either the sea horizon or the circular level bubble which forms the artificial horizon. The altitude is then read on an arc and micrometer drum.

While the marine sextant has become relatively well standardized in construction during the past two centuries of development, the bubble sextant is of too recent origin for the evolvement of a universally accepted standard pattern. Improvements are being made constantly, and it is likely that modifications will continue to be made in the future. Rather than describe each of the many models in detail, it seems wise therefore to confine the description to general terms. Data concerning the use and adjustment of a particular model are supplied by the maker with the instrument.

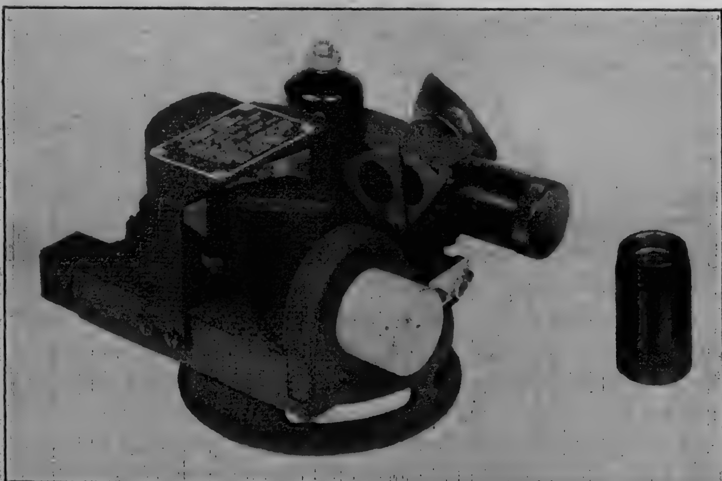
The models being manufactured at present differ from each other both in external appearance and in the internal arrangement of the chain of lenses and prisms or mirrors comprising the optical system. The components of these instruments are customarily enclosed, in contrast to the open pattern of the marine sextant. Lenses and prisms, rather than simple mirrors, are generally used to bring the image of the celestial body into coincidence with the bubble. The size of the bubble is regulated by a knob governing a diaphragm connected to the bubble chamber. Provision is made for illuminating the bubble, the graduations on the altitude arc, and a renewable record disc or pad. Figures 62 and 63 show respectively the Pioneer octant and the Fairchild aircraft sextant, two representative examples of modern instruments of this type.

In operation, the index prism attached to the micrometer drum is rotated in a vertical plane until the image of the celestial body appears in the field of view with the level



Courtesy Pioneer Instrument, Division of Bendix Aviation Corporation

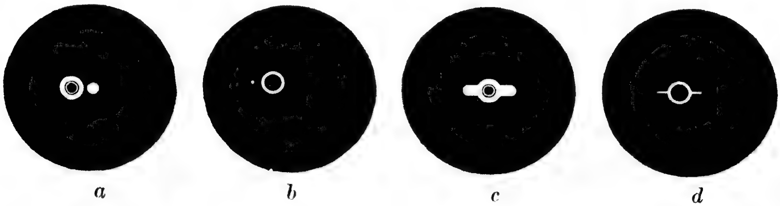
FIG. 62. Pioneer Octant



Courtesy Fairchild Aviation Corporation

FIG. 63. Fairchild Aircraft Sextant

bubble. The coincidence is then perfected, the image of the body being superimposed on the bubble or placed along side on the same horizontal line. At the instant of coincidence the operator presses a marker plunger or trigger which causes a stylus to register a mark on the recording disc attached to the micrometer drum. This graphical representation of an altitude permits the reading of a series of altitudes upon completion of the series rather than after each observation. The altitude of the body may then be



Courtesy Pioneer Instrument, Division of Bendix Aviation Corporation

FIG. 64. Collimation of Celestial Bodies in Pioneer Octant

read on the vernier or counter geared with the micrometer drum. The construction is such that it is not essential that the bubble be in the exact center of the field when the coincidence is perfected. Certain models are provided with an astigmatizer which elongates the image of the sun or star into a band or line of light for more precise setting. Figures 64*a* and 64*b* show the field of the Pioneer octant with coincidence perfected for the sun and a star respectively. Figures 64*c* and 64*d* show the same images with the astigmatizer in place and the images more perfectly centered.

To minimize errors, a series of observations is usually made, as rapidly and as equally spaced in time as possible, and either an average altitude is determined or the median value used. Some models, known as averaging sextants and equipped with a mechanical averaging device, automatically average a series of observations. Where n measurements of the altitude are to be taken, an averaging lever

advances the vernier 1 /nth of the altitude each time so that the final reading is the mean of the series.

When used in air navigation the bubble sextant is subject to large errors unless the plane is in steady, level flight when observations are made, for the bubble is acted upon by the resultant of the gravitational acceleration and any other acceleration which the plane may have at the time. The necessity for steady flying and an extended series of observations, which when averaged will reduce the error which may be present in individual measurements, is apparent.

All observations must be corrected for refraction. When the sea horizon is used corrections must be applied for dip, for semidiameter in the case of sun and moon sights, and for parallax when lunar observations are made. The index error should preferably be eliminated by adjustment. When the artificial horizon is used a Coriolis acceleration correction must be applied to counterbalance the rotational effect of the earth on the airplane. This rotation causes moving objects, winds, ocean currents, or aircraft to be deflected to the right in the northern hemisphere and to the left in the southern. Thus, in the northern hemisphere, a body moving north would be deflected toward the east; a body moving south would be deflected westerly. The amount of this correction for an airplane is dependent upon the speed of the plane and the latitude in which it is flying. In practice, the Coriolis acceleration correction is normally applied not to the observed altitude of a star but to the plotted position of the aircraft as obtained by observations on two or more stars.

70 The Chronometer

Many astronomical observations require that the instant at which the observation is made be known with precision. The *chronometer* is used to determine the time at sea and is also employed in connection with precise observations on land. This instrument is an accurately constructed

timepiece, much heavier and larger than a watch, with a special form of escapement. Chronometers may be regulated for either sidereal or mean time. The beat is usually a half second. Those designed to register the time on a *chronograph* (see Art. 71) are arranged to break an electric circuit at the end of every second or every two seconds. The 60th second is distinguished either by the omission of the break at the previous second or by an extra break, according to the construction of the instrument. Chronometers are usually hung in gimbals to keep them level at all times; this is invariably done when they are taken to sea. It is important that the temperature of the chronometer be kept as nearly uniform as possible because fluctuation in temperature is the greatest source of error.

Two chronometers of the same kind cannot be directly compared with great accuracy, $0^s.1$ or $0^s.2$ being about as close as the difference can be estimated. A sidereal and a solar chronometer, however, may be compared to within a few hundredths of a second. On account of the gain of the sidereal on the solar chronometer, the beats of the two will coincide once in about every $3^m\ 03^s$. If the two are compared at the instant when the beats are apparently coincident, it is only necessary to note the seconds and half seconds as there are no fractions to be estimated. By making several comparisons and reducing them to some common instant of time it is readily seen that the comparison is correct within a few hundredths of a second. The accuracy of the comparison depends upon the fact that the ear can detect a much smaller interval between the two beats than can possibly be estimated when comparing two chronometers whose beats do not coincide.

The chronometer is a delicate instrument and must be handled with great care. It should be wound carefully, and at the same time, each day. The instrument may be classed as semiportable. It is transported to the field for certain surveying observations and is carried by large vessels

at sea, but it is moved with care and only when necessary. It is not taken to the bridge of a ship for sextant observations nor is it carried to the radio room for comparison with time signals. Navigation watches* are used for these purposes and are compared with the chronometer before and after observations. The chronometer is rarely set. Its error is determined from time signals at frequent intervals, and a curve or table showing its rate of gain or loss is prepared. The chronometer correction is then applied to all observed times.

71 Navigation Watches

Navigation watches replace the chronometer in aircraft navigation, in many field observations in surveying, and for many purposes at sea. These watches are precision instruments but are smaller, more readily portable, and, because of a smaller balance and a different escapement, are less delicate than the chronometer. With radio time signals available, these watches are commonly set to the correct time at frequent intervals, to eliminate the need for applying a watch correction to observed times.

Many models have a provision for stopping the second hand so that it may be started at the correct instant as indicated by the radio signal. The minute hand is then set to the correct minute when the second hand reads 60^s. The watch then shows correct time with all hands in the proper interrelation. Other models permit setting the correct second by providing for the rotation of the seconds dial instead of stopping and restarting the second hand. Either the small inner dial or an exterior bezel graduated in seconds is rotated until the second hand gives the correct second as indicated by the time signal. The dial is then locked in position and the minute hand set to conform as before.

Watches have also been developed which read in units

* Sometimes called " hack " watches.

of arc rather than in time units to permit the reduction of observations without a conversion from time to arc. These are most frequently used when the watch is regulated to sidereal time. Stop watches and " time of flight " watches are also useful in converting the instant of observation to chronometer time as indicated in Art. 73.

72 The Chronograph

The chronograph is an instrument for recording the time kept by a chronometer and also any observations the times of which it is desired to determine. The paper on which the record is made is wrapped around a cylinder which is revolved by a clock mechanism at a uniform rate, usually once per minute. The pen which makes the record is placed on the armature of an electromagnet which is mounted on a carriage drawn horizontally by a long screw turned by the same mechanism. The mark made by the pen runs spirally around the drum, which results in a series of straight parallel lines when the paper is laid flat. The chronometer is connected electrically with the electromagnet and records seconds by making notches in the line corresponding to the breaks in the circuit, one centimeter being equivalent to one second. Observations are recorded by the observer by pressing a telegraph key, which also breaks (or makes) the chronograph circuit and makes a mark on the record sheet. When the paper is laid flat the time appears as a linear distance on the sheet and may be scaled off directly with a scale graduated to fit the spacing of the minutes and seconds of the chronograph.

73 Suggestions for Observing with Small Instruments

Engineer's Transit. In using the engineer's transit for astronomical observations, care must be taken to provide a firm support for the tripod. The tripod legs should be set firmly into the ground. Where the ground is shaky, three large pegs may be driven to a firm bearing and the

points of the tripod legs set in depressions in the tops of the pegs. It is well to set the transit in position some time before the observations are to begin; this will allow the instrument to assume the temperature of the air and the tripod legs to come to a firm bearing in the ground. If possible, particularly where night observations are to be taken, the transit station should be so chosen that the ground is comparatively level. Equipment, such as transit boxes, and so forth, should be removed from the vicinity of the instrument to lessen the likelihood of the observer tripping over them in the dark. Tripod legs which are painted in red and white bands, as used extensively in recent years to promote the safety of instrument men engaged on highway surveys, are more clearly visible at night than the conventional varnished legs and are therefore less likely to be brushed against or tripped over. Particular attention should be paid to the leveling of the instrument to insure a firm, permanent setup (see Art. 60). The instrument should be carefully focused on a very distant object and, if the objective slide tends to slip back when the telescope is elevated, correct focus should be maintained by wrapping Scotch tape about the intersection of the objective slide and the telescope barrel.

The observer should be thoroughly familiar with his instrument to enable him to make his observations with nicety and despatch and to avoid accidentally disturbing the settings. This is especially important in night work when the observer must operate his instrument with confidence and surety. Since all celestial targets are apparently moving, pointings must be made with rapidity and assurance, and, once the observer is satisfied with his pointing, he should immediately take his eye from the telescope and commence reading the angles. Further efforts to revise an observation will lead only to confusion. Occasionally difficulty is experienced in getting the image of the celestial body in the field of view. The observer will find that this

is most readily accomplished by sighting over the telescope in the same manner as in training gun sights on a target. At first, the observer may have difficulty with faint stars but experience will prove that equally good pointings can be made on these as on the brighter stars provided both the cross hairs and the star can be seen. When illuminating the cross hairs only enough light should be allowed to enter the telescope to permit the cross hairs to be clearly visible. Sometimes, when it is not possible to see both the stars and the cross hairs at the same time, it is still possible to effect an observation by first illuminating the cross hairs and then removing the light and noting the position of the star in the field. By repeating this process a satisfactory pointing can be made.

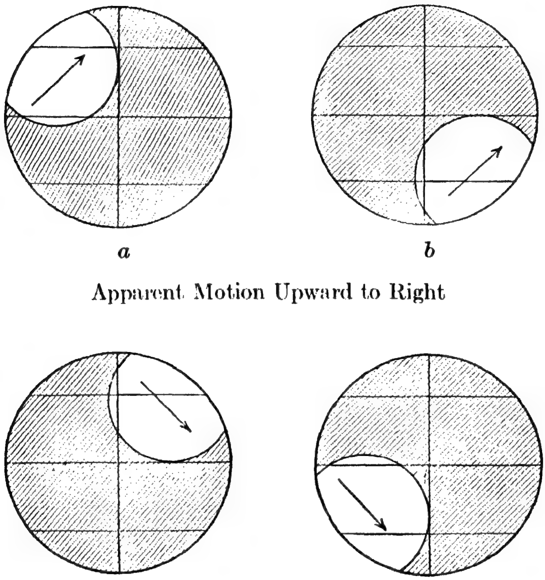
Observations involving the time of meridian transit of stars should be made with the star near the middle of the field, just above or below the horizontal cross hair, to avoid lens errors near the margins of the field. Latitude observations involving altitude measurements should be taken with the star on the horizontal hair near the vertical hair to minimize errors resulting from lack of adjustment of the cross hairs. For observations requiring the measurement of both horizontal and vertical angles the readings must be made at the instant that the star is at the intersection of the two cross hairs. Since the sun and the moon are too large for accurate bisection the observations must be made on the limbs and corrections applied for semidiameter, unless opposite limbs are used, and the results averaged to obtain the horizontal and vertical angles to the center at the mean instant of time. This is usually done in solar observations for time and azimuth. The procedure for such solar observations follows.

It is difficult to manipulate the tangent screws controlling the horizontal and vertical motions of the transit at the same time. Therefore the image of the sun is placed in a quadrant of the field such that one limb may be kept tan-

gent to either the horizontal or vertical cross hair by leading the image by manipulation of the appropriate tangent screw while the other hair is illuminated by the sun. The image will then gradually become tangent to this stationary hair by reason of the apparent motion of the sun. At this instant the time is observed, and the angles are read. Several observations are made in this quadrant. The instrument is then reversed (provided it has a full vertical circle), and an equal number of observations are made in the diagonally opposite quadrant. Both quadrants must be so chosen that the sun will leave the stationary cross hair. If the sun is approaching this hair the cross hair will not be illuminated until the image is past the point of tangency.

The quadrants selected will depend upon whether the observation is made in the morning or afternoon, whether the instrument is erecting or inverting, whether or not a prismatic eyepiece is used, and whether the sun is north or south of the observer's zenith. To avoid memorizing rules, the observer should look at the sun through the colored eyepiece (or at the image on a card held behind the eyepiece), note the apparent direction of motion, and draw a sketch which will indicate at once the proper quadrants to use. Figures 65*a* and 65*b* indicate that the 2nd and 4th quadrants should be used when the sun appears to be rising to the right. In Fig. 65*a* the vertical hair is moved to keep it tangent to the leading edge of the image and the time noted when the image becomes tangent to the horizontal hair. In Fig. 65*b* the horizontal hair is kept tangent to the apparent upper limb and the time noted when the vertical hair becomes tangent. A little thought will indicate that the same quadrants are used if the image appears to move downward to the left, but the moving cross hairs are reversed in the two quadrants. Figures 65*c* and 65*d* indicate that the 1st and 3rd quadrants are used when the image appears to move downward to the right. The horizontal hair is kept tangent to the leading limb in Fig.

65*c*; the vertical hair in Fig. 65*d*. Obviously these same quadrants should be used if the image appears to move upward to the left, but the cross hair which is moved should be reversed in the two quadrants. An analysis of the actual



a *b*

Apparent Motion Upward to Right

c *d*

Apparent Motion Downward to Right

Fig. 65. Positions of Sun's Disc a Few Seconds before Observing

apparent motion of the sun in the forenoon and afternoon (without reference to the optics of the instrument) will indicate whether the limbs observed are the east, west, upper, or lower.

In this observation great care should be exercised that a limb is not tangent to either the upper or lower stadia hair since the sun will not illuminate all three cross hairs at once. Transits provided with a vertical arc cannot be reversed. The index correction must therefore be deter-

mined and applied to the mean altitude. Since the index correction should be read in about the same vertical circle wherein the altitudes were measured, it should be determined after making the last pointing before the setting of the horizontal circle is changed.

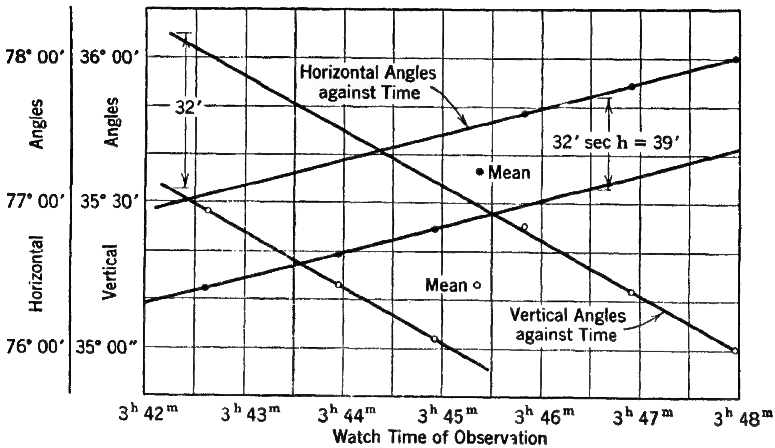


FIG. 66. Plot of Field Measurements — Solar Observation for Time and Azimuth

At least three observations should be taken in each quadrant and the differences in times and angles computed to determine the proportionality of the results. In taking the mean of a series of observations we assume the apparent path of the sun to be a straight line between the first and last observations. If the time interval is short, say less than 10 minutes, this should be sensibly true. Unless the angles, when plotted against the times, give a straight-line variation there is no point in computing the observation. The data given below are typical of a solar observation for time and azimuth. The angle and time differences are tabulated. Their close proportionality is evident and indicates a reliable series of observations. The results are plotted in Fig. 66. The fact that the series on each limb

plots as a straight line and that the lines representing the upper and lower limbs are parallel is indicative of the accuracy of the pointings and timing. The vertical distance between altitude lines should equal 32′, the angular diameter of the sun. The vertical distance between the horizontal angle lines should equal 32′ × sec *h*, or 39′ in this instance. The plot shows that these conditions are very closely met.

Point	*Horizontal Angle*	*Difference*	*Vertical Angle*	*Difference*	*Watch Time*	*Difference*
Reference Mark	0° 00′					
$\underline{\vert\odot}$	76 25		35° 28′		$3^h\ 42^m\ 37^s$ P.M.	
		14′		15′		80^s
	76 39		35 13		43 57	
		10		11		58
	76 49		35 02		44 55	
	(Telescope reversed)					
$\overline{\odot\vert}$	77 37		35 25		45 50	
		12		13		65
	77 49		35 12		46 55	
		11		12		63
	78 00		35 00		47 58	
Reference Mark	0 00					

Where observations involve horizontal angles from a reference mark, as in the above instance, the instrument should always be resighted on the mark following the completion of the celestial observations and the vernier read to make certain that the setting of the lower plate has not been disturbed.

In reading angles at night it is important to hold the light in such a position that the graduations on the circle are plainly visible and may be viewed along the lines of graduation, not obliquely. By changing the position of the eye, it will be found that the reading varies by larger amounts than would be expected when reading in daylight. Small

penlights may be used to advantage since it is possible to place them in such a position that their light shines along the graduations. As indicated in Art. 61, some instruments are equipped with interior lighting systems, presenting the observer with ideal illumination for the verniers. Care must be taken not to touch the exposed vertical circle and vernier so that rapid tarnishing of the graduated surfaces will be avoided. The recorder should hold his light so as to prevent it from shining into the observer's eyes. A headlamp for the recorder is most convenient since it frees both his hands for note keeping and timing.

It is especially important that a program of observations be laid out in detail beforehand to avoid mistakes and to conserve time. Such a list should anticipate exactly what is to be done and the proper order of the several steps. The notes should be prepared in advance insofar as possible so that the recorder will not delay the observer at any time. Observations should be arranged to eliminate instrumental errors by reversing the instrument; but, since instruments used for astronomical work should be in excellent adjustment, it will be possible to make acceptable observations if it is not practicable to reverse the instrument. The index correction must be determined and applied, unless it can be eliminated by the method of observing.

It should be obvious from the above discussion that successful astronomical observations are dependent upon the development by the observer of considerable manual dexterity and technique in handling his instrument with swiftness, sureness, and delicate touch.

Sextant. In handling a sextant, the same nicety and care should go into its operation as when observing with the engineer's transit, and many of the above suggestions apply. The index correction should be determined, if possible, shortly before the observations are to be made. The methods of bringing down the image of a celestial

object to coincidence with the sea horizon have been described in Art. 67. The same methods may be used with a bubble sextant. When reading a vernier sextant, the magnifying microscope should be centered over the coinciding lines of the vernier and the arc to prevent errors of reading caused by parallax. Great care should be exercised to prevent touching either the graduated arc or vernier to minimize tarnishing. Instruments wet by salt spray should be dried and cleansed after use.

Watches and Chronometers. In observations where time is involved it will often be necessary to use an ordinary watch. When there are two men in the party, the recorder can note the time of the observation at a distinctive signal from the observer. To insure accuracy, the observer should warn the recorder when an observation is imminent. Some observers prefer to have the recorder start counting the seconds aloud just before the celestial body is to be observed. The instrument man is then able to note the time to the nearest second. When a chronometer is used, the observer may work alone. In making observations by this method (called the " eye and ear method "), the observer looks at the chronometer, notes the reading at some instant, say at the beginning of some minute, and carries along the beat mentally and without looking at the chronometer. In this way he can note the second and fraction without taking his attention from the star and cross hair. After making his observation he can check his count by again looking at the chronometer to see if the two agree. After a little practice this method can be used easily and accurately. In using a watch it is possible for one observer to make the observation and also note the time, but, because of the rapidity of the ticks (5 per second), it cannot be done with any such precision as when the chronometer is used. The observer must in this case look quickly at his watch and make an allowance for the time lost in looking up and taking the reading. Another method whereby the observer may

note the time accurately is through the use of "time-of-flight" or stop watches. The hand of the usual stop watch completes a circuit of the face in 60 seconds although there are some special time-of-flight watches which take only 3 seconds. From the former it is possible to estimate short intervals of time to about $0^s.2$ and the graduations in some of the latter are to $0^s.01$. If one of these watches is started at some even minute of a chronometer or watch and stopped at the instant of the observation, the time will then be the sum of the starting time and the reading of the stop watch. Some observers prefer to start the watch when the observation is made and stop it at some even minute of chronometer time. It will be obvious that the time of observation will then be the difference between the chronometer time and the stop watch reading.

74 Errors in Horizontal Angles

When measuring horizontal angles with a transit, such, for example, as in determining the azimuth of a line from the polestar, any error in the position of the sight line, or any inclination of the horizontal axis will be found to pro-

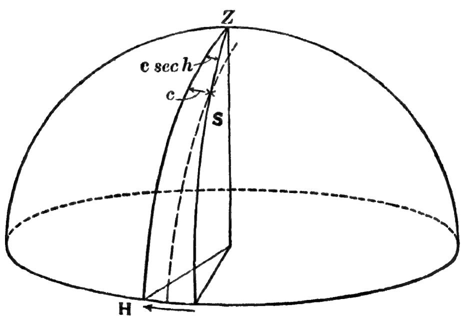


FIG. 67. Line of Sight in Error (Cross-Hair Out)

duce a large error in the result, on account of the high altitude of the star. In ordinary surveying these errors are so small that they are neglected, but in astronomical

work they must either be eliminated or determined and allowed for in the calculations.

In Fig. 67, ZH is the circle traced out by the true collimation axis, and the dotted circle is that traced by the actual line of sight, which is in error by the small angle c. The effect of this on the direction of a star S is the angle SZH.

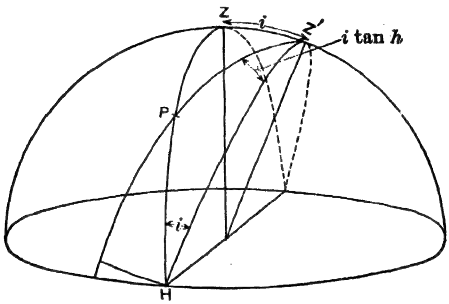


FIG. 68. Plate Levels Adjusted (Bubbles Not Centered)

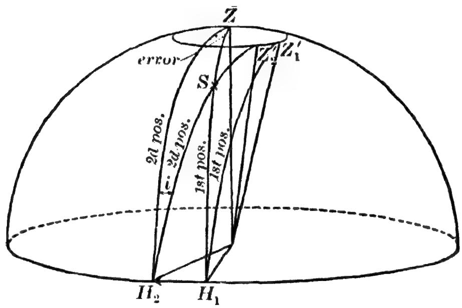


FIG. 69. Plate Levels Correct (Horizontal Axis Out of Adjustment)

In Fig. 68 the vertical axis is not truly vertical but is inclined by the angle i owing to poor leveling. This produces an error in the direction of point P which is equal to the angle $HZ'P$. If the vertical axis is truly vertical but the horizontal axis is inclined to the horizon by the angle i, owing to lack of adjustment, the error in the direction of the point S is the same in amount and equal to the angle H_1ZH_2 in Fig. 69.

The inclination of the horizontal axis may be measured by means of a striding level and a level correction applied to the measured horizontal angle. If w and e are the readings of the west and east ends of the bubble in one position of the level, and w' and e' are the corresponding readings after reversal, the level correction, when the bubble graduations are numbered in both directions from the middle, is given by the equation

$$\text{Level correction} = \frac{d}{4}\{(w + w') - (e + e')\}\tan h$$

When the graduations are numbered continuously in one direction the following equation applies

$$\text{Level correction} = \frac{d}{4}\{(w - w') + (e + e')\}\tan h$$

The primed letters in this latter case refer to the readings taken when the zero graduation is to the west. In both formulae, d is the angular value of a level division, and h is the altitude of the star.

If the azimuth mark is not in the horizon a similar correction must be applied to the readings on the mark. Ordinarily this latter correction will be negligible.

When applying this correction it should be observed that when the west end of the axis is too high the instrument has to be turned too far west (left) when pointing at a star north of the observer. The correction must therefore be added to the measured angle if the mark is west of the star; in other words the reading on a clockwise graduated circle must be increased. If the correction is applied to the computed azimuth of the mark the sign must be reversed (see Art. 115).

Problems

1. Show that if the sight line makes an angle c with the perpendicular to the horizontal axis (Fig. 67), the horizontal angle between two points is in

error by the angle $c \sec h' - c \sec h''$, where h' and h'' are the altitudes of the two points.

2. Show that if the horizontal axis is inclined to the horizon by the angle i (Figs. 68 and 69) the effect upon the azimuth of the sight line is $i \tan h$ and that an angle is in error by $i (\tan h' - \tan h'')$, where h' and h'' are the altitudes of the points.

3. Show diagrammatically the position of the index of the vertical circle vernier of a transit when I.C. is (*a*) $+2'$ and (*b*) $-3'$. Make the sketch reasonably complete to show the relation of the telescope with respect to the vertical circle and vernier.

4. On October 1 a sextant observation was made on the sun to determine the index correction of the instrument. The average reading on the arc was $0° 33' 20''$. The average reading off the arc was $0° 30' 40''$. Determine the index correction and the sun's semidiameter. Compare the sun's semidiameter with the value given in Table IV as a check.

5. Prepare a sketch to show how the solar attachment automatically solves the astronomical triangle to place the telescope of the transit in the true meridian.

6. Prepare sketches similar to those of Fig. 65 showing the proper quadrants for direct telescopic observations on the sun for (*a*) morning observations in lat. 45° N and (*b*) afternoon observations in lat. 45° S. Transits with the following optical systems are to be provided for in each of the previous instances: (1) inverting telescope, (2) erecting telescope, and (3) inverting telescope with prismatic eyepiece. Show by an arrow the apparent motion of the sun in each case, and indicate clearly which cross hair is tangent to the leading limb.

9

The Constellations—Star Identification

75 The Constellations

The civil engineer does not find it as necessary to place great stress on a study of the constellations and rapid methods of star identification as does the navigator. Yet the engineer will find a knowledge of the position of the stars within the constellations most useful. For the navigator, at sea and in the air, such knowledge is practically essential.

In making observations on land we normally identify stars by means of their coordinates. If an engineer has leveled his transit with the telescope in the plane of the meridian and knows approximately his latitude and local time, he can identify stars crossing the meridian by means of the times and altitudes at which they culminate. In fact, latitude, longitude, and time observations are customarily made in this manner on stars which are too faint to locate readily by means of their positions in the constellations. A convenient watch time to begin observing is selected; this is converted into local sidereal time; stars culminating within the range of time set for observing are selected from the *Ephemeris* or a star catalogue by referring to their tabulated right ascensions; the approximate alti-

tudes at culmination are deduced from the rough latitude and tabulated declinations; and the approximate watch times of culmination are figured from the right ascensions. With the altitude laid off on the vertical circle the star will appear in the field of view shortly before the time computed and the required observation may be completed.

Frequently observations for time and azimuth are made on stars east or west of the meridian, near the prime vertical. Even here it is not essential that the name of the star be known at the time of observation. If the observer's latitude and longitude are known approximately, it is only necessary for him to note the observed altitude, the watch time, the magnetic bearing, and the approximate magnitude of the star. With these data, and a knowledge of the declination of the compass, the right ascension and declination of the star may be computed with sufficient accuracy to permit the identification of the star from the tabulated values in the *Ephemeris*. This procedure may be necessary if the sky is overcast and only a few isolated stars can be seen. On the other hand, it is obvious that much labor will be saved if the star can be immediately identified, in a clear sky, from the observer's knowledge of the constellations or from reference to a star chart. Furthermore, when the latitude and longitude are not known, the engineer must make his identification from the constellations.

At sea or in the air the ability to identify stars quickly is particularly essential. At sea, the time available for observing is strictly limited. Since altitudes are measured from the sea horizon, observations must be made when both the horizon and the stars are visible. This confines the work to periods shortly before sunrise and after sunset. To conduct his work most expeditiously the navigator must know his stars, in clear weather, by their positions in the constellations. In foul weather, when the altitude and bearing of an occasional star are caught through flying scud, identification is possible through use of a star finder,

such as the Rude star finder which is described later in this chapter. The aviator, through his use of an artificial horizon as provided by the bubble sextant, is not restricted in his observations to any particular time of night. Because of his rapid flight however, he must compute his position and course with the utmost speed and, from the constellations or star finder, must make a quick identification of the star observed.

76 Method of Naming Stars

The whole sky is divided in an arbitrary manner into irregular areas, all of the stars in any one area being called a *constellation* and given a special name. The individual stars in any constellation are usually distinguished by a name, a Greek letter, or a number. The letters are usually assigned in the order of brightness of the stars, α being the brightest, β the next, and so on. A star is named by stating first its letter and then the name of the constellation in the (Latin) genitive form. For instance, in the constellation *Ursa Minor*, the star α is called α *Ursa Minoris* (*Polaris*); the star *Vega* in the constellation *Lyra* is called α *Lyrae*. When two stars are very close together and have been given the same letter, they are often distinguished by the numbers 1, 2, etc., written above the letter, as, for example, α^2 *Capricorni*, meaning that the star passes the meridian after α^1 *Capricorni*.

77 Magnitudes

The *magnitude* of a star is the measure of its apparent brightness. The ancient Greek astronomers devised an arbitrary scale of six magnitudes to include all stars visible to the naked eye. The twenty brightest stars were designated as first-magnitude stars; those of about the brightness of *Polaris* were placed in the second magnitude group; fainter stars were placed in successive groups in descending order of brightness, with the sixth magnitude embracing those stars just visible to the eye on a clear night.

Our present system of stellar magnitudes devolves from this ancient scale although modern precision necessitates the use of decimals to correctly designate the relative brightness of stars, and the introduction of the telescope requires that the scale be greatly lengthened to include stars far too faint for the eye to detect. Within modern times it was determined that the average first magnitude star was about 100 times as bright as the average sixth magnitude star and that the arithmetical progression in the ancient magnitude scale corresponded closely with a geometrical scale in the brightness ratio. If this brightness ratio is indicated by x, then a fifth magnitude star is x times as bright as one of the sixth magnitude; a fourth magnitude star is x times as bright as a fifth magnitude star and x^2 times as bright as one of the sixth magnitude; and so on. Thus a first magnitude star is x^5 times as bright as a sixth magnitude star. Since this difference in brightness was found to be 100, we have $x^5 = 100$ and $x = 2.512$. That is, the brightness of a star of a particular magnitude is 2.512 times that of the next fainter magnitude. A star this much brighter than a first magnitude star would have a magnitude of 0. Still brighter stars have negative magnitudes. Thus the magnitude of *Sirius* is -1.58 while that of the sun is -26.7. The brightness ratio corresponding to a tenth of a magnitude is that ratio such that when multiplied by itself ten times the product will equal 2.512. The ratio for hundredths of a magnitude is found in a similar manner.

The difference in brightness of stars may be found readily by use of logarithms. The brightness ratio between magnitudes is 2.512 or $\sqrt[5]{100}$. The logarithm of this is $\frac{2}{5}$ or 0.4. Therefore the difference in brightness between two stars may be found by multiplying the difference in their magnitudes by 0.4 and taking from a logarithmic table the antilogarithm corresponding to this product. Conversely, we can find the difference in magnitudes if we know the difference in brightness.

With telescopic stars we must distinguish between visual and photographic magnitudes. If the brightness is determined visually it will have one value; if determined by photometric means it will have another since the eye and the photographic plate are not equally sensitive to all colors of light rays.

78 Constellations near the North Pole

The stars of the greatest importance to the surveyor are those near the pole. In the northern hemisphere the pole is marked by a second-magnitude star, called the *polestar*, *Polaris*, or *α Ursae Minoris,* which is about 1° 00′ distant from the pole at the present time. This distance is now decreasing at the rate of about one-third of a minute per year so that for several centuries this star will be close to the celestial north pole. On the same side of the pole as *Polaris,* but much farther from it there is a constellation called *Cassiopeia,* the five brightest stars of which form a rather unsymmetrical letter W (Fig. 70). The lower left-hand star of this constellation, the one at the bottom of the first stroke of the W, is called δ and is of importance to the surveyor because it is very nearly on the hour circle passing through *Polaris* and the pole; in other words its right ascension is nearly the same as that of *Polaris.* On the opposite side of the pole from *Cassiopeia* is *Ursa Major,* or the Big Dipper, a rather conspicuous constellation. The star ζ, which is at the bend in the dipper handle, is also nearly on the same hour circle as *Polaris* and *δ Cassiopeiae.* If a line is drawn on the sphere between *δ Cassiopeiae* and *ζ Ursae Majoris,* it will pass nearly through *Polaris* and the pole and will show at once the position of *Polaris* in its diurnal circle. The two stars in the bowl of the Big Dipper on the side farthest from the handle are in a line which, if prolonged, would pass near *Polaris.* These stars are therefore called the *pointers* and may be used to find the polestar. There is no other star near *Polaris* which is likely to be

MAP I

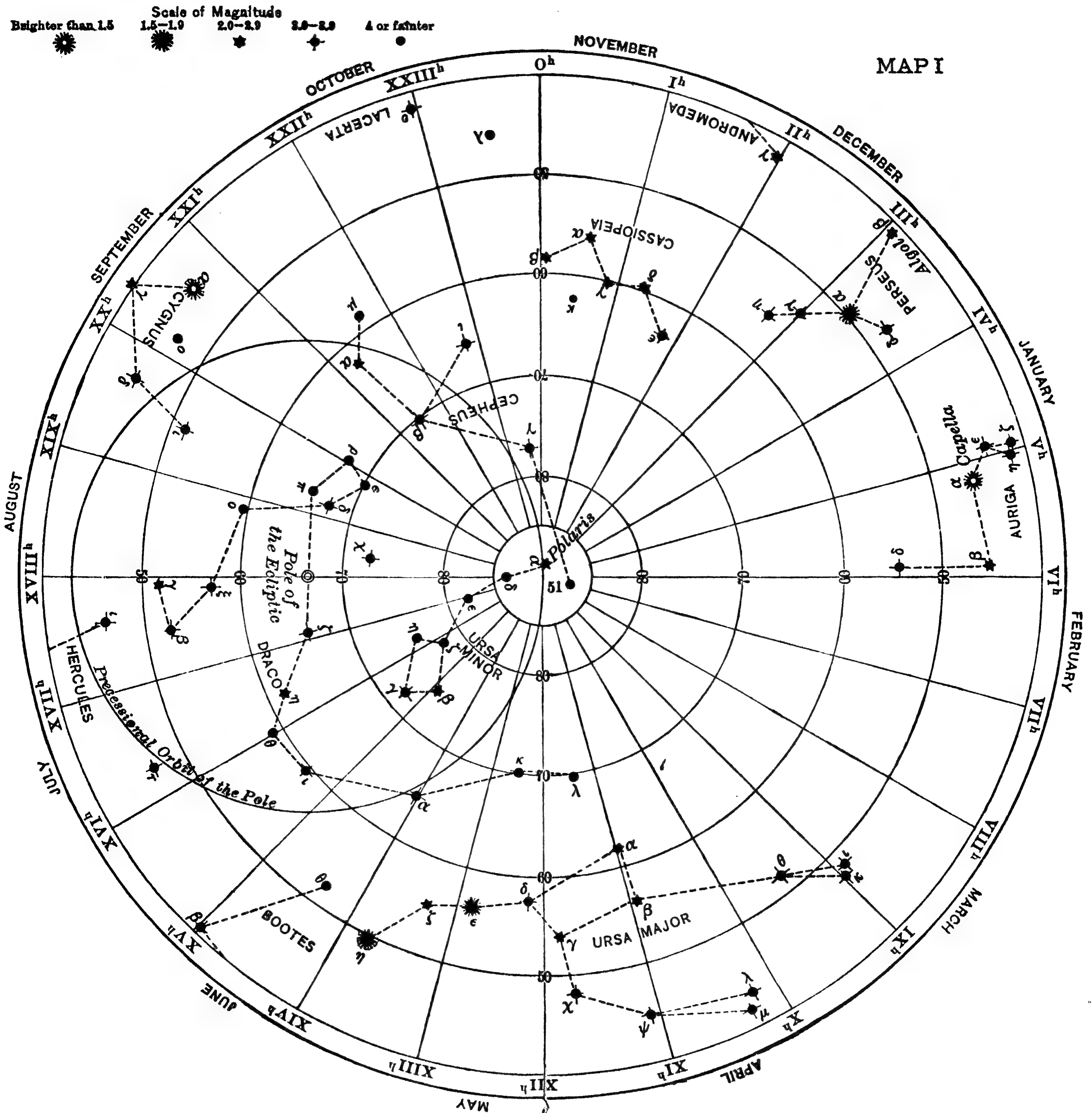


FIG. 70. Constellations about the North Pole

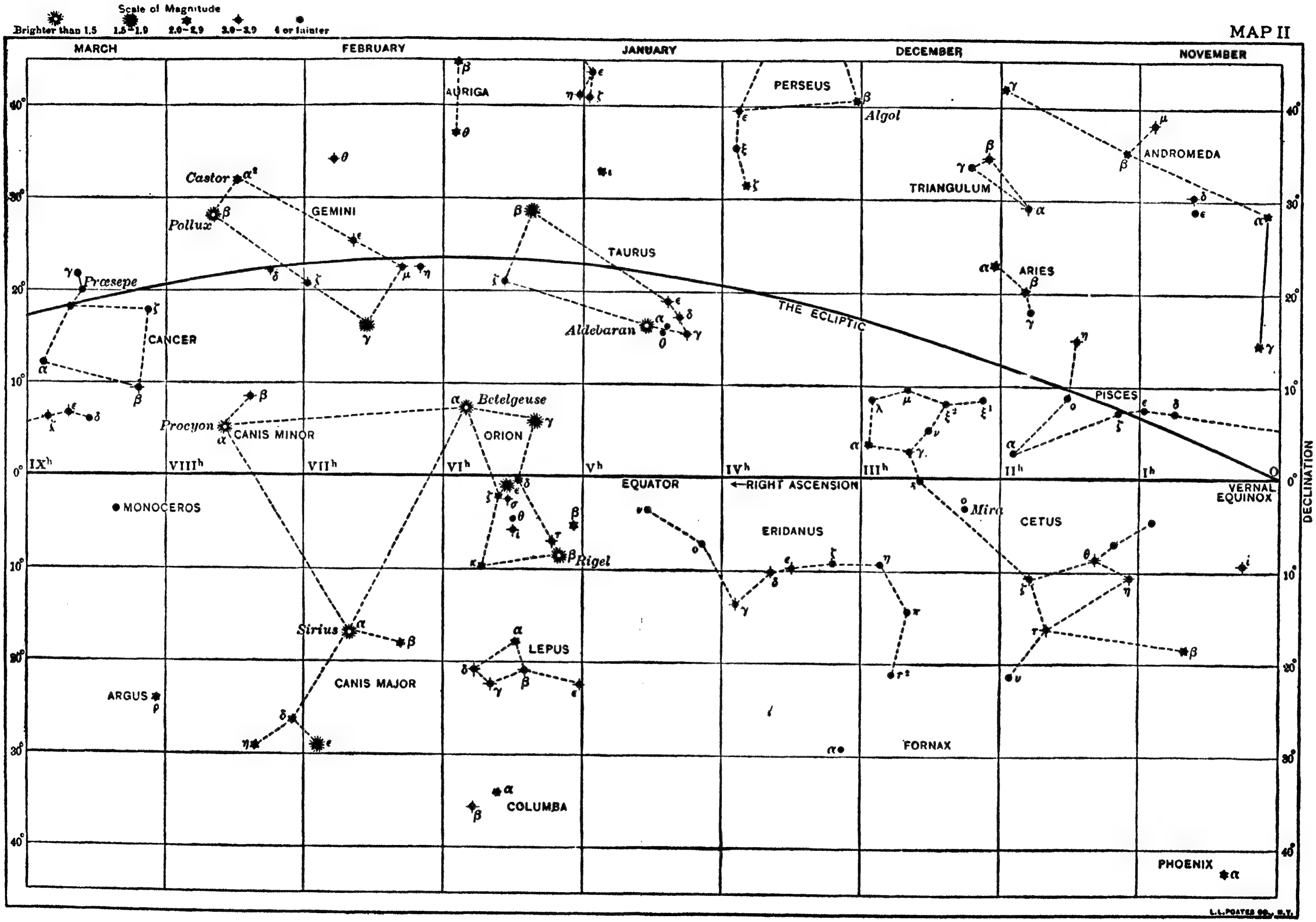


Fig. 71. Principal Fixed Stars between Declinations 45° North and 45° South

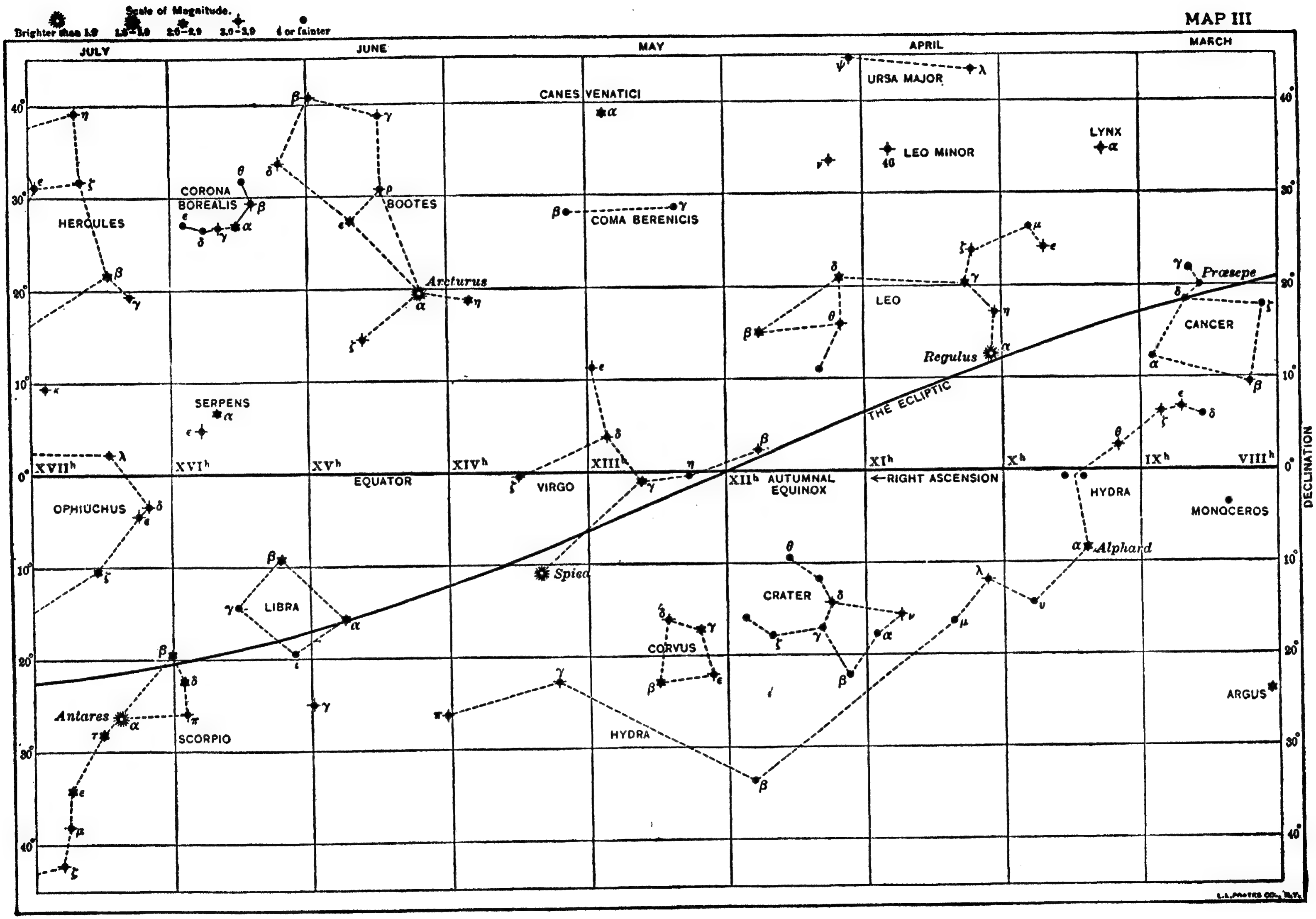


FIG. 72. Principal Fixed Stars between Declinations 45° North and 45° South

MAP IV

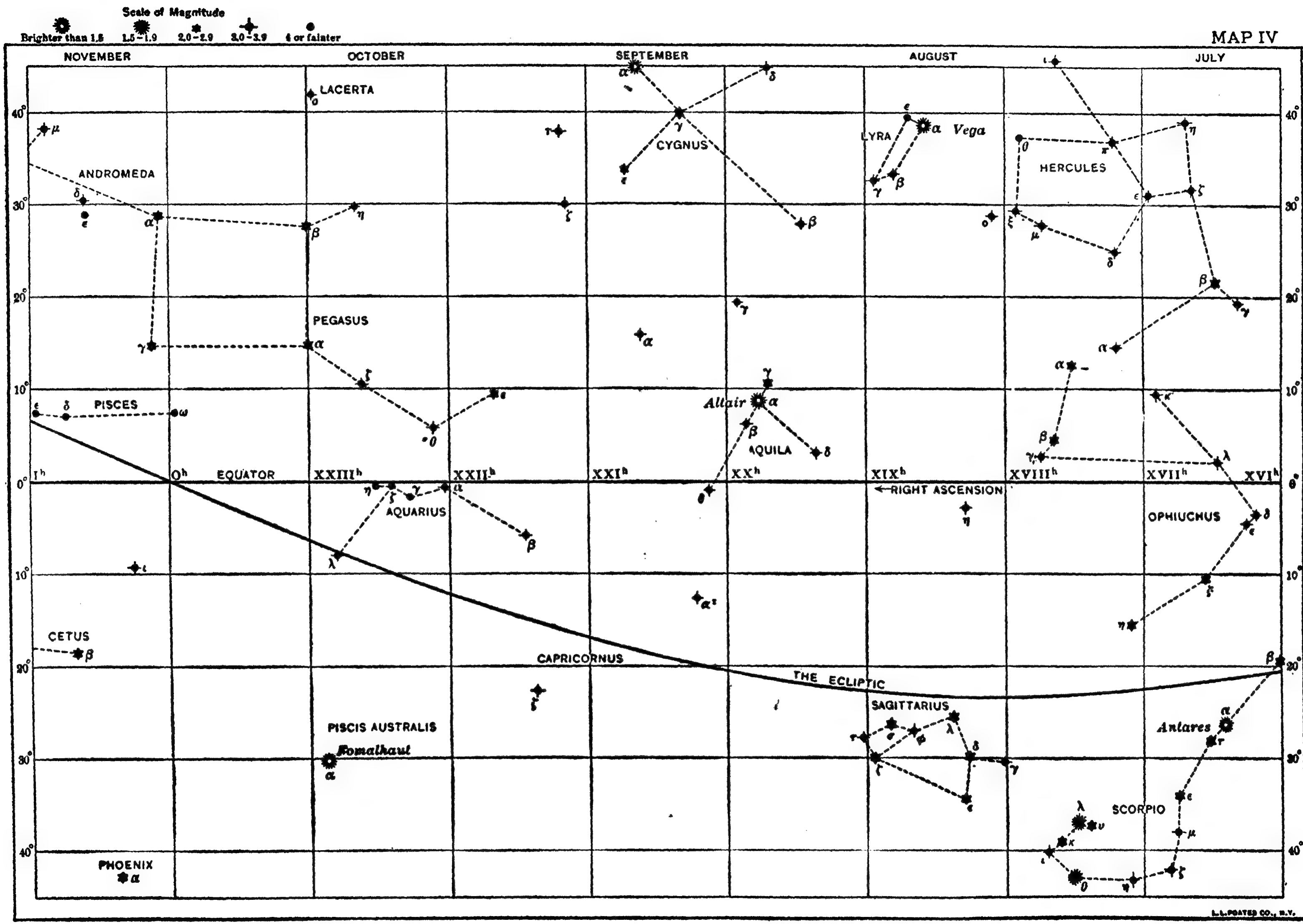


FIG. 73. Principal Fixed Stars between Declinations 45° North and 45° South

MAP V

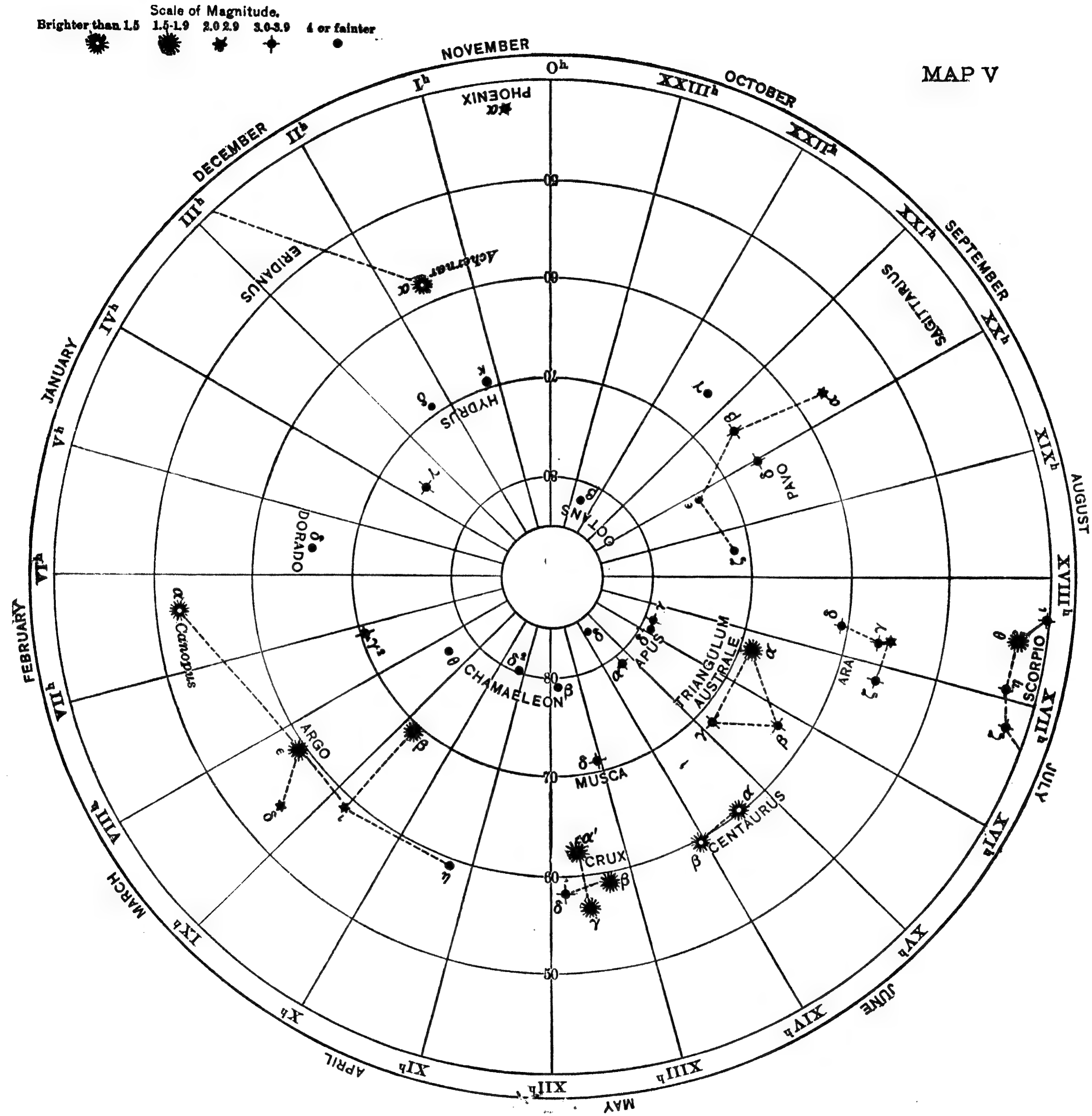


Fig. 74. Constellations about the South Pole

confused with it. Another star which should be remembered is β *Cassiopeiae*, the one at the upper right-hand corner of the W. Its right ascension is very nearly 0^h, and therefore the hour circle through it passes nearly through the equinox. It is possible then, by simply glancing at β *Cassiopeiae* and the polestar, to estimate approximately the local sidereal time. When β *Cassiopeia* is vertically above the polestar it is nearly 0^h sidereal time; when the star is below the polestar it is 12^h sidereal time; halfway between these positions, left and right, it is 6^h and 18^h respectively. In intermediate positions the hour angle of the star (= sidereal time) may be roughly estimated.

79 Constellations near the Equator

The principal constellations within 45° of the equator are shown in Figs. 71 to 73. Hour circles are drawn for each hour of right ascension and parallels are drawn for each 10° of declination. The approximate declination and right ascension of a star may be obtained by scaling the coordinates from the chart. The position of the ecliptic, or sun's path in the sky, is shown as a curved line. The moon and the planets are always found near this circle because the planes of their orbits have only a small inclination to the earth's orbit. A belt extending about 8° each side of the ecliptic is called the *Zodiac*, and all the members of the solar system will always be found within this belt. The constellations along this belt, which have given their names to the twelve "signs of the Zodiac," are *Aries, Taurus, Gemini, Cancer, Leo, Virgo, Libra, Scorpio, Sagittarius, Capricornus, Aquarius,* and *Pisces.* These constellations were named many centuries ago, and the names have been retained, both for the constellations themselves and also for the positions in the ecliptic which they occupied at that time. But because of the continuous westward motion of the equinox, the "signs" no longer correspond to the constellations of the same name. For

example, the *sign of Aries* extends from the equinoctial point to a point on the ecliptic 30° eastward, but the constellation actually occupying this space at present is *Pisces.* In Figs. 70 to 72 the constellations are shown as seen by an observer on the earth, not as they would appear on a celestial globe. Because of the form of projection used in these maps there is some distortion, but, if the observer faces south and holds the page up at an altitude equal to his colatitude, the map represents the constellations very nearly as they will appear to him. The portion of the map to be used in any month is that marked with the name of the month at the top; for example, the stars under the word "February" are those passing the meridian in the middle of February at about 9 P.M. For other hours in the evening the stars on the meridian will be those at a corresponding distance right or left, according as the time is earlier or later than 9 P.M. The approximate right ascension of a point on the meridian may be found at any time in the following manner. First compute the right ascension of the sun by allowing 2^h per month, or more nearly 4^m per day for every day since March 23, remembering that the right ascension of the sun is always increasing. Add this right ascension $+ 12^h$ to the local civil time, and the result is the sidereal time or right ascension of a star on the meridian.

EXAMPLE. On October 10 the R.A. of the sun is $6 \times 2^h + 17 \times 4^m = 13^h\,08^m$. R.A. of sun $+ 12^h$ is $25^h\,08^m$, or $1^h\,08^m$. At 9^h P.M. the L.C.T. is 21^h. $1^h\,08^m + 21^h = 22^h\,08^m$. A star having an R.A. of $22^h\,08^m$ would therefore be close to the meridian at 9 P.M.

80 Constellations near the South Pole

Figure 74 shows the stars about the south celestial pole. There is no bright star near the south pole so that the convenient methods of determining the meridian by observations on a close circumpolar star are not practicable in the southern hemisphere.

81 The Planets

In using the star maps, the student should be on the lookout for planets. These cannot be placed on the maps because their positions are rapidly changing. If a bright star is seen near the ecliptic, and its position does not correspond to that of a star on the map, it is a planet. The planet Venus is very bright and is never very far east or west of the sun; it will therefore be seen a little before sunrise or a little after sunset. Mars, Jupiter, and Saturn move in orbits which are outside of that of the earth and therefore appear to us to make a complete circuit of the heavens. Mars makes one revolution around the sun in 1 year 10 months, Jupiter in about 12 years, and Saturn in $29\frac{1}{2}$ years. Jupiter is the brightest and when looked at through a small telescope shows a disc like that of the full moon; four satellites can usually be seen lying nearly in a straight line. Saturn is not as large as Jupiter, but in a telescope of moderate power its rings can be distinguished; in a low-power telescope the planet appears to be elliptical in form. Mars is reddish in color and shows a disc.

82 Star Identification

Tables and diagrams for rapid star identification depend upon tabular or graphical solutions of the astronomical triangle. As we have seen in Chapter 4, the three sides of the astronomical triangle PZS (Fig. 30, p. 48) are PZ, $90° -$ latitude; PS, $90° -$ declination; and ZS, $90° -$ altitude. The three angles are the azimuth of the star, Z; the hour angle, t, at the pole; and the parallactic angle S. If any three parts of this triangle are known, the missing sides or angles may be determined.

The navigator, observing a star through breaks in the clouds, measures its altitude and approximate azimuth. He also knows his approximate latitude and longitude, from dead reckoning determinations of position. With his lati-

tude, and the altitude and azimuth of the star roughly known, he has sufficient data to determine the declination and local hour angle of the star. From his approximate longitude and the local hour angle he can determine the star's right ascension or its Greenwich hour angle. These coordinates enable him to identify the star from the tabulated values in the *Nautical Almanac.* The surveyor may follow a similar procedure in identifying stars observed near the prime vertical.

To minimize the time required at sea or in the air for this reduction, solutions of the astronomical triangle for a large number of values of latitude, altitude, and azimuth have been computed and either tabulated or shown graphically on star-finder diagrams. Typical of tabular forms are the star identification tables found in Hydrographic Office publication 127 and those in H.O. 214, *Tables of Computed Altitude and Azimuth.* The latter is published in nine volumes, each covering a belt of 10° of north or south latitude. The navigator enters the table at the proper page for his dead reckoning latitude and, under the observed altitude and opposite the observed azimuth, takes out the declination and local hour angle of the star.

EXAMPLE. On April 25, 1947, in dead reckoning position 40° 04′ N and 70° 15′ W, a star of about the first magnitude was observed in the southeast. Its altitude was 27° 30′ and its azimuth was 134° $\frac{3}{4}$ from the north. The +5 zone time was $20^h\ 10^m$. Identify the star.

In H.O. 214, Vol. V, under latitude 40°, p. 26, we find that a star, having the altitude and azimuth shown, has decl. 11° and H.A. 40°. The declination is in italics, indicating that it is of contrary name to the latitude. Since the latitude is north the declination is south or minus. If we now convert the L.H.A. to R.A. or to G.H.A. we can enter the *Nautical Almanac* with known coordinates and pick out the star which was observed. Both computations follow.

For the approximate right ascension

Zone time	$20^h\ 10^m$	April 25, 1947
Zone difference	+5	
G.C.T.	1 10	April 26, 1947
R.A.M.S. + 12^h	14 12 .6	
Table VI (*Nautical Almanac*)	0 .2	(See Table III)
G.S.T.	15 23	
Long. W	4 41	
L.S.T.	10 42	
H.A. of star	2 40 E	
R.A. of star	$13^h\ 22^m$	

On pp. 162–163 of the 1947 *Nautical Almanac* we find that the star with R.A. $13^h\ 22^m$ and decl. $-11°$ is α *Virginis* (*Spica*).

For the approximate G.H.A.

L.H.A., 40° E	320°	
Long. W	70 15	
Sum	390 15	
Subtract 360°	360 00	
G.H.A. at $1^h\ 10^m$ G.C.T., 4/26	30 15	
Nautical Almanac correction for $1^h\ 10^m$	17 33	(See p. 214,
G.H.A. at 0^h G.C.T., 4/26	12° 42′	1947 *Almanac*)

From p. 180 of the 1947 *Nautical Almanac* we find as before that the star with approximate G.H.A. 12° 42′ and decl. $-11°$ is *Spica*.

83 Rude Star Finder and Identifier

The "Rude Star Finder and Identifier" is typical of graphical, mechanical devices for spotting and identifying stars. It is published in two forms by the Hydrographic Office. H.O. 2102-A is a large finder designed for shipboard use. H.O. 2102-C is a small finder for the use of either aircraft or ships. In either case it consists of a star base with the principal navigational stars of the northern sky on one side, those of the southern sky on the other side, and a series of templates for 10° intervals of latitude which may be placed on the base. The navigator selects the template closest to his dead reckoning latitude, places it on the base so that the hole in the template is over the pin in the center of the base, and rotates it until the arrow

on the 0° line is opposite the mark on the outer edge of the base corresponding to the local sidereal time or to the local hour angle of the equinox, depending on which form of finder is used. In this position visible stars more than 10° above the horizon are found within the transparent grid of the template. Their approximate altitudes and azimuths can be read from the circles of equal altitude and azimuth curves shown on the template if stars suitable for observation are to be selected. When the identity of a star is unknown, it may be determined by spotting the observed altitude and azimuth on the diagram.

Questions

1. What bright star will be on or near the meridian at about 7^h P.M. local time on September 10, and nearly overhead in lat. 40° N?
2. At about what hour, local time, will Orion be on the meridian on November 1?
3. Are any of the stars of the Southern Cross visible from points within the continental limits of the United States? If so, where?

Part II
Engineering Astronomy

10

Observations for Latitude

84 Astronomical Observations with the Engineer's Transit — Latitude

In this chapter and the two immediately following, the more common methods of determining latitude, time, longitude, and azimuth with the engineer's transit are given. The methods outlined are suitable for the majority of observations which civil engineers are called upon to make. Geodetic methods, involving the use of instruments of high precision, are not included. These are more properly described in texts on geodesy and geodetic surveying.

Observations for latitude are included in this chapter. The latitude of an observer on the surface of the earth is the angular distance of his position north or south of the equator, measured along his meridian of longitude. In Art. 16 it was defined astronomically as the declination of the observer's zenith. In this definition the latitude is referenced to the coordinates of the equatorial systems. Referenced to the horizon system, the latitude may be defined as the zenith distance of the celestial equator or, the same thing, the altitude of the celestial pole, both coordinates being measured along the meridian. The purpose of observations for latitude is to find any one of these equivalent quantities.

85 Latitude by a Circumpolar Star at Culmination

This method may be used with any circumpolar star, but *Polaris* is the best one to use, when it is practicable to do so, because it is of the second magnitude while all of the other close circumpolars are quite faint. The observation consists in measuring the altitude of the star when it is a maximum or a minimum, or, in other words, when it is on the observer's meridian. This altitude may be obtained by trial, and it is not necessary to know the exact instant when the star is on the meridian. The approximate time when the star is at culmination may be obtained from Table V or by Art. 38 and Eq. 46. It is not necessary to know the time with accuracy, but it will save unnecessary waiting if the time is known approximately. In the absence of any definite knowledge of the time of culmination, the position of the polestar with respect to the meridian may be estimated by noting the positions of the constellations. When δ *Cassiopeiae* is directly above or below *Polaris* the latter is at upper or lower culmination. The observation should be begun sometime before one of these positions is reached. The horizontal cross hair of the transit should be set on the star* and the motion of the star followed by means of the tangent screw of the horizontal axis. When the desired maximum or minimum is reached the vertical arc is read. The index correction should then be determined. If the instrument has a complete vertical circle and the time of culmination is known approximately, it will be well to eliminate instrumental errors by taking a second altitude with the instrument reversed, provided that neither observation is made more than 4^m or 5^m from the time of culmination. If the star is a faint one, and therefore diffi-

* The image of a star would be practically a point of light in a perfect telescope, but, because of the imperfections in the corrections for spherical and chromatic aberration, the image is irregular in shape and has an appreciable width. The image of the star should be bisected with the horizontal cross hair.

cult to find, it may be necessary to compute its approximate altitude (using the best known value for the latitude) and set off this altitude on the vertical arc. The star may be found by moving the telescope slowly right and left until the star comes into the field of view. *Polaris* can usually be found in this manner some time before dark when it cannot be seen with the unaided eye. It is especially important to focus the telescope carefully before attempting to find the star, for the slightest error of focus may render the star invisible. The focus may be adjusted by looking at a distant terrestrial object or, better still, by sighting at the moon or at a planet if one is visible. If observations are to be made frequently with a particular transit, it is well to have a reference mark scratched on the telescope tube so that the objective may be set at once at the proper focus.

The latitude may be computed from Eqs. 3 or 4. The true altitude h is derived from the reading of the vertical circle by applying the index correction with the proper sign and then subtracting the refraction correction (Table I). The polar distance is found by taking from the *Ephemeris* the apparent declination of the star and subtracting this from 90°.

EXAMPLE 1. The observed altitude of *Polaris* at U.C. January 10, 1947 is 43° 37′; I.C. +30″; decl. +89° 01′ 06″.

Vertical circle	43° 37′ 00″
I.C.	+30
Sum	43 37 30
Refr.	−1 00
True alt.	43 36 30
Polar distance	00 58 54
Lat.	42° 37′ 36″ N

Since the vertical circle reads directly only to 1′ the resulting value for the latitude must be considered as containing a maximum error of 30″.

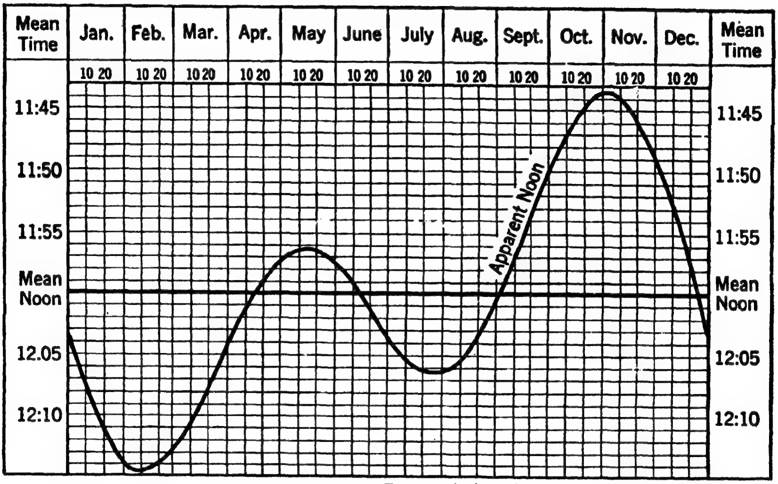
EXAMPLE 2. The observed altitude of 51 H. *Cephei* at L.C. on a certain date is 39° 33′ 30″; I.C. 0″; decl. +87° 08′ 21″.

Observed alt.	39° 33′ 30″
Refr.	−1 09
True alt.	39 32 21
Polar distance	2 51 39
Lat.	42° 24′ 00″ N

86 Latitude by Altitude of Sun at Noon

The altitude of the sun at local apparent noon (meridian passage) may be determined by placing the line of sight of the transit in the plane of the meridian and observing the altitude of the upper or lower limb of the sun when it is on the vertical cross hair. Usually the direction of the meridian is not known so that the maximum altitude of the sun is observed and assumed to be the same as the meridian altitude. On account of the sun's changing declination the maximum altitude is not quite the same as the meridian altitude; the difference is quite small, however, usually a fraction of a second and may be entirely neglected for observations made with the engineer's transit or the sextant. Since the standard time of local apparent noon varies considerably during the year, it is advisable to determine the watch time of apparent noon well in advance to make certain of being in readiness when the sun transits and to avoid unnecessary delay. Figure 75 shows graphically the local mean time of local apparent noon. By applying a correction for the difference in longitude between the local meridian and the standard meridian this curve may be used to indicate the watch time at which the engineer should be ready to begin observing. It should be noted that this curve is simply an adaptation of Fig. 35. The watch time at which the sun will pass the meridian may also be computed by converting 12^h local apparent time into standard or local mean time (whichever is used) as shown in Arts. 29, 33, and 42. Shortly before the time of transit, observing should begin. The maximum altitude of the upper or lower limb is found by trial, the horizontal cross hair being kept tangent to the limb as long as it con-

tinues to rise. When the observed limb begins to drop below the cross hair, the altitude is read from the vertical arc, and the index correction is determined. The true altitude of the center of the sun is then found by applying the corrections for index error, refraction, semidiameter, and parallax. In order to compute the latitude it is necessary to know the sun's declination at the instant the altitude



By permission of Professor Charles O. Roth, Jr.

FIG. 75. Mean Time of Apparent Noon

was taken. If the longitude of the place is known the sun's declination may be corrected as follows: if the Greenwich time or the standard time is noted at the instant of observation the number of hours since 0^h G.C.T. is known at once. If the time has not been observed it may be derived from the known longitude of the place since the sun is on the meridian when the local apparent time is 12^h. Applying the longitude with the correct sign for east or west longitude we obtain Greenwich apparent time. This is converted into Greenwich civil time. The declination as tabulated in the *Ephemeris* for Greenwich 0^h is then corrected for

the time interval since 0^h, using a chord interpolation. If the *Nautical Almanac* is used, the declination will be found tabulated for each 2 hours of civil time and the hourly difference shown. The time need not be computed with great accuracy since an error of 1^m will never cause an error greater than 1″ in the computed declination. The Greenwich civil time corresponding to local apparent noon, used in obtaining the sun's declination at this instant, may be obtained more rapidly from the Greenwich hour angle of the sun than by the use of the equation of time as outlined above (see Eq. 49 and Example 1, Art. 42).

With the altitude and declination known the latitude may be computed by applying Eq. 1 or, preferably, from an analysis of a sketch similar to Fig. 29.

EXAMPLE 1. The observed maximum altitude of the sun's upper limb on January 19, 1947, was 29° 01′ at a place in long. 74° 10′ W, sun bearing south of the zenith; I.C. +30″. Determine the latitude of the observer.

Observed alt.	29° 01′ 00″
I.C.	+30
Sum	29 01 30
Refr.	−01 43
Difference	28 59 47
s. d.	−16 17
Difference	28 43 30
Parallax	+08
Alt.	28 43 38
Decl.	20 24 30
Colat.	49 08 08
Lat.	40° 51′ 52″ N

L.A.T.	$12^h\ 00^m\ 00^s$
Long. W	4 56 40
G.A.T.	16 56 40
Eq. T.	−10 42.9 (add)
G.C.T.	17 07 22.9 = $17^h.123$

Decl. at 16^h G.C.T.	−20° 25′.1
Decl. at 18^h G.C.T.	−20 24 .0
Hourly difference	0′.55
Corrected decl.	−20° 24′.5

In this solution the corrections for refraction, semidiameter, and parallax have been taken from Tables I and IV in the Appendix and the equation of time and the sun's declination from the 1947 *Nautical Almanac*. The latitude, within the precision of measurements, should be correct to within 30″.

EXAMPLE 2. The observed meridian altitude of the sun's lower limb on July 13, 1947, was 41° 10′ 30″ at a place in long. 20° 10′ 30″ E, the sun bearing north of the observer. The instrument had no index correction. Determine the latitude of the observer.

Observed alt.	41° 10′ 30″	L.A.T.	$12^h\ 00^m\ 00^s$
Refr.	−1 05	Long. E	1 20 42
Difference	41 09 25	G.A.T.	10 39 18
s.d.	+15 46	Eq. T.	−05 30.1 (add)
Sum	41 25 11	G.C.T.	10 44 48.1 = $10^h.75$
Parallax	+07		
Corrected alt.	41 25 18	Decl. at 10^h G.C.T.	+21° 55′.9
Decl.	21 55 38	12^h G.C.T.	+21 55 .2
Colat.	63 20 56	Hourly difference	0 .35
Lat.	26° 39′ 04″ S		
		Corrected decl.	+21° 55′ 38″

To the nearest ½ minute the latitude is 26° 39′ 00″ S.

Alternative solutions for the G.C.T. used in determining the sun's declination in Examples 1 and 2 are shown below. They involve the use of the G.H.A. method and consist of applications of Eqs. 49 and 48 respectively.

Example 1. January 19, 1947		Example 2. July 13, 1947	
Sun's G.H.A. = long. W		Sun's G.H.A. = −long. E	
Sun's G.H.A.	74° 10′.0	G.H.A. = −20° 10′.5 =	339° 49′.5
G.H.A. at 16^h	57 19 .5	G.H.A. at 10^h	328 37 .5
Remainder	16 50 .5	Remainder	11 12 .0
Correction $01^h\ 07^m$	16 45 .0	Correction 44^m	11 00 .0
Remainder	05. 5	Remainder	12 .0
Correction 22^s	05 .5	Correction 48^s	12 .0
G.C.T.	$17^h\ 07^m\ 22^s$	G.C.T.	$10^h\ 44^m\ 48^s .0$

While all problems involving the determination of latitude from the meridian altitude of a heavenly body may be solved by the application of Eq. 1, the uncomprehending use of a formula may be conducive to error. In the solution of problems such as those illustrated previously the student has no prior knowledge of the position. He does not know whether the observation was made in the northern or southern hemisphere. While the practising engineer usually has a very close idea of his position, both the student and the engineer will find that a sketch analysis as described below will aid materially in the visualization of the problem. If carefully followed, errors should be eliminated. Draw a sketch of the visible meridian section of the celestial sphere as shown in Fig. 76. Mark the north and south points

of the horizon and the zenith by the letters *N*, *S*, and *Z*. Next show the heavenly body, marked *A*, at the proper altitude above the north point of the horizon if the body is north of the zenith, otherwise above the south point of the horizon. Then determine from the *Epheremis* or *Almanac* the declination of the body. If the declination is positive the body is north of the equator, and the position of the equator *E* should be plotted at an angle equal to the declination toward the south point of the horizon. When

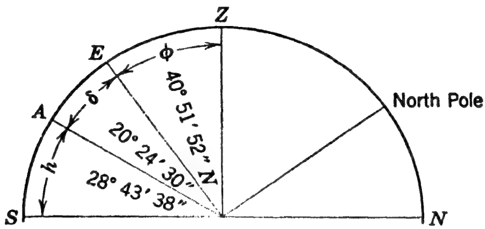


FIG. 76. Latitude by Meridian Transit of Sun, Example 1

the declination is negative the equator should be plotted north of the body. A protractor need not be used in plotting the angles but sufficient care should be taken so that the equator and the zenith will show in their correct relative positions. The elevated pole may then be shown at right angles to the equator. This will be particularly helpful in solving an observation on a circumpolar star. We now have a diagram which has been prepared in a logical sequence, showing, in order, the north and south points of the horizon, the zenith, the body, and, finally, the equator and the elevated pole in their proper relative positions. The solution will be evident at once, and there will be no question whether the zenith is north or south of the equator.

Thus Fig. 76, *with points placed on the drawing in the order indicated,* shows the correct diagram for Example 1. *SA* is the altitude of the sun and *AE* is the declination. *EZ*, the latitude, is equal to 90° minus the sum of the altitude and declination (without regard to sign). The figure in-

dicates that the zenith is north of the equator and that the latitude is therefore north. Figure 77, prepared in the same manner, shows clearly the situation in Example 2 and indicates the south latitude in which the observation was made. Such sketches will apply equally well to stars near the equator as covered in the next article. When applied

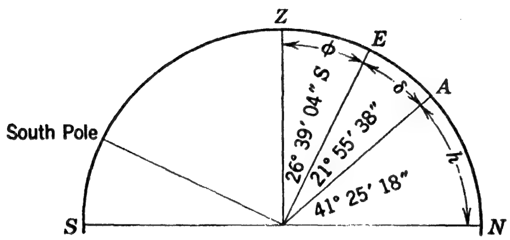


FIG. 77. Latitude by Meridian Transit of Sun, Example 2

to observations on circumpolar stars at either upper or lower culmination a slight change in procedure should be made. After plotting the position of the star, determine the polar distance by subtracting the declination from 90°. Then plot the elevated pole at this angle, toward the zenith if the star was observed at lower culmination and toward the horizon if the observation was at upper culmination. The equator may then be shown 90° from the pole, and the sketch will be complete.

87 Latitude by the Meridian Altitude of a Southern* Star

The latitude may be found from the observed maximum altitude of a star which culminates south of the zenith, by the method of the preceding article, except that the parallax and semidiameter corrections become zero and that it is not necessary to note the time of the observation since the declination of the star changes so slowly. In measuring the altitude, the star's image is bisected with the horizontal cross hair, and the maximum is found by trial as when

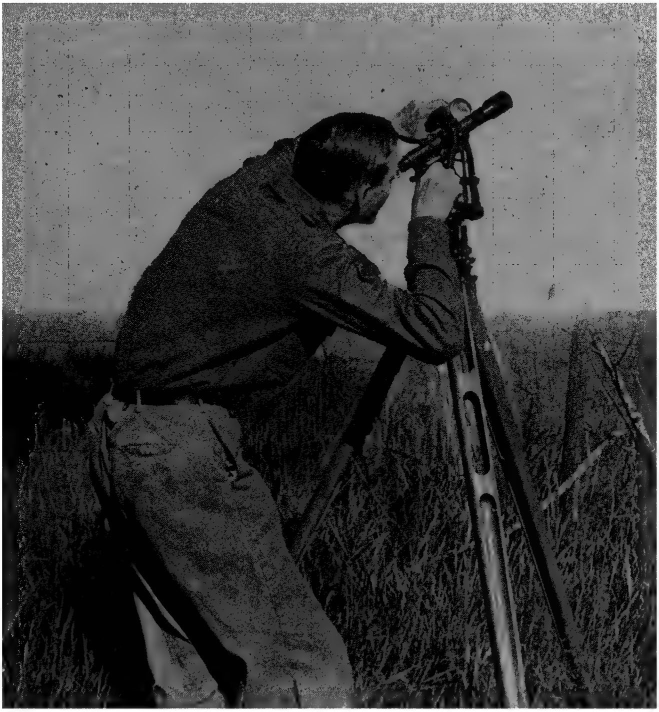


Fig. 78.

observing on the sun. For the method of finding the time at which a star will pass the meridian see Art. 92.

Example 1. The observed meridian altitude of *Spica* (α *Virginis*) was 38° 26′ 30″ on May 30, 1947, star bearing south; I.C. +30″. Determine the latitude.

Observed alt.	38° 26′ 30″	
IC.	+30	
Sum	38 27 00	
Refr.	−1 12	
Alt.	38 25 48	
Decl.	−10 53 18	From *Almanac*
Colat.	49 19 06	
Lat.	40° 40′ 54″ N	

The method applies equally well in the southern hemisphere with observations made on stars north of the zenith.

EXAMPLE 2. The observed meridian altitude of *Vega* (α *Lyrae*) was 34° 42′ 30″ on August 18, 1947, star bearing north; I.C. −30″. Determine the latitude.

Observed alt.	34° 42′ 30″	
I.C.	−30	
Difference	34 42 00	
Refr.	−1 22	
True alt.	34 40 38	
Decl.	+38 44 10	From *Almanac*
Colat.	73 24 48	
Lat.	16° 35′ 12″ S	

The student should make sketches for the above examples.

Constant errors in the measured altitudes may be eliminated by combining the results obtained from circumpolar stars with those from southern stars. An error which makes the latitude too great in one case will make it too small by the same amount in the other case.

88 Latitude by Circum-Meridian Altitudes

Latitude, from a single observed altitude of a star, is determined with maximum accuracy and simplicity of computation when the star is on the meridian. If, however, a series of altitudes are measured when the star is near the meridian and each altitude reduced by computation to the meridian altitude, the series will give a latitude determination upon which greater reliance can be placed than upon the result of a single measurement.

To derive formulae for making the reduction to the meridian we employ Eq. 8.

$$\sin h = \sin \phi \sin \delta + \cos \phi \cos \delta \cos t \tag{8}$$

Substituting $1 - 2 \sin^2 t/2$ for $\cos t$, we have

$$\sin h = \sin \phi \sin \delta + \cos \phi \cos \delta - 2 \cos \phi \cos \delta \sin^2 t/2$$

or

$$\sin h = \cos (\phi - \delta) - 2 \cos \phi \cos \delta \sin^2 t/2$$

Denoting the meridian altitude $90° - (\phi - \delta)$ by h_m and replacing $\cos (\phi - \delta)$ by $\sin h_m$, the equation may be written

$$\sin h_m = \sin h + 2 \cos \phi \cos \delta \sin^2 t/2 \tag{71}$$

If the time is noted when the altitude is measured the value of t may be computed, provided that the chronometer correction is known. With an approximate value of ϕ known or assumed the second term may be computed and the meridian altitude h_m found through its sine. If the latitude computed from h_m differs appreciably from the preliminary value used a second computation should be made, using the new value for the latitude. For a series of observations on a star, Eq. 71 will yield reliable results even when t is large since the declination of the star is substantially constant. For solar reductions, however, the equation should be restricted to observations made within a few minutes of culmination. When the sun's hour angle is large its declination at the instant of observation may differ appreciably from its declination at the instant of culmination. No provision has been made for the effect of any change in declination in the derivation of Eq. 71.

A more commonly used reduction formula may be derived by writing Eq. 71 in the form

$$\sin h_m - \sin h = 2 \cos \phi \cos \delta \sin^2 t/2$$

Since, by trigonometry,

$$\sin h_m - \sin h = 2 \cos \tfrac{1}{2}(h_m + h) \sin \tfrac{1}{2}(h_m - h)$$

we may write

$$\sin \tfrac{1}{2}(h_m - h) = \cos \phi \cos \delta \sin^2 t/2 \sec \tfrac{1}{2}(h_m + h) \tag{72}$$

Since $h_m - h$ is small, we may replace $\sin \frac{1}{2}(h_m - h)$ by $\frac{1}{2}(h_m - h) \sin 1''$; and also replace $\frac{1}{2}(h_m + h)$ by $h =$

$90° - \zeta$. Eq. 72 then becomes

$$h_m = h + \cos\phi \cos\delta \csc\zeta \, \frac{2\sin^2 t/2}{\sin 1''} \tag{73}$$

Placing $A = \cos\phi \cos\delta \csc\zeta$ and $m = \dfrac{2\sin^2 t/2}{\sin 1''}$

we have

$$h_m = h + Am^* \tag{74}$$

The latitude is then found by methods previously described.

When the observations are made within a few minutes of culmination the zenith distances will not vary greatly and the value of A may be taken as constant for a series of measured altitudes of the sun or of a star. Using an approximate value of ϕ and meridian values of δ and ζ, the corresponding value of A may normally be computed with sufficient precision by slide rule. Values of m will be found in Table X. The approximations made in deriving Eq. 74 introduce errors which become appreciable when the value of t is more than about 10 minutes of time.

In applying this method observations should be begun about 10 minutes before culmination and a series of altitudes measured in quick succession to give several on each side of the meridian. While it is not essential that the series be symmetrical with respect to the meridian, such symmetry is helpful in determining the approximate value of ϕ used in computing A. By inspection or by plotting the observed altitudes the maximum altitude may be determined closely and a very nice approximation of the latitude quickly computed. Where possible, the telescope should be reversed midway in the series. Refraction errors may be minimized by taking one series on a southern star and a second on a star at about the same altitude north of the zenith. If the altitudes are low and vary considerably individual refraction corrections should be applied to each,

* When a series of altitudes of a star is measured with the star near lower culmination the term Am is subtractive.

but where the change in altitude is insufficient to affect the refraction correction it is simpler to apply this quantity to the mean value of h_m. The watch is read at each pointing and the watch correction obtained from radio signals. The watch time of culmination is then computed, and the values of the hour angle t are determined by taking the differences between the time of transit and the times of observations. For solar observations these will be correctly given in solar units. For star observations these values of t *should be reduced to sidereal time intervals* by applying a Table III correction.

EXAMPLE 1. Determine the latitude from the following series of circum-meridian altitudes of *Nunki* (σ *Sagittarii*) on September 15, 1944, at a place in long. $4^h\ 56^m\ 45^s$ W, star bearing south.

Observation Number	Telescope	Vertical Circle	Watch
1	Direct	22° 53′ 45″	$08^h\ 00^m\ 20^s$ P.M.
2		54 45	02 42
3		55 15	04 27
4		55 30	06 01
5		55 45	07 38
6	Reversed	55 45	10 47
7		55 30	12 12
8		55 15	13 44
9		54 45	15 17
10		54 15	16 50

Radio signals indicate that the watch is 2^s slow on E.D.S.T. From the *Ephemeris* we obtain the right ascension and declination of *Nunki* as $18^h\ 51^m\ 49^s.5$ and $-26°\ 22'\ 01''$ respectively.

The maximum observed altitude is, by inspection, 22° 55′ 45″. From this we determine the approximate latitude and the value of A.

Observed h_m	22° 55′ 45″	$A = \cos\phi \cos\delta \csc\zeta$
Refr.	−2 15	
Approximate h_m	22 53 30	$= \dfrac{\cos 40°\ 44'.5 \cos 26°\ 22'}{\sin 67°\ 06'.5}$
Decl.	−26 22 01	
Colat.	49 15 31	
Lat.	40 44 29 N	
Zenith dist.	67° 06′ 30″	$= \dfrac{0.7577 \times 0.8960}{0.9212} = 0.737$

We then compute the watch time of culmination.

L.S.T.	$18^h\ 51^m\ 49^s.5$	R.A. star
Long. W	4 56 45	
G.S.T.	23 48 34 .5	
R.A.M.S. $+12^h$ at 0^h September 16	23 39 15 .7	
Sid. Int.	00 09 18 .8	
Table II	01 .5	
G.C.T.	00 09 17 .3	September 16
Long. W	04 00 00	
E.D.S.T.	08 09 17	P.M. September 15
Watch slow	02	
Watch time of transit	$08^h\ 09^m\ 15^s$	P.M. September 15

The watch time of transit may also be computed from the G.H.A. of the star as tabulated in the *Nautical Almanac*. Making use of Eq. 49 and recognizing that the L.H.A. will be zero at the instant of culmination we find the G.H.A. of the star to be equal to the west longitude. An inspection of the data indicates that the G.C.T. corresponding to the instant of local meridian transit will fall on September 16. We then have from the *Almanac*

G.H.A. *Nunki*	74° 11′.25	
G.H.A. at 0^h September 16	71 51 .5	
Remainder	02 19 .75	
Correction for 09^m	02 15 .4	
Remainder	04 .35	
Correction for $17^s.2$	04 .35	
G.C.T.	$00^h\ 09^m\ 17^s.2$	September 16

This substantially checks the value determined from the local sidereal time From this we proceed as before and obtain the watch time of transit as $08^h\ 09^m\ 15^s$ P.M. September 15.

The differences between the observed times and the watch time of culmination, with Table III corrections added, give values of t as indicated below. The remainder of the computation is shown in the same tabular form. Values of m are from Table X.

Observation	Observed h	t	A	m	Am	h_m
1	22°53′ 45″	8^m 56^s	0.737	156″.67	115″	22° 55′ 40″
2	54 45	6 34		84. 66	62	47
3	55 15	4 49		45 .55	34	49
4	55 30	3 15		20 .74	15	45
5	55 45	1 37		5 .13	4	49
6	55 45	1 32		4. 62	3	48
7	55 30	2 57		17 .08	13	43
8	55 15	4 30		39 .76	29	44
9	54 45	6 03		71 .86	53	38
10	54 15	7 36		113 .40	84	39
				Mean h_m		22° 55′ 44″
				Refr.		−2 15
				Alt.		22 53 29
				Decl.		−26 22 01
				Colat.		49 15 30
				Lat.		40° 44′ 30″ N

89 Latitude by Altitude of Polaris When the Time Is Known

The latitude may be found conveniently from an observed altitude of *Polaris* taken at any time provided that the watch correction is approximately known. *Polaris* is less than one degree from the pole, and small errors in the time have a relatively small effect upon the result. It is advisable to take several altitudes in quick succession and to note the time at each pointing on the star. Unless the observations extend over a long period, say more than 10 minutes of time, it will be sufficiently accurate to take the mean of the altitudes and the mean of the times and treat this as the result of a single observation. The telescope should be reversed midway in the series if possible, otherwise the index correction should be carefully determined and applied.

The hour angle t of the star must be computed for the instant of observation. This is done according to the methods given in Chapter 5. In the following example the watch is set to Eastern daylight saving time. This is

converted into local sidereal time (using the known longitude). The hour angle t is the difference between the sidereal time and the star's right ascension.

The local hour angle of *Polaris* may also be obtained directly in units of arc by changing the known Eastern daylight saving time to Greenwich civil time, then taking the Greenwich hour angle of *Polaris* corresponding to this instant from the *Nautical Almanac*, and finally subtracting the west longitude from the Greenwich hour angle to obtain the local hour angle of the star.

The latitude is computed by the formula

$$\phi = h - p \cos t + \tfrac{1}{2}p^2 \sin^2 t \tan h \sin 1'' \qquad (75)$$

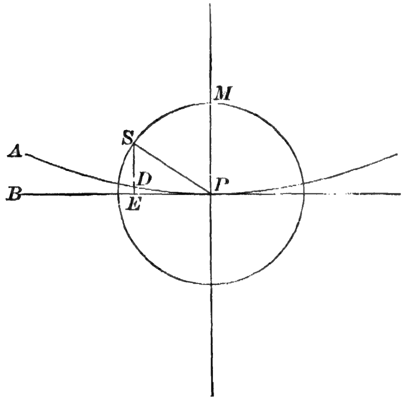


Fig. 79.

the polar distance p being in seconds. For the derivation of this formula see Chauvenet's *Spherical and Practical Astronomy*, Vol. I, p. 253.

In Fig. 79, P is the pole, S the star, MS the hour angle, and PDA the parallel of altitude through the pole. The point D is therefore at the same altitude as the pole. The term $p \cos t$ is approximately the distance from S to E, a point on the 6-hour circle PB. The distance desired is SD, the difference between the altitude of S and the alti-

tude of the pole. The last term of the formula represents very nearly this distance DE. When S is above the pole DE diminishes SE; when S is below the pole it increases it.

EXAMPLE 1. Determine the latitude of the observer from the following observation on *Polaris*, May 14, 1947, in long. 74° 11′ 10″ W. Radio signals indicate that the watch is 8^s fast on E.D.S.T.

Telescope	Vertical Angle	Watch
Direct	39° 52′ 30″	$09^h\ 08^m\ 06^s$ P.M.
	39 52 30	09 09 44
Reversed	39 52 00	09 11 30
	39 52 00	09 12 40
Means	39° 52′ 15″	$09^h\ 10^m\ 30^s$ P.M.

Watch	$09^h\ 10^m\ 30^s$ P.M.	From *Ephemeris*	
Watch fast	08	R.A. *Polaris*	$01^h\ 45^m\ 32^s.89$
E.D.S.T.	09 10 22 P.M.	Decl. *Polaris*	+89° 00′ 42″.94
$+12^h$	12 00 00	p = 00° 59′ 17″ = 3557″	
A.C.T.	21 10 22		
Long. W	04 00 00		
G.C.T.	01 10 22 May 15	Obs. alt.	39° 52′ 15″
R.A.M.S. $+12^h$	15 27 30 .88	Refr.	−01 08
Table III	11 .56	Alt.	39° 51′ 07″
G.S.T.	16 38 04 .44		
Long. W	04 56 44 .67		
L.S.T.	11 41 19 .77		
R.A. *Polaris*	01 45 32 .89		
H.A. *Polaris*	$09^h\ 55^m\ 46^s.88$ = 148° 56′ 43″		

$$\phi = h - p \cos t + \tfrac{1}{2} p^2 \sin^2 t \tan h \sin 1'' \tag{75}$$

Second Term		*Third Term*	
log p''	3.5510839	log ($\frac{1}{2}$ sin 1″)	4.3845 − 10
log cos t	9.9328181^n − 10	2 log sin t	9.4251 − 10
log 2nd term	3.4839000	log tan h	9.9215 − 10
2nd term	−3047″	2 log p''	7.1022
	= −00° 50′ 47″	log 3rd term	0.8333
		3rd term	+6″.81

Alt.	39° 51′ 07″
2nd term	+50 47
3rd term	+07
Lat.	40° 42′ 01″ N

The above computation may be greatly shortened by use of Tables I and I*a* of the *Ephemeris* or of Table I of the *Nautical Almanac*. Table I of the *Ephemeris* gives the total correction to be applied to the altitude for every 3^m of hour angle and for every 10″ of declination when the altitude of *Polaris* is 45°. For other altitudes a supplementary correction may be taken from Table I*a*. Thus, entering Table I with the declination of +89° 00′ 43″ and the hour angle of 09^h 55^m 47^s as arguments, we obtain by double interpolation the correction of +50′ 56″. Entering Table I*a* with the approximate altitude of 40° and the approximate hour angle of 10^h as arguments, we obtain a supplemental correction of −01″.

Table I of the *Nautical Almanac* gives the total correction for every 10^m of local sidereal time. Entering this table with the local sidereal time of 11^h 41^m 20^s as the argument we obtain a total correction of +50′ 34″. Because the *Nautical Almanac* table gives the correction to the nearest $\frac{1}{10}$ minute only, and because the table is based on average values of δ and α, the interpolated value will not, in general, check exactly with that obtained from the *Ephemeris* and will not be as accurate. The computations using these tables are tabulated below.

Alt.	39° 51′ 07″
Correction, Table I, *Ephemeris*	+50 56
	40 42 03
Correction, Table I*a*, *Ephemeris*	−01
Lat.	40° 42′ 02″ N

This substantially checks the result obtained by formula.

Alt.	39° 51′ 07″
Correction, Table I, *Nautical Almanac*	+50 34
Lat.	40° 41′ 41″ N

Questions and Problems

1. Observed maximum altitude of the sun's lower limb April 27 of a certain year is 61° 28′, bearing south; I.C. +30″; E.S.T. 11^h 42^m A.M. Sun's decl. April 27 at 0^h G.C.T. +13° 35′ 51″.3; the variation per hour +48″.19;

April 28, $+13° 55' 01''.0$; variation per hour $+47''.62$; s.d. $15' 55''.03$. Compute the latitude.

2. Observed maximum altitude of the sun's lower limb December 5 of of a certain year is $30° 10'$, bearing south; long. $73°$ W; Eq. T. $+09^m 22^s$. Sun's decl. December 5 at 0^h G.C.T. $-22° 16' 54''.0$; variation per hour $-19''.85$; December 6, $-22° 24' 37''.5$; variation per hour $-18''.77$; s.d. $16' 15''.84$. Compute the latitude.

3. The noon altitude of the sun's lower limb, observed at sea October 1 of a certain year is $40° 30' 20''$, bearing south; height of eye 30 feet; long. $35° 10'$ W; Eq. T. $+10^m 03^s.56$. Sun's decl. October 1 at 0^h G.C.T. $-2° 53' 38''.2$; variation per hour $-58''.28$; October 2, $-3° 16' 56''.1$; variation per hour $-58''.20$; s. d. $16' 00''.57$. Compute the latitude.

4. The observed meridian altitude of δ *Crateris* is $33° 24'$, bearing south; I.C. $+30''$; decl. of star $-14° 17' 37''$. Compute the latitude.

5. Observed (circum-meridian) altitude of α *Ceti* at $3^h 08^m 49^s$ L.S.T. $= 51° 21'$, bearing south; I.C. $-1'$; R.A. of α *Ceti* $2^h 57^m 24^s.8$; decl. $+3° 43' 22''$. Compute the latitude.

6. Observed altitude of *Polaris* $41° 41' 30''$; chronometer time $9^h 44^m 38^s.5$ (local sidereal); chronometer correction -34^s; R.A. $1^h 25^m 42^s$; decl. $+88° 49' 29''$. Compute the latitude.

7. Show by sketch the following three points: (1) *Polaris* at greatest elongation, (2) *Polaris* on the 6-hour circle, and (3) *Polaris* at the same altitude as the pole. (See Art. 89, p. 204, and Fig. 31, p. 54.)

8. Draw a sketch (like Fig. 21) showing why the sun's maximum altitude is not the same as the meridian altitude.

9. Observed altitude of *Polaris* at U.C. on a certain date $39° 31' 30''$; at L.C. on the same date obs. alt. $37° 23' 00''$; I.C. $00''$. Compute the latitude of the observer.

10. The following meridian observation on the sun for latitude was made with a sextant, using an artificial horizon, on April 7, 1909. I.C. $+1' 00''$; refr. and parallax correction, corrected for a temperature of $-25°$ F, $08' 20''$; corrected decl. of the sun $+6° 46' 17''$. The sun bore south of the zenith. Determine the latitude. Name the observer.

Limb	Observed Angle
Lower	$13° 18' 20''$
Upper	14 21 30
Lower	13 18 00
Upper	14 21 50

11. Observed altitude of β *Hydri* $48° 00' 00''$ at U.C. on a certain date, the star bearing south of the observer; I.C. $+1' 00''$; decl. $-77° 35' 10''$. Determine the latitude.

In the following problems take the date as being for the current year.

Take all necessary data not given from the current *Ephemeris* or *Nautical Almanac.*

12. Compute the latitude from the following observation on *Polaris* at L.C., May 16. Obs. alt. 39° 42′; I.C. 0′.

13. Observed maximum altitude of the sun's lower limb 48° 16′ 30″ on June 30, sun bearing north; long. $2^h\ 10^m\ 30^s$ E; I.C. +30″. Compute the latitude.

14. Observed meridian altitude of *α Virginis* 42° 16′ on June 10, star bearing south; I.C. +1′. Compute the latitude.

15. Observed meridian altitude of *α Argus* 37° 21′ 00″ on December 1, star bearing south; I.C. +30″. Determine the latitude.

16. Observed meridian altitude of *α Eridani* 44° 13′ on August 1, star bearing south; I.C. 0. Compute the latitude.

17. Determine the latitude from the following observation on *Polaris,* the time being known. Check result by the use of Table I of the *Ephemeris.* Obs. alt. 43° 28′. Data: December 2; long. $4^h\ 54^m\ 20^s$ W; E.S.T. of observation $9^h\ 54^m\ 20^s$ P.M.; I.C. −1′.

11

Observations for Time and Longitude

90 Time and Longitude

It has been pointed out in Arts. 16 and 31 that time and longitude are closely interrelated and that the difference in the local times between two places is equal to the difference in their longitudes. If an observer knows the longitude of his station he can, from observations for local time, determine the chronometer or watch correction. Conversely, if the correction to reduce the watch time to that of some standard meridian is obtained from radio time signals or by other means, observations for local time permit the determination of the local longitude. Because of this intimate relationship, the two types of observations are considered in the same chapter.

Since no natural circle of reference, such as is evidenced by the relation of latitude to the equator, is implied by the nature of time and longitude it becomes necessary to select an arbitrary initial circle. In consequence, the meridian passing through the observatory at Greenwich, England, has been almost universally adopted as the reference meridian for both time and longitude. Thus we often refer to Greenwich civil time as Universal time, and we base our longitudes on that of Greenwich.

91 Observations for Local Time

Observations for determining the local time at any place at any instant usually consist in finding the error of a timepiece on the kind of time which it is supposed to keep. To find the solar time it is necessary to determine the hour angle of the sun's center. To find the sidereal time the hour angle of the vernal equinox must be measured. Sometimes these quantities cannot be measured directly so that it is often necessary to measure other coordinates and to calculate the desired hour angle from these measurements. The *chronometer correction* or *watch correction* is the amount to be added algebraically to the reading of the timepiece to give the true time at the instant. It is *positive* when the chronometer is *slow* and *negative* when it is *fast*. The *rate* is the amount the timepiece gains or loses per day; it is positive when it is losing and negative when it is gaining.

92 Time by Transit of a Star — General Method

The most direct and simple means of determining the time is by observing transits of apparently rapidly moving stars across the meridian. If the line of sight of a transit is placed so as to revolve in the plane of the meridian and if the instant is noted when some known star crosses the vertical cross hair, then, since $S = \alpha + t$ (Eq. 38) and, in this case, $t = 0$, the local sidereal time is the same as the right ascension of the star given in the *Ephemeris* for the date (see Art. 38). If the chronometer is supposedly keeping Greenwich sidereal time it is necessary to apply only the local longitude to the right ascension of the star to obtain the true Greenwich sidereal time; the chronometer correction is then the difference between this true sidereal time and the chronometer reading at the instant of observation. If the timepiece keeps mean solar time for a standard meridian we convert the local sidereal time (right ascension of the star) into standard time by the method of Art. 41. The

watch correction is then the difference between the true standard time and the watch time of observation.

The application of this method obviously requires a knowledge of the local longitude and a previous determination of the direction of the meridian (see Chapter 12 for azimuth observations). An approximate value of the latitude should also be known, to permit the telescope to be set at the proper altitude to bring the star in the field of view at the time of culmination. For reasons which will be discussed in the next article the method is ill-adapted for use in high latitudes.

A more detailed description of the method of making such observations will follow a discussion of the selection of the stars to be observed.

93 Selecting Stars for Transit Observations

Before work in the field is begun the observer should prepare a list of stars suitable for transit observations. This list should include the name or number of each star, its magnitude, the approximate time of transit, and its meridian altitude or its zenith distance. The right ascensions of consecutive stars in the list should differ by sufficient intervals to give the observer time to make and record an observation and prepare for the next one.

The stars selected should preferably have small declinations for, given an accurately established meridian and a well adjusted instrument, the precision of transit observations for the determination of time depends primarily upon the apparent diurnal rates of travel of the stars chosen. It is clear that the more rapid the apparent motion of the star the more precise will be the determination of the instant of transit. Since the apparent rates of travel of stars vary with the cosines of their declinations, the equatorial stars will yield the best results, and the close circumpolars will be the most unsatisfactory.

Irrespective of the stars selected, the observer in polar

regions will encounter greater difficulty in obtaining precise results than one in an intermediate or low latitude. As we go poleward the accuracy of the usual methods of establishing the meridian diminishes. Thus the observer in a high latitude, seeking to minimize the effect of azimuth errors, may select stars transiting near his zenith, only to find the precision of his results reduced by the slow motion of these close circumpolars. If, on the other hand, the observer choses stars nearer to the equator, the apparent gain in precision obtained from their more rapid movement is likely to be offset in large measure by the uncertainty in azimuth.

Very faint stars should not be selected unless the telescope is of high power and good definition; those smaller than the fifth magnitude are rather difficult to observe with a small transit, especially as it is difficult to maintain the illumination at the point where both the cross hairs and the star may be seen.

The selection of stars will also be governed somewhat by a consideration of the effect of the different instrumental errors (see Art. 74). For stars near the zenith the azimuth error approaches zero, while the inclination error is a maximum; for stars near the horizon the azimuth error is a maximum, and the inclination error is a minimum. If the azimuth of the instrument is uncertain and the inclination can be accurately determined, stars having high altitudes should be preferred. On the other hand, if the level parallel to the axis is not a sensitive one and is in poor adjustment and if the sight line can be placed accurately in the meridian, which is usually the case with an engineer's transit, low stars will give the more accurate results. With the engineer's transit the choice of stars is somewhat limited, however, because it is not practicable to sight the telescope at much greater altitudes than 70° with the use of the prismatic eyepiece and 50° to 55° without this attachment.

Following is a sample list of stars selected for observations

in latitude 40° 43′ 24″ N, longitude 74° 17′ 20″ W, on June 2, 1947, between the hours of 9^h and 11^h P.M., Eastern daylight saving time. Because of the presence at low altitudes of smoke, haze, and other conditions unfavorable to observations, a minimum altitude of 20° has been selected. The maximum altitude without a prismatic eyepiece has been taken as 55°. The minimum time interval between star culminations has been set at $2^m.5$.

We first determine the local sidereal time corresponding to the hour set for beginning observations.

E.D.S.T.	09^h	00^m	00^s	P.M. June 2, 1947
G.C.T.	01	00	00	June 3, 1947
R.A.M.S. + 12^h at Greenwich 0^h 6/3/47	16	42	25	
Table III			10	
G.S.T.	17	42	35	
Long. W	04	57	09	
L.S.T.	12^h	45^m	26^s	

The right ascension of that star which would transit the local meridian at exactly 9^h P.M., Eastern daylight saving time, will therefore be $12^h\,45^m\,26^s$. At 11^h P.M., the corresponding local sidereal time will be $14^h\,45^m\,46^s$ since sidereal time units show an increase of about 10^s per hour of solar time. Stars selected must therefore fall within the above limits of right ascension.

We next determine the limiting declinations. The colatitude, to the nearest minute, is 49° 17′. This is the meridian altitude of a star on the equator. For the limiting altitudes of 20° and 55° this gives a declination range from −29° 17′ to +05° 43′.*

With these data we enter the table of "Mean Places of Stars" in the 1947 *Ephemeris*. In searching for stars we examine the right ascension first. Since the stars are arranged in the list in the order of increasing right ascension it is necessary to find only the right ascension for the

* Corrections for refraction have not been included since values based on these figures serve solely as *finder angles*.

time of beginning the observations and then follow down the list. We next check off those stars whose declinations fall within the prescribed limits. We finally note the magnitudes, rejecting those stars of undesirable magnitude for observing with the instrument at hand.

Following this procedure, we find twenty-one stars transiting during the hours specified. Of these, only four fall within the declination range set. All of these are of suitable magnitude. The data for these four stars from this table are tabulated below.

Catalogue Number	Star	Magnitude	Right Ascension	Declination
495	γ *Hydrae*	3.33	$13^h\ 16^m\ 02^s.10$	$-22°\ 53'32''.9$
498	α *Virginis*	1.21	13 22 23 .83	−10 53 07 .0
501	ζ *Virginis*	3.44	13 31 59 .39	−00 19 32 .3
519	π *Hydrae*	3.48	14 03 20 .80	−26 25 40 .5

If the list of " Mean Places of Stars " referred to is consulted it will be noticed that γ *Virginis* transits about $6\frac{1}{2}$ minutes before 9^h P.M. and that α^2 *Librae* transits about 2 minutes past 11^h P.M. These might be included to increase the number of observations. Recent editions of the *Ephemeris* include too few stars for an extended series of time observations. To prepare a more complete list use the annual volume, *Apparent Places of Fundamental Stars,* published by the British Admiralty.

After selecting the stars to be observed the approximate watch times of transit must be determined. Since we have computed the right ascension of a star which would be on the meridian at 9^h P.M. to be $12^h\ 45^m\ 26^s$, the differences between this value and the right ascensions of the stars listed may be added to 9^h P.M. to give the respective times of transit. Strictly speaking, the right ascensions for the given date should be used and a Table II correction applied to give a mean time interval. These refinements are un-

necessary since presumably the watch has an error, and we need the time only roughly so that we may know about when to look for the star through the telescope. The approximate altitudes at culmination are then found from the known latitude and the tabulated declinations. In this computation it is unnecessary to apply a refraction correction since it is necessary only to know the altitude to within about 5′ to be sure the star will appear in the field of view. The data required in the field are tabulated below. The magnitude is included to give an idea of the brightness of the star, since occasionally more than one star may be in the field of view at a time.

Catalogue Number	Star	Magnitude	Approximate E.D.S.T.	Approximate Altitude
495	γ *Hydrae*	3.33	$09^h\ 30^m\ 36^s$ P.M.	26° 23′
498	α *Virginis*	1.21	09 36 58	38 23
501	ζ *Virginis*	3.44	09 46 33	48 57
519	π *Hydrae*	3.48	10 17 55	22 51

We next look in the table of " Apparent Places of Stars " to obtain the right ascensions for the date. These may be obtained by simple interpolation between the values given for every 10 days. The mean places given above may be in error for any particular date by several seconds. With the correct right ascensions known the Eastern daylight saving time of transit may be computed by the method of Art. 41 and the observed watch time of transit subtracted from this to give the watch correction. If the declinations are also taken from the apparent place table they may be combined with the observed altitudes, which have been corrected for refraction, to check the latitude as explained in Chapter 10. This necessitates placing the image of the star on the horizontal cross hair immediately after culmination and reading and recording the altitude as well as the watch time of transit.

94 Time by Transit of a Star — Observing Procedure

With the star list prepared the transit should be set up over the station as indicated in Art. 73. It is important that the horizontal axis be accurately leveled; the plate level which is parallel to this axis should be adjusted and centered carefully or else a striding level should be used. The vertical cross hair is then sighted on a meridian mark which has been previously established. If the instrument is in adjustment the sight line will then define the plane of the meridian as the telescope is rotated about its horizontal axis. Any errors in the adjustment will be eliminated if the instrument is used in both the direct and reversed positions, provided that the altitudes of the stars observed in the two positions are equal. With a more complete star list than is provided by the present *Ephemeris* it is usually possible to select stars whose altitudes are so nearly equal that the elimination of errors will be practically complete.

The approximate altitude of the first star on the list is then laid off. In the engineer's transit the field of view is usually about 1° so that a star near the equator will be seen in the field about 2^m before it reaches the vertical cross hair. About this time before the computed time of transit the observer begins to look for the star in the field. If more than one star is visible in the field, a knowledge of the magnitude will usually enable the observer to choose the correct star. Should the wrong star be observed the watch correction will be in error. This will be evident if several stars are used in the series of observations and the erroneous value may be discarded.

Near culmination the star's path is so nearly horizontal that it will appear to be parallel to the horizontal cross hair from one side of the field to the other. It is best to place the image just above or below this cross hair. When the star passes the vertical cross hair the time should be noted as accurately as possible. A stop watch is convenient for use in conjunction with the pocket watch when these ob-

servations are made. When the chronometer is used the "eye and ear method" is the best (see Art. 73). If it is desired to determine the latitude from this same observation, the observer has only to set the horizontal cross hair on the star immediately after making the time observation, and the reading of the vertical arc will give the star's apparent altitude at culmination (see Art. 87).

Example. In this example the watch correction has been computed from observations on each of the stars in the list which was prepared in Art. 93 and the mean watch correction determined from the series. The positions from the apparent place tables for the given local date, June 2, 1947 (G.C.T. = June $3^d.04$), are as follows:

Catalogue Number	Star	Right Ascension	Declination
495	*γ Hydrae*	$13^h\ 16^m\ 03^s.20$	−22° 53′ 45″.0
498	*α Virginis*	13 22 24 .95	−10 53 15 .9
501	*ζ Virginis*	13 32 00 .58	−00 19 38 .5
519	*π Hydrae*	14 03 22 .16	−26 25 53 .4

The results of the field measurements are tabulated below. The index correction of the instrument was 00′. All stars observed were south of the zenith.

Star	Watch Time of Transit	Observed Altitude
γ Hydrae	$09^h\ 30^m\ 30^s$ P.M. (E.D.S.T.)	26° 26′
α Virginis	09 36 51	38 26
ζ Virginis	09 46 25	48 59
π Hydrae	10 17 41	22 54

The watch corrections are then computed, using the right ascension of the star as the local sidereal time in each instance.

Star	*γ Hydrae*	*α Virginis*	*ζ Virginis*	*π Hydrae*
L.S.T.	$13^h\ 16^m\ 03^s.20$	$13^h\ 22^m\ 24^s.95$	$13^h\ 32^m\ 00^s.58$	$14^h\ 03^m\ 22^s.16$
Long. W	04 57 09 .33	04 57 09 .33	04 57 09 .33	04 57 09 .33
G.S.T.	18 13 12 .53	18 19 34 .28	18 29 09 .91	19 00 31 .49
R.A.M.S. + 12^h	16 42 25 .45	16 42 25 .45	16 42 25 .45	16 42 25 .45 June 3
Sid. int.	01 30 47 .08	01 37 08 .83	01 46 44 .46	02 18 06 .04
Table II	14 .87	15 .92	17 .49	22 .62
G.C.T.	01 30 32 .21	01 36 52 .91	01 46 26 .97	02 17 43 .43 June 3
E.D.S.T.	09 30 32 .2	09 36 52 .9	09 46 27 .0	10 17 43 .4 P.M.
Watch	09 30 30	09 36 51	09 46 25	10 17 41 June 2
Watch correction	$+2^s.2$	$+1^s.9$	$+2^s.0$	$+2^s.4$
Mean (watch slow)				$+2^s.1$

The computations for the latitude check, as made from the recorded altitudes, follow:

Star	γ *Hydrae*	α *Virginis*	ζ *Virginis*	π *Hydrae*
Obs. alt.	26° 26′ 00″	38° 26′ 00″	48° 59′ 00″	22° 54′ 00″
Refr.	01 55	01 12	50	02 15
Alt.	26 24 05	38 24 48	48 58 10	22 51 45
Decl.	−22 52 27	−10 51 58	−00 18 22	−26 24 38
Colat.	49 16 32	49 16 46	49 16 32	49 16 23
Lat.	40° 43′ 28″ N	40° 43′ 14″ N	40° 43′ 28″ N	40° 43′ 37″ N
Mean lat.				40° 43′ 27″ N

95 Time by Transit of Sun

The apparent solar time may be determined directly by observing the watch times when the west and the east limbs of the sun cross the meridian. The mean of the two readings is the watch time of the instant of local apparent noon, or 12^h apparent time. This 12^h is to be converted into local civil time and then into standard time. If only one edge of the sun's disc can be observed the time of transit of the center may be found by adding or subtracting the *time of semidiameter passing the meridian.* This is given in the *Ephemeris* for Washington apparent noon. The tabulated values are in sidereal time but may be reduced to mean time by applying a Table II correction.

Example. Determine the watch correction from the following observation on the sun at culmination, June 3, 1947, in long. $4^h\,44^m\,22^s$ W. The observed time of transit of the east limb of the sun was $12^h\,43^m\,25^s$ P.M. approximate E.D.S.T.

s.t.s.d.p.	$01^m\,08^s.48$
Table II correction	$-00\ .19$
Mean solar time of s.d.p.	$01^m\,08^s.29$
Watch time of transit of east limb	$12^h\,43^m\,25^s$ P.M.
Mean solar time of s.d.p.	$-01\ \ 08\ .3$
Watch time of transit of sun's center	$12^h\,42^m\,16^s.7$ P.M.
L.A.T.	$12^h\,00^m\,00^s.0$
Long. W	$4\ \ 44\ \ 22$
G.A.T.	$16^h\,44^m\,22^s$

G.A.T.	$16^h\ 44^m\ 22^s$
Eq. T.	+02 04 .8
G.C.T.	16 42 17 .2
Long. W	4 00 00 .0
E.D.S.T.	12 42 17 .2 P.M.
Watch time of transit	12 42 16 .7
Watch correction (slow)	$+0^s.5$

96 Time by an Altitude of the Sun

The apparent solar time may be determined by measuring the altitude of the sun when it is not near the meridian and then solving the astronomical triangle for the angle at the pole, which is the hour angle of the sun east or west of the meridian. The west hour angle of the sun is the local apparent time since noon. An east hour angle must be subtracted from 12^h to obtain the local apparent time since midnight. The observation is made by measuring several altitudes in quick succession and noting the corresponding instants of time. The mean of the observed altitudes is assumed to correspond to the mean of the observed times; that is, the curvature of the path of the sun is neglected. The error caused by neglecting the correction for curvature is very small provided the sun is not near the meridian and the series of observations extends over only a few minutes of time, say 10^m. The measurement of altitude must, of course, be made to the upper or the lower limb and a correction applied for the semidiameter. The observations may be made in two sets, half the altitudes being taken on the upper limb and half on the lower limb, in which case no semidiameter correction is required. The telescope should be reversed between the two sets if the instrument has a complete vertical circle. The reader should refer to Art. 73 for a discussion of solar pointings. The mean of the altitudes must be corrected for index error, refraction, and parallax, and for semidiameter if only one limb is observed. The declination at 0^h Greenwich civil time on the date in question must be corrected

to obtain the declination of the sun at the instant of observation. If the watch used is keeping standard time the Greenwich time of observation is found at once (see Art. 33) and the declination readily corrected for the interval since 0^h. Provided that the watch is not more than 2^m or 3^m in error the resulting error in computing the declination will not exceed $2''$ or $3''$, which is usually negligible in observations with small instruments. If the standard time is not known (watch seriously in error) but the longitude is known, the Greenwich time could be computed if the local time were known. Since the local time is the quantity sought the only way of obtaining it is first to compute the hour angle t using an approximate value of the declination. From this result an approximate value of the Greenwich civil time may be computed. The declination may now be computed more accurately. A recomputation of the hour angle t, using this new value of the declination, may be considered final unless the declination used the first time was very much in error.

In order to compute the hour angle, the latitude of the place must be known. This may be obtained from a reliable map or may be observed by the methods of Chapter 10. The precision with which the latitude must be known depends upon how precisely the altitudes are to be read and also upon the time at which the observation is made. When the sun is near the prime vertical the effect of an error in the latitude is small.

The value of the hour angle is computed by applying any of the formulae for t in Art. 20. This hour angle is converted into time units; if the sun is west of the meridian this is the local apparent time since noon, but if the sun is east of the meridian this time interval is to be subtracted from 12^h to obtain the local apparent time. This apparent time is next converted into mean (civil) time by applying the (corrected) equation of time or by use of the Greenwich hour angle method (Art. 42). The local or the Greenwich

time is then converted into standard time by means of the longitude difference. The difference between the computed time and the time read on the watch is the watch correction. This observation is often combined with the observation on the sun for azimuth, the watch readings and altitude readings being common to both.

EXAMPLE. Determine the watch correction from the following solar observation for time and azimuth at N.J.G.C.S. Monument 501, June 16, 1947; lat. 40° 43′ 21″ N, long. 74° 17′ 49″ W.*

Object Observed	Telescope	Horizontal Angle	Vertical Angle	Watch Time (E.D.S.T.)
Monument 500	*D*	000° 00′		
		150 57	52° 23′	03h 38m 47s P.M.
☉|		(15′)	(13′)	(71s)
		151 12	52 10	03 39 58
		(19)	(16)	(86)
		151 31	51 54	03 41 24
|☉	*R*	151 15	50 53	03 44 07
		(17)	(15)	(80)
		151 32	50 38	03 45 27
		(16)	(14)	(73)
		151 48	50 24	03 46 40
Monument 500		000 00		
Means		151° 22′ 30″	51° 23′ 40″	03h 42m 43s.8

The differences between observed values are tabulated in parentheses. The proportionality of the results is evident.

Approximate E.D.S.T.	03h 42m 44s P.M.	Obs. alt.	51° 23′ 40″
12h	12	Refr.	−46
Approximate A.C.T.	15 42 44		51 22 54
Long. W	04 00 00	Parallax	+06
Approximate G.C.T.	19 42 44	Alt.	51 23 00
	= 19h.712		

Decl. at Greenwich 0h	+ 23° 18′ 42″.9
Correction	+ 01 57 .5
Decl.	+ 23 20 40

* The azimuth computation for this observation will be found in Art. 116, Chapter 12.

Polar distance	66° 39′ 20″	$s - p = 12° 43′ 30″$
Lat.	40 43 21	$s - \phi = 38\ 39\ 29$
Alt.	51 23 00	$s - h = 27\ 59\ 50$
$2s$	158 45 41	
s	79 22 50	

For a check, the hour angle has been computed from Eq. 19, using logarithms, and from Eq. 20, using natural tables and a calculating machine. The figures follow.

$$\tan \tfrac{1}{2}t = \sqrt{\frac{\cos s \sin (s - h)}{\cos (s - p) \sin (s - \phi)}}$$

$$\cos t = \frac{\sin h - \sin \phi \sin \delta}{\cos \phi \cos \delta}$$

log cos s	9.2654898 − 10	sin h		0.78133896
log sin $(s - h)$	9.6715697 − 10			
log sec $(s - p)$	0.0108001	sin ϕ	0.65239607	
log csc $(s - \phi)$	0.2043485	sin δ	0.39625782	
	2)19.1522081 − 20	product		0.25851704
log tan $\frac{1}{2}t$	9.5761040 − 10	numerator		0.52282192
$\frac{1}{2}t$	20° 38′ 46″	cos ϕ	0.75787820	
t	41° 17′ 32″	cos δ	0.91813928	
t	$02^h\ 45^m\ 10^s.1$	product		0.69583774
	(since noon)	cos t		0.75135609
		t		41° 17′ 31″

L.A.T.	$14^h\ 45^m\ 10^s.1$
Long. W	04 57 11 .3
G.A.T.	19 42 21 .4
Eq. T.	−26 .4
G.C.T.	19 42 47 .8
E.D.S.T.	03 42 47 .8 P.M.
Watch	03 42 43 .8
Watch correction	$+04^s.0$ (slow)

The most favorable conditions for an accurate determination of time by this method are when the sun is on the prime vertical and the observer is on the equator. When the sun is in the east or west it is rising or falling at its most rapid rate and an error in the altitude produces less error in the calculated hour angle than the same error would produce if the sun were near the meridian. The nearer the observer is to the equator the greater is the inclination of the sun's

path to the horizon and consequently the greater its rise or fall per second of time. If the observer were at the equator and the declination were zero the sun would rise or fall 1′ in 4^s of time. In the preceding example the fall is 1′ in about $5^s.3$ of time. When the observer is near the pole the method is practically useless.

Observations on the sun when it is very close to the horizon, however, should be avoided, even when the sun is near the prime vertical, because the difference between the actual and the tabulated refraction correction, resulting from variations in temperature and pressure of the air, is likely to be large. Observations should not be made when the altitude is less than 10° if it can be avoided.

97 Time by the Altitude of a Star

The method of the preceding article may be applied equally well to an observation on a star. In this case the parallax and semidiameter corrections are zero. If the star is west of the meridian the computed hour angle is the star's true hour angle; if the star is east of the meridian the computed hour angle must be subtracted from 24^h. The sidereal time is then found by adding the right ascension of the star to its hour angle (see Art. 37). If mean time is desired the sidereal time thus found is to be converted into mean solar time by the method of Art. 41. Since it is easy to select stars in almost any position it is desirable to eliminate errors in the measured altitudes by taking two observations, one on a star which is nearly due east, the other on one about due west. The mean of these two results will be nearly free from instrumental errors and also from errors in the assumed value of the observer's latitude. Planets may be used instead of stars in this observation, but in this case it will be necessary to know the Greenwich civil time with sufficient accuracy for correcting the tabulated values of the right ascension and declination of the planet.

EXAMPLE. Determine the watch correction from the following observation on *Pollux* (β *Geminorum*), February 1, 1947, star in east. Lat. 40° 43′ 40″ N, long. $4^h\ 56^m\ 15^s$ W; obs. alt. 28° 01′ 30″; I.C. −30″; obs. time $05^h\ 49^m\ 06^s$ P.M. approximate E.S.T.

From the *Ephemeris* we find the right ascension and declination of *Pollux* to be $07^h\ 42^m\ 05^s.2$ and +28° 09′ 23″ respectively. R.A.M.S. + 12^h at Greenwich 0^h on this date is $08^h\ 41^m\ 25^s.8$.

$$\cos t = \frac{\sin h - \sin \phi \sin \delta}{\cos \phi \cos \delta} \tag{20}$$

Obs. alt.	28° 01′ 30″		
Index	−30		
	28 01 00		
Refr.	01 47		
h	27 59 13	sin h 0.46927036	
ϕ	40 43 40	sin ϕ 0.65246588	cos ϕ 0.75781810
δ	28 09 23	sin δ 0.47187982	cos δ 0.88166288

$$\cos t = \frac{0.46927036 - 0.65246588 \times 0.47187982}{0.75781810 \times 0.88166288} = 0.24154348$$

$$t = 76° \ 01' \ 20'' \text{ E} = 05^h\ 04^m\ 05^s.3$$

H.A. of *Pollux*	$18^h\ 55^m\ 54^s.7$
R.A. of *Pollux*	07 42 05 .2
L.S.T.	26 37 59 .9
Long. W	04 56 15
G.S.T.	31 34 14 .9
R.A.M.S. + 12^h	08 41 25 .8
Sid. int.	22 52 49 .1
Table II	03 44 .9
G.C.T.	22 49 04 .2
E.S.T.	05 49 04 .2 P.M.
Watch	05 49 06
Watch correction	$-1^s.8$ (fast)

98 Effect of Errors in Altitude and Latitude upon Time Determinations

In determining time by the preceding methods involving the measurement of the altitude of the sun or of a star, an error in the computed local hour angle may result from errors in the measured altitude or in the value accepted for the observer's latitude. The magnitude of the resulting error will depend not only upon the magnitude of each

contributing error but also upon the position of the celestial body and the latitudinal position of the observer.

In order to determine the exact effect upon t of any error in the altitude h let us differentiate Eq. 8 with respect to h, the quantities ϕ and δ being regarded as constant.

$$\sin h = \sin \phi \sin \delta + \cos \phi \cos \delta \cos t \tag{8}$$

Differentiating, $\cos h = 0 - \cos \phi \cos \delta \sin t \, dt/dh$ from which

$$\frac{dt}{dh} = -\frac{\cos h}{\cos \phi \cos \delta \sin t}$$

$$= -\frac{1}{\cos \phi \sin Z} \text{ (by Eq. 12)} \tag{76}$$

An inspection of this equation shows that when $Z = 90°$ or $270°$ $\sin Z$ is a maximum, and dt/dh is a minimum for any given value of ϕ. It also shows that the smaller the latitude the greater is its cosine and consequently the smaller the value of dt/dh. It is clear that an error in altitude will have the least effect on the hour angle when the body is on the prime vertical and the observer is on the equator. The negative sign shows that the hour angle decreases as the altitude increases. When Z is zero (body on meridian) or when ϕ is $90°$ (observer at pole), the value of dt/dh is infinite, and t cannot be found from the observed altitude.

If we now substitute the value of $\sin Z$ from Eq. 15 in Eq. 76, we have

$$\frac{dt}{dh} = -\frac{1}{\cos \delta \sin S} \tag{77}$$

This equation indicates that the nearer the parallactic angle S is to $90°$ the smaller will be the effect on t of an error in h. While S is $90°$ for any star at elongation, the equation further shows that the error will be least for a star having a declination of $0°$. Close circumpolar stars are therefore unsatisfactory for time observations even when

at or near elongation. Theoretically a star having declination zero, observed when on the prime vertical ($S = 90°$ in this case), would be most suitable. The altitude would also be zero, however, and refraction would be most uncertain. Equatorial stars should, therefore, be observed when far enough from the prime vertical to have sufficient altitude so that uncertainties in the refraction correction will be too small to have an important effect on the accuracy of result.

The effect of an error in the latitude may be found by differentiating Eq. 8 with respect to ϕ. The result is

$$0 = \cos\phi \sin\delta + \cos\delta\left(-\cos\phi \sin t \frac{dt}{d\phi} - \cos t \sin\phi\right)$$

$$\cos\phi \cos\delta \sin t \frac{dt}{d\phi} = \cos\phi \sin\delta - \sin\phi \cos\delta \cos t$$

$$= \cos h \cos Z \text{ (by Eq. 11)}$$

therefore

$$\frac{dt}{d\phi} = \frac{\cos h \cos Z}{\cos\phi \cos\delta \sin t}$$

$$= \frac{\cos Z}{\sin Z \cos\phi} \text{ (by Eq. 12)}$$

$$= \frac{1}{\cos\phi \tan Z} \tag{78}$$

This shows that when $Z = 90°$ or $270°$ an error in ϕ has no effect on t since $dt/d\phi = 0$. In other words, the most favorable position of the body is on the prime vertical. It also shows that the method is most accurate when the observer is on the equator.

99 Time by Equal Altitudes of a Star

If the altitude of a star is observed when it is east of the meridian at a certain altitude and if the same star is again observed at the same altitude when west of the meridian,

the mean of the two observed times is the watch reading for the instant of transit of the star, provided that the watch is so closely regulated that it may be safely assumed that the timepiece does not gain or lose during the interval between observations. It is not necessary to know the actual value of the altitude employed, but it is essential that the two altitudes should be equal. The right ascension of the star (local sidereal time at culmination) is converted to standard time and compared with the mean of the two observed times to obtain the watch correction. The disadvantage of the method is that the interval between the two observations is inconveniently long.

100 Time Service

The standard time used in the United States and possessions is determined by means of star transits at the United States Naval Observatory in Washington and is broadcast by naval radio stations at Annapolis, Mare Island, Pearl Harbor, and Balboa at stated hours of Greenwich time as indicated in Table E. Service is also provided by telegraph to the telegraph companies and firms and individuals providing wires to the transmitting room at the Observatory. The data in Table E are the latest available from the Observatory and show, under the call letters of each radio station, the frequencies in kilocycles used in transmitting via these stations. Hours shown are Greenwich civil time. Signals are transmitted beginning 5 minutes before the hour and end on the hour.

Until a few years ago meridian transit telescopes were used for highly precise time determinations at the Naval Observatory. More recently the photographic zenith tube has been used for this purpose. The following description is quoted from information supplied by the Superintendent of the Observatory. "The telescope is rigidly fixed in a vertical position and therefore cannot photograph any objects except those which pass very near the zenith. At

TABLE E

UNITED STATES NAVAL OBSERVATORY TIME TRANSMISSION

Hour G.C.T.	*NSS*	*NSS*	*NSS*	*NSS*	*NPG*	*NPM*	*NPM*	*NPM*	*NBA*
0	122	4390	9425	12630	115				
2	122	4390	9425	12630					
3					Note 1				
4	122	4390	9425	12630		Note 2		17370	
5									Note 3
6	122	4390	9425	12630					
8	122	4390	9425	12630	Note 1				
10	122	4390	9425	12630					
12	122	4390	9425	12630					
14	122	4390	9425	12630					
15					Note 1				
16	122	4390	9425	12630		Note 2	4525		
17					115				Note 3
18	122	4390	9425	12630					
20	122	4390	9425	12630	115	Note 2	4525		
22	122	4390	9425	12630					

NSS U. S. Naval Radio Station, Annapolis, Maryland
NPG U. S. Naval Radio Station, Mare Island, California
NPM U. S. Naval Radio Station, Pearl Harbor, Territory of Hawaii
NBA U. S. Naval Radio Station, Balboa, Canal Zone

Note 1. 115, 9255, and 12,540 kc.
Note 2. 16.68 (or 56 kc. standby), 9050, and 13,575 kc.
Note 3. 148, 5005, and 11,080 kc.

the lower end of the tube is a basin filled with mercury. The light from a star passes through a lens at the upper end of the instrument, continues down through the tube, is reflected from the mercury surface, and comes to focus on a small photographic plate located just under the lens. The location of the plate and the curves of the lens are such that the lens and plate may be tilted as a unit, through a small angle, without sensibly altering the position of the image on the plate. If the plate and the lens are rotated through 180°, the distance on the plate between the images of the star, before and after reversal, corresponds to twice the zenith distance of the star. Were it possible to take

both photographs in the same instant, when a star was exactly in the zenith, the two images would coincide, and the time of meridian transit would be the time when the photograph was taken. In actual practice the plate is driven from west to east so as to keep pace with the motion of the star's image, and the clock time at which the plate is in certain positions is automatically recorded. The images obtained before and after reversal do not coincide, but by measurement of the distances of the images it is possible to determine the positions of the stars during exposure and to deduce the times of transit."

Electric chronographs record graphically both the ticks of the standard sidereal clocks and the clock times at which the photographic plate is in certain positions. From this record the clock times of star transits can be determined within a small fraction of a second and compared with the theoretical times of transit to give the clock error for each clock used. These standard clocks are never disturbed, reset, nor interfered with except for necessary repairs. Their rates are checked by periodic astronomical observations, and the short period variations in the rates are so small that the clock rates may be predicted to within a few thousandths of a second per day.

The actual time signals are transmitted by sending clocks which are rated to mean solar time. These are compared with the standard sidereal clocks twice daily and their errors determined chronographically. The sending clocks are then set to correct time by an automatic device which accelerates or retards their rates until chronographic comparisons show them to be correct.

The transmission of time signals begins at $55^m\ 00^s$ of some hour and continues through the last second of the hour. Signals are sent out at the beginning of each second during this 5-minute period, except that there is no signal on the 29th second of each minute nor on certain seconds at the ends of the minutes, as indicated in Fig. 80. The

number of signals transmitted after the first omitted signal near the end of each minute indicates the number of minutes remaining to be broadcast during the hour (the 60-second signal being taken as the first second of the next minute). The last signal transmitted (on the hour) is much longer than the others.

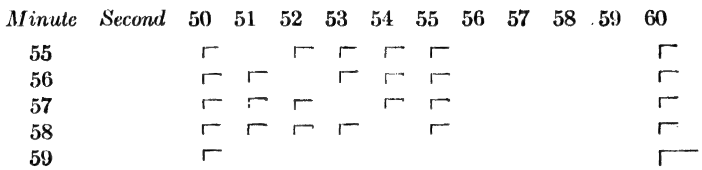


Fig. 80. Time Signals

In order to test and keep record of the errors in these signals they are received and recorded on a chronograph at the Observatory. Thus the error of the sending clock and the error of the signal are on record and may be obtained for use in accurate work. The average error of the signals, as transmitted on 113 kilocycles, is about 1/100 second.

101 Determination of Longitude

Since the difference in longitude between two places is equal to the difference in their local times, the observer who seeks to determine his longitude has a dual problem to solve. He must determine his local time at a particular instant by making observations such as those described in the earlier articles of this chapter. He must also be able to ascertain for this same instant the local time on the reference (Greenwich) meridian or on some other meridian of known longitude.

Since an error of only 1^s in time determination results in a corresponding error of $15''$ in longitude (about 1522 feet at the equator), it is evident that the close determination of longitude involves the use of timepieces of precision

and also of satisfactory methods of communicating the time, at the reference meridian, to the observer. Thus the history of the measurement of longitude is related not only to the development of telescopes and timepieces but to progressive improvements in communications as well.

Prior to the invention of modern telegraphic and radio communications it might at first appear that the simplest method by which the astronomer or navigator could determine the standard meridian (Greenwich) time at which his local time observations were made would be to have with him a timepiece regulated to Greenwich time. The comparison between Greenwich and local times would then indicate his longitude. This method requires a timer having certain definite characteristics. It must be portable; it must not require a stable support, for it must be adapted for use on either land or sea; it should preferably have a low rate of gain or loss; and its rate must be substantially constant under a wide variety of climactic conditions.

It required many centuries of time and the inventive genius of many men to devise such a timer. The determination of longitude at sea became of prime import during the epoch of intensive geographical exploration which was inaugurated shortly before the first voyage of Columbus. The safety of a vessel and her entire complement frequently depended upon close knowledge of her position. Yet less than two centuries have elapsed since the invention of the chronometer by Harrison, a Yorkshire carpenter and watch maker, made possible the determination of longitude at sea by the method of transporting a timepiece. Latitude measurement had been possible for centuries, but no rapid method of fixing a ship's position, both in latitude and longitude, was evolved until this device became generally available to mariners some years after its introduction in 1761. The difficulties inherent in the problem may perhaps best be indicated by reviewing briefly the history of the development of timepieces.

102 Development of Timers

The first time indicator was undoubtedly the sundial, which seems to have been used in Babylonia as early as 2000 B.C. Keeping apparent solar time, requiring a stable support and fixed inclination of gnomon for a given latitude, and useless at night or in cloudy weather it was not adapted to the problem, even though more accurate than many of the early clocks. The sundial was followed by the clepsydra, or water clock, of somewhat uncertain age and equally ill-adapted to the problem of longitude determination.

The date of invention of the earliest mechanical clock is unknown. Doubtless it was the result of many inventions, over many years. We do know, however, that such a mechanical clock was produced by Henry De Vick, of Wurtemburg, for Charles V of France about 1360. This clock, perhaps the best timer of the fourteenth century, is considered by Milham to have kept time within about two hours per day. Indeed, all early clocks seem to have been valued more for their exterior ornamentation than for their accuracy. It was not until the pendulum was applied to clocks, about 1658, that reasonable time-keeping accuracy was obtained. However, these pendulum clocks were not portable and were useless for longitude determinations involving transportation of the timepiece.

These early clocks were all weight-driven, somewhat ponderous affairs. The invention of the mainspring for driving power (Henlein, circa 1500) made portable timers possible. These clock watches were no more accurate than the clocks of the period; indeed the variation in the tension of the spring introduced a new source of error. Their accuracy was increased, however, by the successive inventions of the stackfreed and the fusee and still further improved by the introduction of the balance and balance spring, improved types of escapement, and the use of jeweled bearings.

During this period the vital need for devising methods

for the determination of longitude at sea became increasingly apparent as shipwreck followed shipwreck. Research was encouraged by the offer of monetary rewards by a number of governments, beginning with a handsome annuity promised by Philip III of Spain in 1598 to "the discoverer of the longitude." Many other offers of prizes followed. In 1714 the British Parliament passed an act providing a reward of £10,000 for any method of determining the longitude of a ship within 1°; £15,000 if the method was good to within 40′; and £20,000 if good to within 30′. A Board of Longitude was appointed to carry out the provisions of the act. Its successive members functioned until 1828 and disbursed over £100,000 during this period.

Although the reward was large, it was many years before it was won. In 1761 Harrison appeared before the Board with his fourth timepiece, the result of a lifetime of effort. In a test the chronometer, as it later became known, was transported from Portsmouth to Port Royal, Jamaica, aboard the "Deptford," which sailed November 18, 1761, and reached its destination 61 days later. Returned to England on the "Merlin," the timepiece reached Portsmouth, April 2, 1762. In this period its error was found to amount to 65 seconds. This was well within the limit stipulated and was confirmed by a second test. The £20,000 reward was paid to Harrison, in several installments, between 1762 and 1769. Further improvements by others, particularly in the form of escapement, gave us, by the beginning of the nineteenth century, the marine chronometer in substantially the form so widely used today. Thus, after many centuries of effort, the determination of longitude by transportation of the timepiece became feasible.

103 Longitude by Transportation of Timepiece

From the time of Harrison until the invention of the electrical telegraph and the laying of the transoceanic cables the accurate determination of longitude on land

was based on this method of transportation of timepieces. It still forms the basis of calculations at sea, with the added use of radio signals to periodically check the rate of the chronometer, although the use of radio beacons and radar now permits the application of the principles of triangulation to fixing the position of a ship.

As applied to land observations, the error of the chronometer with reference to the meridian of a station of known longitude is found by observing the local time at this first station. The rate of the timepiece should be determined by making other observations at the same place on later dates. The chronometer is then carried to the second station (of unknown longitude) and its error determined with reference to this meridian. If the chronometer runs perfectly the two watch corrections will differ by just the difference in longitude. Assume that the first observation is made at the easterly station and the second at the westerly station. To correct for rate, let r be the daily rate in seconds, $+$ when losing, $-$ when gaining, c_e the chronometer correction at the easterly station, c_w the chronometer correction at the westerly station, d the number of days between the observations, and T the chronometer reading at the second observation. Then the difference in the longitude is found as follows:

$$\text{Local time at west station} = T + c_w$$

$$\text{Local time at east station} = T + c_e + dr$$

$$\text{Difference in time} = \text{Difference in Longitude} = c_e + dr + c_w \qquad (79)$$

The same result will be obtained if the stations are occupied in the reverse order.

If the error of a mean-time chronometer or watch is found by star observations, it is necessary to know the longitudes accurately enough to correct the sun's right ascension. If

a sidereal chronometer is used and its error found on local sidereal time, no correction is necessary.

In order to obtain a check on the rate of the timepiece the observer should, if possible, return to the first station and again determine the local time. If the rate is uniform the error in its determination will be eliminated by taking the mean of the results.

Today this method is useful chiefly at sea and in exploration surveys. It is not so accurate as are modern telegraphic and radio methods but will yield good results if several chronometers are used and several round trips are made between stations. When so applied the method may become tremendously expensive. For example, Doolittle records that Struve determined the difference in longitude between Pulkova and Altona in 1843 by carrying sixty-eight chronometers nine times in one direction and eight in the other. Reports of the United States Coast and Geodetic Survey for the years 1853, 1854, and 1856 also record observations to determine the longitude of Cambridge, Mass. Over a period of years fifty-odd chronometers were transported many times between Boston and Liverpool. The final results of voyages in the two directions agreed within $0^s.15$. It is evident that savings in time and money are secured from the use of submarine cable or radio signals, since these methods of communication became available.

104 Other Early Methods and Attempts to Determine Longitude

The scientists who for centuries labored with the problem of longitude did not limit their efforts to the improvement of timepieces. Many other proposals were put forth. Some were impractical, others have been used; however, none of them was simple, rapid, or facile either in observation or computation.

Early efforts were made to utilize the known change in variation of the magnetic compass with change in longitude. These attempts failed in practice because of (1) lack of knowledge of magnetic variation in all parts of the world; (2) the presence of diurnal, annual, and secular changes in the variation; (3) the fact that the changes to be observed are small; and (4) the absence of precision instruments, particularly at sea, for observing these changes.

Soon after his discovery of four of Jupiter's satellites, in 1610, Galileo proposed that the movements of these bodies could be utilized in longitude determinations. These satellites are frequently eclipsed by the parent planet, and the Greenwich times at which these eclipses occur can be computed and tabulated for a long time in advance. Thus, a comparison between the local time of such an eclipse and the tabulated Greenwich time of its occurrence will give the longitude of the observer. The eclipses are not instantaneous, however, and close determinations by this method are difficult on land and impractical at sea. Furthermore, for part of the year, Jupiter is above the horizon only during daylight hours when observations cannot be made.

Observations of a similar character, such as those of occultations of the stars by the moon and eclipses of the sun and the moon, have been proposed. They are worthless at sea because of the difficulty of making sufficiently accurate observations from an unstable deck, and they are of limited utility on land. Eclipses, in particular, occur so seldom as to render such methods of little value.

Two lunar observations, both feasible on land and one at sea, deserve somewhat fuller consideration. The rapid motion of the moon among the stars is evident from Fig. 4*b*. This motion, with reference to the fixed stars, makes possible the determination of longitude by the methods of lunar transits and lunar distances. These will be discussed briefly in the following two articles.

105 Longitude by Transit of the Moon

Because of the eastward motion of the moon among the stars, the local time at which the moon culminates over successive meridians will become later and later as we proceed westward from Greenwich. This retardation amounts to a little over 2^m for each $15°$ of longitude. Because of the eccentricity of the moon's orbit the retardation is not constant. The right ascension of the moon is tabulated in the *Ephemeris* for each hour of Greenwich civil time. If the right ascension at the instant of local transit can be ascertained, the Greenwich time may be obtained by interpolation and compared with the local time to give the longitude difference.

The method is of no value at sea because of the impracticability of determining the exact instant of meridian transit. On land it may be useful in exploration surveys where great precision is not required. Since the right ascension of the moon increases about 2^s in every 1^m of time it is evident that any error in determining the right ascension will produce an error about thirty times as great in the longitude. Highly precise results cannot, therefore, be expected. However, the observation has one distinct advantage. If, through accident to the timepiece, an exploring party loses all knowledge of Greenwich time, its time may be recovered with fair accuracy by this method. Less necessary today than before the advent of radio, situations are still conceivable where it would prove of value.

The observation consists in placing the transit in the plane of the meridian and noting the time of transit of the moon's bright limb and also of several stars whose declinations are nearly the same as that of the moon. The table of "moon culminations" in the *Ephemeris* shows which limb may be observed. The observed interval of time between the instants of the moon's transit and a star's transit (reduced to sidereal time if necessary) added to or subtracted from the star's right ascension gives the right

ascension of the moon's limb. A value of the right ascension is obtained from each star observed and the mean value used. To obtain the right ascension of the moon's center it is necessary to apply to the right ascension of the limb a correction, taken from the *Ephemeris*, called the "sidereal time of semidiameter passing meridian." In computing this correction the increase in right ascension during this short interval has been allowed for so that the result is not the right ascension of the center at the instant of transit of the limb but rather at the instant of transit of the center. If the west limb was observed this correction must be added; if the east limb, it must be subtracted. The result is the right ascension of the center at the instant of transit, which is also the local sidereal time at that instant. Then the Greenwich civil time corresponding to this instant is found by interpolation in the *Ephemeris* table which gives the moon's right ascension for each hour of Greenwich civil time. This civil time is then converted to the corresponding instant of Greenwich sidereal time. The difference between the Greenwich and the local sidereal time gives the local longitude.

In preparing for observations of the moon's transit the table of moon culminations in the *Ephemeris* should be consulted to determine whether an observation can be made and to find the approximate time of transit. The time of transit may be computed roughly from the tabular time of transit at either Greenwich or Washington by applying a correction for the approximate longitude. The apparent altitude of the moon should be computed and allowance made for parallax. The moon's parallax is so large that the moon would not be in the field of view if this correction were neglected. The horizontal parallax multiplied by the cosine of the altitude is the required correction. The moon will appear lower than it would if seen from the center of the earth. The correction is therefore subtractive.

EXAMPLE. Compute the longitude from the following lunar observation, June 6, 1941.

Object	E.D.S.T. of Transit	Right Ascension
η *Centauri*	$10^h\ 39^m\ 38^s$ P.M.	$14^h\ 31^m\ 48^s.45$
☽ West Limb	10 49 46	
α^2 *Librae*	10 55 26	14 47 39 .21
σ *Librae*	11 08 25	15 00 39 .55
β *Librae*	11 21 35	15 13 52 .26

Star	η *Centauri*	α^2 *Librae*	σ *Librae*	β *Librae*
Transit star	$10^h\ 39^m\ 38^s$	$10^h\ 55^m\ 26^s$	$11^h\ 08^m\ 25^s$	$11^h\ 21^m\ 35^s$
Transit moon	10 49 46	10 49 46	10 49 46	10 49 46
Interval	10 08	05 40	18 39	31 49
Table III	01.7	00.9	03.1	05.2
Sid. int.	10 09.7	05 40.9	18 42.1	31 54.2
R.A. star	14 31 48.45	14 47 39.21	15 00 39.55	15 13 52.26
R.A. limb	14 41 58.15	14 41 58.31	14 41 57.45	14 41 58.06
s.t.s.d.p.	01 10.65	01 10.65	01 10.65	01 10.65
R.A. moon	14 43 08.80	14 43 08.96	14 43 08.10	14 43 08.71

Mean R.A. moon's center	$14^h\ 43^m\ 08^s.64$	L.S.T.
R.A. moon at Greenwich 02^h June 7	14 41 02 .85	*Ephemeris*
R.A. moon at Greenwich 03^h June 7	14 43 31 .13	*Ephemeris*
Tabular difference	02 28 .28	$= 148^s.28$

Correction $= 125.79 \times 3600/148.28 = 3054^s = 50^m\ 54^s$

G.C.T. of observation	$02^h\ 50^m\ 54^s$	June 7
R.A.M.S + 12^h	16 59 59 .759	
Table III	19 .713	
	08 .214	
	00. 148	
G.S.T.	19 51 21 .83	
L.S.T.	14 43 08 .64	
Long. W	$05^h\ 08^m\ 13^s.19$	$= 77°\ 03'\ 17''.8$ W

106 Longitude by Lunar Distances

From physical observations of the motion of the moon the path of this body may be calculated, and tables can be prepared showing her position in the heavens at various

instants for a long time in advance. Furthermore, since the corresponding positions of the sun and stars at these same instants can be calculated, it is possible to tabulate the angular distances of the moon from the sun and the stars at various instants of Greenwich time. If we then observe the angular distance between the moon and a heavenly body for which the lunar distances have been tabulated, it will be possible to interpolate in the table to determine the Greenwich civil time of observation. This, when compared with the local time, will yield the difference in longitude. The lunar distances may be observed, either on land or sea, with the sextant.

Simple in theory, the method suffers from the arduousness of the calculations and the fact that any error in determining the angular distance is multiplied to many times that error in the resulting longitude. The observed lunar distance must be corrected for instrumental errors and for the semidiameters, refractions, and parallaxes of the two bodies. Certain of these corrections depend upon the altitudes of the bodies which should preferably be measured by other observers at the same time that the lunar distance is measured.

The method appears to have been first proposed by Werner in 1514, but knowledge of the moon's motion was too rudimentary at the time to permit its application. The desire to make use of this principle, however, was primarily responsible for the establishment of the Greenwich Observatory. When Maskelyne inaugurated the *British Nautical Almanac* in 1767, he tabulated lunar distances of the sun and seven stars for each 3 hours of Greenwich time. Such tabulations continued to appear in the *Almanac* until 1907. By this date the telegraph was available for use on land, and sea voyages required so much less time that chronometer errors did not assume large proportions so that the method had ceased to have practical value. Thus longitude by lunar distances became practicable at about the same time

that the chronometer made longitude determination possible by transportation of the timepiece. For a time the use of lunar distances rivaled the method of transportation of the timepiece, and for many years it was used to check approximately the error of the chronometer on long sea voyages. The advent of the radio has relegated the method to a position of solely historical interest in the annals of longitude determination.

107 Longitude by Telegraph

The development of the telegraph made possible the economical precise determination of longitude on land. While precision methods are not, in general, included within the scope of this book, our historical treatment of longitude would not be complete without brief reference to the telegraphic and wireless methods.

When the telegraph is used for communication between stations the local sidereal time is accurately determined by star transits observed at both the station of known longitude and the point whose position is to be determined. The observations are made with large portable transits and recorded on chronographs which are connected with break-circuit chronometers. The stars observed are selected in such a manner as to permit the determination of the instrumental errors so that the effect of these errors may be eliminated from the results. Half the number are observed with the axis in one position and the other half with the axis reversed. This determines the error of the sight line. In each half set some of the stars are north of the zenith and some south. The differences in times of transit of these two groups measure the azimuth error. The inclination error of the horizontal axis of the instrument is measured with a striding level.

After the corrections to the chronometers have been accurately determined the two chronographs are switched into the main-line circuit and signals sent either by making

or breaking the circuit a number of times by the use of a telegraph key. These signals are recorded on both chronographs. In order to eliminate the error resulting from the time required for transmission of the signal, the signals are sent first in the direction from east to west and then in the reverse direction. The mean of the two results is free from the error, provided that it remains constant during the interval. Personal errors are nearly eliminated by use of the transit micrometer. After all the observations have been corrected for azimuth, collimation, and level errors, and the error of each chronometer on local sidereal time is known, each signal sent over the main line will be found to correspond to a certain instant of sidereal time at the east station and to a different instant of sidereal time at the west station. The difference between the two is the difference in longitude expressed in time units.

108 Wireless Longitude

In the early years of the twentieth century wireless communication began to be utilized in longitude work. By 1922 marked progress had been made in the application of the method, and longitude determinations by the United States Coast and Geodetic Survey have largely depended upon wireless communication since that date. The local time at a station is compared with that at Washington or Greenwich by receiving the standard time signals by radio. With observations to be made at the local station only, the work involved is roughly half that required for the telegraph method.

109 Longitude by Time Signals

In well-mapped regions the surveyor can usually take his local longitude from a map as closely as he can obtain it instrumentally. Where good maps are not available, however, he can determine his approximate longitude, provided that he can obtain the error of his watch on standard

time by means of radio or telegraphic time signals. Any of the methods given in the first part of this chapter may be used to determine the local time for comparison with the standard time. We may, for example, obtain the local time by observing the meridian transit of a star. The longitude is then obtained by reversing the procedure followed in Art. 94.

EXAMPLE. Determine the longitude from the following observation on γ *Hydrae*, June 2, 1947. The watch time of transit was $09^h\ 30^m\ 24^s$ P.M. Radio time signals received both before and after observation indicate that the watch was 03^s slow of E.D.S.T.

Watch	$09^h\ 30^m\ 24^s$		P.M. June 2
Watch slow	03		
E.D.S.T.	09 30 27		P.M. June 2
Long. W	04		
G.C.T.	01 30 27		June 3
R.A.M.S. + 12^h	16 42 25 .45		
Table III	14 .86		
G.S.T.	18 13 07 .31		
L.S.T.	13 16 03 .20		R.A. γ *Hydrae*
Local long.	$04^h\ 57^m\ 04^s.11$	=	74° 16′ 02″ W

Questions and Problems

1. Compute the E.S.T. of transit of *Regulus* (α *Leonis*) over the meridian 71° 06′.0 west of Greenwich on March 3 of a certain year; R.A. $10^h\ 04^m\ 23^s.8$ R.A.M.S. $+12^h$ at 0^h G.C.T. March 3, $10^h\ 41^m\ 00^s.26$; March 4, $10^h\ 44^m\ 56^s.81$.

2. At what time (E.S.T.) will the center of the sun be on the meridian on April 1 of a certain year when Eq. T. at 0^h G.C.T. April 1 = $-04^m\ 12^s.47$; variation per hour $+0^s.755$; long. 71° 06′.0 W?

3. Using the *Ephemeris* for the current year, prepare a table of stars suitable for time observations at culmination between the hours of 8:00 P.M. and 10:00 P.M., E.S.T., December 12, at a place in lat. 40° 43′ 24″ N and long. 74° 17′ 20″ W. Minimum and maximum altitudes of stars are 20° and 55° respectively. Minimum interval between stars is to be $02^m\ 30^s$. List the star numbers, names, magnitudes, right ascensions and declinations for the given date, approximate altitudes, and approximate E.S.T. times of transit.

4. The watch time of transit of β *Crateris* on May 22, 1938 was $8^h\ 05^m\ 38^s$ P.M., approximate E.D.S.T.; long. of the observer 74° 10′ 43″ W; R.A. $11^h\ 08^m\ 38^s.757$; R.A.M.S. + 12^h at Greenwich 0^h on that date $15^h\ 55^m\ 51^s.119$. Determine the watch correction.

5. Determine the watch correction from the following observation on the sun at culmination, July 1, 1941. Long. $4^h\ 44^m\ 22^s$ W; obs. time of transit of east limb of sun $12^h\ 49^m\ 11^s$ P.M., approximate E.D.S.T. Eq. T. at Greenwich 0^h, July 1, $-03^m\ 31^s.68$; July 2, $-03^m\ 43^s.34$. s.t.s.d.p. $01^m\ 08^s.74$.

6. Determine the error of the watch from the following observation on *Achernar* (*α Eridani*) at equal altitudes on December 6, 1941. Lat. 18° 00′ S, long. 70° 15′ W. Observation east of the meridian taken at $06^h\ 44^m\ 30^s$ P.M., approximate E.S.T. Observation west of the meridian taken at $09^h\ 45^m\ 10^s$ P.M., approximate E.S.T. R.A. of star $01^h\ 35^m\ 33^s.73$. R.A.M.S. + 12^h at Greenwich 0^h December 7, $05^h\ 01^m\ 29^s.214$. Assume no gain or loss of watch during the interval between observations.

7. Compute the watch correction from the following observation on the sun for time and azimuth, March 4, 1942. Lat. 40° 44′ 20″ N, long. 74° 18′ 00″ W; R.A.M.S. + 12^h at Greenwich 0^h March 4, $10^h\ 44^m\ 29^s.539$. Decl. of sun at Greenwich 0^h March 4, −06° 45′ 47″.9; March 5, −06° 22′ 44″.9. Eq. T. at Greenwich 0^h, March 4, $-12^m\ 01^s.42$; March 5, $-11^m\ 48^s.32$.

Point	Horizontal Angle	Vertical Angle	Approximate E.D.S.T.
Hub *E*	00° 00′ 00″		
$\underline{\odot}\vert$	217 21 00	29° 56′ 00″	$03^h\ 54^m\ 00^s$ P.M.
	217 38 00	29 45 00	03 55 12
	217 54 00	29 36 00	03 56 17
$\vert\overline{\odot}$	217 42 00	28 46 30	03 58 07
	217 52 30	28 40 00	03 58 52
	218 05 30	28 32 00	03 59 50
Hub *E*	00 00 00		

8. Determine the watch correction from the following observation on *Mirach* (*β Andromedae*) January 30, 1941. Lat. 44° 45′ 00″ N, long. 67° 30′ 00″ W; obs. alt. 43° 33′, star in west; obs. time $08^h\ 00^m\ 31^s$ P.M., approximate E.S.T.; star's decl. + 35° 18′ 36″; R.A. $01^h\ 06^m\ 25^s.52$. R.A.M.S. + 12^h at Greenwich 0^h January 31, $08^h\ 39^m\ 17^s.49$.

9. Figure 81 shows a portion of a chronograph record providing a comparison of Naval Observatory time signals and a chronometer rated to Greenwich sidereal time. The time signals are those sent out before and at 0^h G.C.T., April 27, 1947. Those signals sloping down to the left are impressed by the chronometer; the signals sloping down to the right are the time signals. Time identifications at the top of the figure relate to the G.S.T. of the chronometer. Time indications beneath the figure refer to the G.C.T. signals. Determine from the figure the chronometer reading at the instant of 0^h G.C.T. R.A.M.S. + 12^h at Greenwich 0^h was $14^h\ 16^m\ 32^s.88$. Determine the chronometer correction in sidereal time units.

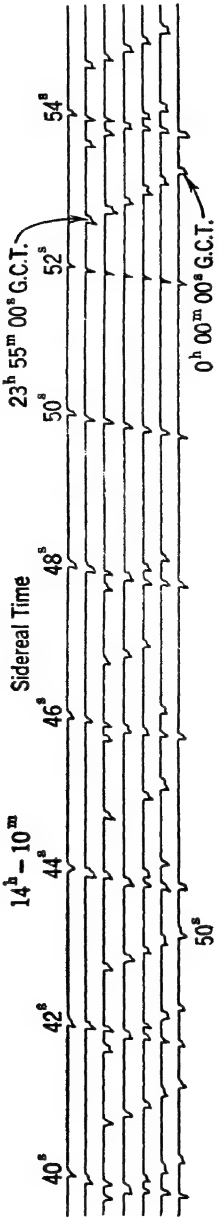


FIG. 81. Chronograph Record

10. On February 1, 1941, the transit of *Canopus* (*α Carinae*) was observed, chronometer reading $09^h\ 15^m\ 38^s$ P.M., approximate 75th meridian time. The rating curve maintained for the chronometer indicated that, at the instant of observation, the chronometer was 1.5 seconds slow. The observed meridian altitude of the star was 47° 20′ 30″, the star bearing south of the zenith. I.C. −30″; R.A. $06^h\ 22^m\ 40^s.53$; decl. −52° 40′ 10″; R.A.M.S. + 12^h at Greenwich 0^h February 2, $08^h\ 47^m\ 10^s.587$. Determine the latitude and longitude of the observer.

11. Compute the longitude from the following lunar observation made during the course of an exploration survey in South Australia.

Object	Observed Time of Transit	Right Ascension
γ Capricorni	$09^h\ 01^m\ 52^s$ P.M.	$21^h\ 35^m\ 23^s.76$
δ Capricorni	09 08 49	21 42 21 .83
Moon's west limb	09 14 23	
μ Capricorni	09 15 07	21 48 40 .54
ι Aquari	09 28 16	22 01 51 .69

The s.t. of moon's s.d.p. = $01^m\ 04^s.76$; R.A. of moon: at Greenwich 12^h = $21^h\ 48^m\ 44^s.20$, variation per minute, $1^s.9560$; at Greenwich 13^h = $21^h\ 50^m\ 41^s.41$, variation per minute $1^s.9510$. R.A.M.S. + 12^h at Greenwich 0^h = $00^h\ 31^m\ 35^s.15$.

12. Compute the longitude of the observer from the following solar observation made on February 26. Use *Ephemeris* data for that date for the current year. Lat. 40° 44′ 30″ N. Radio time signals indicate that the watch is 2^s slow of E.S.T.

Point	Horizontal Angle	Vertical Angle	Watch Time
Mark	00° 00′		
Right and	80 35	20° 01′	$03^h\ 48^m\ 19^s$ P.M.
Lower	80 43	19 54	03 49 04
Limbs	80 52	19 45	03 49 54
Left and	80 46	18 54	03 52 09
Upper	80 58	18 43	03 53 11
Limbs	81 08	18 34	03 54 07
Mark	00 00		

13. On February 17 the transit of *α Canis Minoris* was observed, watch reading $09^h\ 49^m\ 50^s$ P.M., approximate E.S.T. Radio time signals indicated that the watch was 15^s fast. Using *Ephemeris* data for the current year, compute the longitude of the observer.

14. Which limb of the moon can be observed for longitude by meridian transit if the observation is taken just before daybreak?

15. At about what local civil time will the moon transit when it is at first quarter?

16. Determine the watch correction from the following observation on the sun for time and azimuth, February 26. Lat. 40° 44′ 30″ N, long. 74° 11′ 10″ W. Use *Ephemeris* data for current year.

Point	Horizontal Angle	Vertical Angle	Approximate E.S.T
Mark	00° 00′		
Right and	75 16	24° 03′	$03^h\ 22^m\ 35^s$ P.M.
Lower	75 27	23 56	03 23 24
Limbs	75 36	23 50	03 24 08
Left and	75 15	23 07	03 25 16
Upper	75 25	23 00	03 26 04
Limbs	75 33	22 55	03 26 29
Mark	00 00		

17. Determine the watch correction from the following observation on *Castor* (*α Geminorum*) on December 4. Lat. 42° 21′ N, long. 76° 10′ W; obs. alt. 37° 10′, star in east; obs. time $10^h\ 14^m\ 30^s$ P.M. approximate E.S.T. Use *Ephemeris* data for current year.

18. Determine the longitude of the observer from the following observation on the sun at culmination, December 3. Observed time of transit of east limb of sun $11^h\ 59^m\ 21^s$ A.M. Radio signals indicate that the watch is 08^s slow on Central standard time. Use *Ephemeris* data for current year.

19. Compute the longitude from the following lunar observations.

Object	Observed Time of Transit	Right Ascension
θ Aquarii	$05^h\ 16^m\ 04^s$ P.M.	$22^h\ 11^m\ 27^s.6$
π Aquarii	05 24 40	22 20 04 .6
Moon's west limb	05 32 27	
λ Aquarii	05 51 47	22 47 18 .3

The s.t.s.d.p. is $60^s.3$. At G.C.T. 22^h the moon's R.A. was $22^h\ 27^m\ 53^s.3$; variation per minute $1^s.9800$; R.A.M.S. + 12^h at Greenwich 0^h $04^h\ 36^m\ 29^s.7$.

20. On April 2 of a certain year the transit of *ζ Hydrae* was observed; watch reading $7^h\ 52^m\ 31^s$ P.M. At 10^h P.M. radio time signals indicated that the watch was 03^s fast. R.A.M.S. + 12^h at Greenwich 0^h on that date $12^h\ 39^m\ 16^s.82$; R.A. of the star $08^h\ 51^m\ 26^s.41$. Compute the longitude of the observer. Watch is keeping G.C.T.

12

Observations for Azimuth

110 Determination of Azimuth

The determination of the azimuth of a line or of the direction of the true meridian is of frequent occurrence in the practice of the surveyor and is unquestionably the most important to him of all the astronomical observations. In geodetic surveys, in which the triangulation stations are located by means of their latitudes and longitudes, the precise determination of astronomical position is of as great importance as the orientation; but in general engineering practice, in topographical work, and in route surveying, the azimuth observation is the one which is most frequently required.

Too much stress cannot be laid on the desirability of employing the true meridian and true azimuths* for all kinds of surveys. The use of the magnetic meridian or of an arbitrary reference line may save a little trouble at the time but is likely to lay up trouble for the future. As surveys are extended and connected and as lines are resurveyed the importance of having used the true meridian in the original surveys becomes increasingly evident. The

* The term *true azimuths* in this statement includes the use of grid azimuths where surveys are tied in with a state coordinate system since these azimuths are based on true north at the central meridian of the coordinate system (see Art. 125).

expense incident to the observations is insignificant compared to that involved in connecting arbitrarily oriented surveys. In route surveys, in particular, where long traverses are frequently run without angle closure, azimuth observations, with proper regard to meridian convergence (see Art. 124), constitute the most satisfactory check on angular measurements.

111 Azimuth Mark

When an observation is made at night it is frequently inconvenient or impossible to sight directly at the object whose azimuth is to be determined; it is therefore necessary in such cases to determine the azimuth of a special *azimuth mark*, which can be seen both at night and in the day, and then to measure the angle between this mark and the first object by daylight. The azimuth mark may consist of a lamp placed inside a box having a small hole cut in the side through which the light may be seen, or it may consist of a small flashlight bulb, with socket rigidly attached to a stout post, powered by one or two dry cells. In either case the visible portion of the light source should be small enough and far enough from the instrument so that the size of the angle subtended by the light will have an insignificant effect upon the accuracy of the result. The mark should preferably be so far from the instrument that the focus of the telescope will not have to be altered when changing from the star to the mark. Occasionally the topography near the station may be such that it will be impossible to place the mark as far away as desirable, but every effort should be made to secure a long sight line.

112 Azimuth of Polaris at Greatest Elongation

The simplest method of determining the direction of the meridian with accuracy is by means of an observation of the *polestar*, or any other close circumpolar, when it is at its greatest elongation (see Art. 20). The appearance of the

constellations at the time of this observation on *Polaris* may be seen by referring to the star map (Fig. 70) or to Fig. 82. When the *polestar* is westerly of the pole the Big Dipper is in the east and Cassiopeia is west of *Polaris*.

The time of elongation may be determined by first computing the local hour angle of the star at the instant of elongation by applying Eq. 35. Two procedures are then open to us: (1) We may convert the local hour angle to

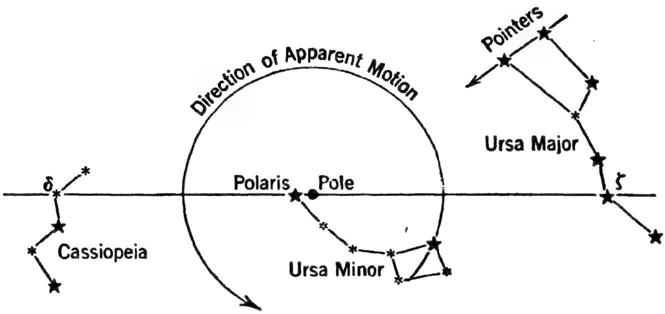


FIG. 82. Constellations Near the North Pole. *Polaris* at Western Elongation

Greenwich hour angle by applying the longitude and then determine from the *Nautical Almanac* the instant of Greenwich civil time corresponding to this Greenwich hour angle; (2) We may convert the local hour angle to time units, add to it the right ascension of the star to obtain the local sidereal time, and convert the latter to standard time. In either case the student may encounter difficulty because, when he begins the computation, he does not know the Greenwich date on which the elongation, specified for a certain local date, will fall. The difficulty may be resolved by rigid analysis and painstaking attention to dates. It may also be resolved by computing an approximate time of elongation, using Table V of this text or Table VII of the *Ephemeris*. This will suffice for most purposes since, as a rule, we require the time accurately enough only to be

prepared for the observation shortly before elongation occurs. If we require the time of elongation more closely we may then proceed by either of the two methods outlined above since the approximate computation will indicate clearly the Greenwich date on which the specified elongation occurs. In the following example we shall compute the time of elongation by the use of Table V and then by application of each of the other methods.

EXAMPLE 1. Find the E.S.T. of the eastern elongation of *Polaris* on October 1, 1947, in lat. 40° 43′ N, long. 74° 11′ W.

Table V indicates that, on September 28, 1947, the L.C.T. of U.C. was $01^h\ 23^m\ 02^s$ for an observer in long. 90° W. Table V*a* indicates that, for the latitude given, eastern elongation precedes culmination by $05^h\ 55^m.6$. It is evident that the eastern elongation occurring on October 1 immediately preceded the culmination of October 2. We therefore apply a correction for 4 days from Table V*a*, note *b*. The longitude correction (see Table V*a*, note *c*) will be $+\ 09^s$, and the conversion to E.S.T. will be $-03^m\ 16^s$. Tabulated, these figures yield the following result:

90° W L.C.T. of U.C. 9/28/1947	$01^h\ 23^m\ 02^s$
Correction for 4 days at $-3^m\ 55^s$ (note *b*)	−15 40
90° W L.C.T. of U.C. 10/2/1947	01 07 22
Correction for long. (note *c*)	+ 09
L.C.T. of U.C. 10/2/1947	01 07 31
Correction for standard time (note *d*)	−03 16
E.S.T. of U.C. at local place 10/2/1947	01 04 15
Elongation precedes culmination by	05 55 36
E.S.T. of eastern elongation at local place 10/1/1947	$07^h\ 08^m\ 39^s$ P.M.

If preferred, Table VII of the *Ephemeris* may be used in place of Table V of this book. The computations from Table VII follow.

G.C.T. of U.C., 9/27/1947	$01^h\ 27^m\ 56^s$
Correction for 5 days (5) ($-3^m\ 55^s.3$)	−19 36
G.C.T. of U.C., 10/2/1947	01 08 20
Correction to local meridian (4.95 × 9.80)	− 49
L.C.T. of U.C. 10/2/1947	01 07 31
Correction for standard time	−03 16
E.S.T. of U.C. at local place 10/2/1947	01 04 15
Elongation preceded culmination by	05 55 36
E.S.T. of eastern elongation at local place 10/1/1947	$07^h\ 08^m\ 39^s$ P.M.

It is clear that the G.C.T. corresponding to this instant will be $00^h\ 08^m\ 39^s$ on October 2. This enables us to take the right ascension and declination of *Polaris* from the *Ephemeris* for this date. The values are $01^h\ 47^m\ 59^s.62$ and $+89°\ 00'\ 55''.26$ respectively.*

Applying Eq. 35 we have

$$\cos t = \tan \phi \cot \delta$$

log tan 40° 43′ 00″	9.9348225
log cot 89 00 55	8.2351697
log cos t	8.1699922

$$t = 89°\ 09'\ 08''\ \text{E} = 270°\ 50'\ 52'' = 18^h\ 03^m\ 23^s.5$$

G.H.A. Method

L.H.A.	270° 50′ 52″	
Long. W	74 11 00	
G.HA.	345 01.9	
G.H.A. at 0^h	342 52.3	Oct. 2
Remainder	2 09.6	
Correction 08^m	2 00.3	
Remainder	9.3	
Correction 37^s	9.3	
Remainder	0.0	
G.C.T.	$00^h\ 08^m\ 37^s$	Oct. 2
E.S.T.	07 08 37 P.M.	Oct. 1

Sidereal Time Method

H.A. *Polaris*	$18^h\ 03^m\ 23^s.5$	
R.A. *Polaris*	01 47 59 .6	
L.S.T.	19 51 23 .1	
Long. W	04 56 44 .0	
G.S.T.	00 48 07 .1	
R.A.M.S. + 12^h	00 39 28 .7	Oct. 2
Sid. int.	00 08 38 .4	
Table II	1 .4	
G.C.T.	00 08 37 .0	Oct. 2
E.S.T.	07 08 37 .0 P.M.	Oct. 1

The results of these two solutions check each other and agree with the approximate solution of Table V within 2 seconds.

In making the observation the transit should be set over the station well in advance of the time of elongation to permit the tripod legs to come to rest on stable ground. Some 5^m before the time of elongation the instrument should be carefully leveled, preferably by means of the telescope bubble, the vernier of the horizontal circle set at zero (or the initial vernier reading taken), and the transit sighted along the line of desired azimuth, that is, a sight taken on the azimuth mark or on the next station of the traverse

* It should be pointed out that the coordinates of *Polaris* and other close circumpolars vary quite rapidly. If the time of elongation is to be computed exactly the Greenwich civil time must first be determined closely enough to permit precise interpolation for the star's right ascension and declination.

or triangulation net. The horizontal angle is then turned clockwise from the mark to the star; its value being determined with greater precision by the method of repetition. Near the time of elongation the star will appear to move almost vertically. On account of the very slow change in azimuth of the star there will be ample time to make several repetitions before the error in azimuth amounts to more than $1''$ or $2''$. In latitude $40°$ the azimuth will change no more than $0'.1$ during the 10^m period immediately preceding or following elongation. The change in azimuth varies nearly as the square of the time interval from elongation. Half of the repetitions should be made with the telescope direct and half with the telescope reversed to eliminate instrumental errors.

Rather than measure the angle from the mark to the star at night, the observer may prefer to bisect the star, lower the telescope, and set a stake several hundred feet north of the transit in line with the star; then reverse the telescope, bisect the star again, and set a second point on the stake. If there are errors of adjustment (line of collimation and horizontal axis) the two points will not coincide; the mean of the two points will be free from these errors. The angle between the traverse line and the stake can later be measured by daylight.

The angle between the meridian and the line to such a stake (or the azimuth of the star at the instant of elongation) is found by the equation

$$\sin Z_n = \sin p \sec \phi \tag{36}$$

where Z_n is the azimuth from the north (toward the east for eastern elongation, toward the west for western elongation); p is the polar distance of the star; and ϕ is the latitude of the place. The polar distance may be obtained by taking the declination from the *Ephemeris* and subtracting it from $90°$; or it may be taken from Table F if an error of $\frac{1}{2}'$ is permissible. The latitude ϕ may be taken from a map or

found by observation (see Chapter 10). The latitude does not have to be known with great precision; a differentiation of Eq. 36 will show that an error of $1'$ in ϕ causes an error of only about $1''$ in Z_n for *Polaris* for latitudes within the United States.

TABLE F

MEAN POLAR DISTANCE OF POLARIS

Year	Mean Polar Distance	Year	Mean Polar Distance
1948	00° 58′ 52″	1955	00° 56′ 48″
1949	00 58 34	1956	00 56 30
1950	00 58 16	1957	00 56 12
1951	00 57 58	1958	00 55 55
1952	00 57 41	1959	00 55 37
1953	00 57 23	1960	00 55 19
1954	00 57 05	1961	00 55 02

The apparent polar distance for these dates may differ by as much as 27″ from the above values as a result of nutation and aberration. (From data supplied by the Director, United States Naval Observatory.)

The preceding method is general and may be applied to any circumpolar star. For *Polaris,* whose polar distance in 1947 was about 59′, it is usually sufficiently accurate to use the approximate formula

$$Z_n'' = p'' \sec \phi \tag{80}$$

where Z_n'' and p'' are expressed in seconds of arc.

This computed angle (Z_n) may then be combined with the measured angle from the mark to the star to determine the azimuth of the mark. If it is desired to set a meridian stake due north of the transit station and a stake has been set vertically beneath the position of the star at elongation it is necessary only to tape the distance to this latter stake from the transit and to compute, from Z_n, the perpendicular offset to be laid off to place a mark in the plane of the meridian.

Example 2. A transit was set up at triangulation station Newark and sighted on a light shown from station South Mountain. The horizontal angle from South Mountain to *Polaris* at eastern elongation was measured (clockwise) by repetition and found to be 34° 10′ 35″. Date of observation, October 1, 1947. Position of station Newark, lat. 40° 43′ 10″ N, long. 74° 11′ 00″ W. Compute the true bearing of the line Newark-South Mountain.

Computations carried out earlier in this article indicate that the E.S.T. of eastern elongation on this date is $07^h\ 08^m\ 37^s$ P.M. and that the declination of *Polaris* at this instant, as taken from the *Ephemeris*, is +89° 00′ 55″. The polar distance is therefore 00° 59′ 05″ = 3545″.

By Eq. 36		By Eq. 80	
log sin p	8.2351697	log p''	3.5496162
log sec ϕ	0.1203806	log sec ϕ	0.1203806
log sin Z_n	8.3555503	log Z_n''	3.6699968
Z_n	01° 17′ 58″	Z_n''	4677″
		Z_n	01° 17′ 57″

Angle, South Mountain to *Polaris*	34° 10′ 35″
Azimuth of *Polaris* (east)	01 17 58
True bearing, Newark-South Mountain	N 32° 52′ 37″ W

113 Observations near Elongation

If observations are made on a close circumpolar star within a few minutes of elongation the azimuth of the star at the instant of pointing may be reduced to its value at elongation if the time of observation is known. The formula for computing the correction is

$$C = 112.5 \times 3600 \times \sin 1'' \times \tan Z_e \times (T - T_e)^2 \quad (81)$$

where

Z_e = the azimuth at greatest elongation
T = the observed time
T_e = the time of elongation

$T - T_e$ must be expressed in sidereal minutes. The correction is in seconds of angle. Values of this correction are given in Table V*a* in the *Ephemeris* for each minute up to 25, or in Table VI of this book.

Example. On March 10, 1947, it was intended to make an observation on *Polaris* at western elongation. At the time of elongation the star was

obscured by clouds. Fifteen minutes after elongation a horizontal angle of 74° 12′ 20″ was measured clockwise from mark to star. Lat. 40° 43′ 20″ N, long. $2^h\ 30^m\ 36^s$ E. Compute the azimuth of the mark.

From the *Ephemeris*, decl. of *Polaris* + 89° 01′ 02″, giving 00° 58′ 58″ for the polar distance. Equation 36 yields 01° 17′ 48″ for the azimuth at elongation. $T - T_e$ equals 15 solar minutes or $15^m\ 02^s.5$ in sidereal units. The Table VI correction is 10″. Subtracting this from the azimuth at elongation we obtain 01° 17′ 38″ as the azimuth of *Polaris* (west of north) at the instant of observation. Combining this latter value with the measured horizontal angle we have the bearing of the mark N 75° 29′ 58″ W. Its azimuth (from the south point clockwise) is 104° 30′ 02″.

114 Azimuth by Elongations in the Southern Hemisphere

The method described in the preceding article may be applied to stars near the south pole, but, since there are no bright stars within about 20° of this pole, the observation is not quite so simple, and the results are somewhat less accurate. As the polar distance increases the altitude of the star at elongation increases, and the diurnal motion becomes more rapid. The increase in altitude causes greater inconvenience in making the pointings and also magnifies the effect of instrumental errors. Because of the rapid motion of the star it is important to know beforehand both the time at which elongation will occur and the altitude of the star at this instant.

The time of elongation is computed by application of Eq. 35 to obtain the local hour angle of the star. This, combined with the star's right ascension, gives the local sidereal time. The latter, in turn, is converted into watch time. The altitude may be found by the formula

$$\sin h = \frac{\sin \phi}{\sin \delta} = \sin \phi \sec p \tag{82}$$

There is usually time enough to reverse the transit and make one observation in each position of the axis, without serious error, if the first is taken when the star is slightly below eastern elongation, or the same amount above western elongation

Two stars, *Miaplacidus*,* and α *Trianguli Australis*, both brighter than the second magnitude, are reasonably well positioned for such observations in southern latitudes. An eastern or western elongation of one or the other of these will occur at a reasonably convenient time for observation at any season of the year.

EXAMPLE 1. An observer near Asuncion, Paraguay, in lat. 25° 15′ S, long. 57° 30′ W, desired to make an azimuth observation at elongation on October 12, 1947. Determine whether β *Carinae* or α *Trianguli Australis* was most favorably situated for observation. Compute the $+4^h$ zone time of elongation and the star's altitude at this instant.

At this season it may be assumed that it will be sufficiently dark for observations by 8 P.M. and that observations may be carried on conveniently for some time thereafter. Assume 8:30 P.M. as a reasonable time.

Zone $+ 4^h$ time	$20^h\ 30^m$	October 12
Long. W	04 00	
G.C.T.	00 30	October 13
R.A.M.S. $+ 12^h$	01 23	
Approximate G.S.T.	01 53	
Long. W	03 50	
Approximate L.S.T.	22 03	

Both stars have declinations of about $-69°$. Applying Eq. 35, we have

$$\cos t = \tan \phi \cot \delta = 0.472 \times 0.384 = 0.181.\ t = 79°\ 34'.$$

t at western elongation $= 5^h\ 18^m$; at eastern elongation $= 18^h\ 42^m$.

	β *Carinae* Western Elongation	β *Carinae* Eastern Elongation	α *Trianguli Australis* Western Elongation	α *Trianguli Australis* Eastern Elongation
R.A.	$09^h\ 13^m$	$09^h\ 13^m$	$16^h\ 43^m$	$16^h\ 43^m$
H.A.	05 18	18 42	05 18	18 42
L.S.T.	14 31	03 55	22 01	11 25

It is evident that the western elongation of α *Trianguli Australis* will occur at a convenient hour for observation. With the approximate time of elonga-

* *Miaplacidus* is star β of the ancient constellation *Argo Navis*, the Ship of the Argonauts. This constellation covered a large area of the southern sky and has been subdivided by modern astronomers into four parts of the ship: *Carina*, the keel; *Malus*, the mast; *Puppis*, the stern; and *Vela*, the sails. Another and earlier subdivision is *Pyxis*, the mariner's compass. Thus *Miaplacidus* appears in the *Ephemeris* as β *Carinae:* it is shown in the *Nautical Almanac*, which follows the older terminology, as β *Argus*.

tion known we may then interpolate in the *Ephemeris* for the right ascension and declination of the star at this instant, obtaining $16^h\ 43^m\ 02^s.5$ and $-68°\ 56'\ 19''.4$ respectively. Then, by Eq. 35, $\cos t = \tan\phi \cot\delta$

		R.A. star	$16^h\ 43^m\ 02^s.5$	
		H.A. star	05 18 08 .6	
$\log\tan\phi$	9.6736020	L.S.T.	22 01 11 .1	
$\log\cot\delta$	9.5855641	Long. W	03 50 00 .0	
$\log\cos t$	9.2591661	G.S.T.	01 51 11 .1	
t	79° 32′ 09″	R.A.M.S. $+ 12^h$	01 22 50 .8	October 13
	$= 05^h\ 18^m\ 08^s.6$	Sid. int.	00 28 20 .3	
		Table II	4 .6	
		G.C.T.	00 28 15 .7	October 13
		$+ 4^h$ zone T	20 28 15 .7	October 12
			$= 08^h\ 28^m\ 15^s.7$ P.M.	

This gives the watch time of western elongation of *α Trianguli Australis*, as required.

By Eq. 82, $\sin h = \sin\phi/\sin\delta$, we have

$$0.42683/0.93316 = 0.45740 = \sin h,\ h = 27°\ 13'.$$

If this angle of 27° 13′ is laid off the observer should have no difficulty in picking up the star west of the meridian since *α Trianguli Australis* is brighter than the second magnitude. The apparent altitude at elongation would be the value shown plus the refraction.

Example 2. At the place, date, and time, indicated in Example 1, the observer, occupying station Pilcomayo, sighted on station Carapegua and turned the horizontal angle from Carapegua to *α Trianguli Australis*. One half of the doubled angle, turned clockwise from mark to star, was 78° 25′ 40″. Compute the true bearing of the line Pilcomayo-Carapegua.

By Eq. 36, $\sin Z = \sin p \sec\phi$ ($p = 21°\ 03'\ 40''.6$).

$\log\sin p$	9.5555373	
$\log\sec\phi$	0.0436130	
$\log\sin Z$	9.5991503	
Z (star)	23° 24′ 41″	(west of south)
Horizontal angle	78 25 40	
Bearing	S 55° 00′ 59″ E	(Line Pilcomayo-Carapegua)

115 Azimuth Observation on a Circumpolar Star at Any Hour Angle

The most precise determination of azimuth may be made by measuring the horizontal angle between an azimuth mark and a circumpolar star, the hour angle of the star

at each pointing being known. If the sidereal time is determined accurately, by any of the methods given in Chapter 11, the hour angle of the star may be found at once by Eq. 38, and the azimuth of the star at the instant may be computed. Since the close circumpolars move very slowly in azimuth and errors in the observed times will thus have but a small effect upon the computed azimuths, it is evident that only such stars should be used if precise results are sought.

Certain advantages accrue from observing a circumpolar star at any hour angle rather than at elongation. First, the number of observations may be increased materially and greater accuracy thereby secured. Second, the observation may be made at any convenient time, without the necessity of computing the time of elongation and of waiting for such elongation to occur. Third, when the observation is made on the relatively bright *polestar*, it is usually possible to sight the star during twilight when terrestrial objects may still be seen distinctly and no illumination of the field of the telescope is necessary.

Polaris is the only close circumpolar whose position is currently published in the *Ephemeris*. Since it is the brightest circumpolar in either hemisphere it should be used in preference to others whenever practicable. The positions of other circumpolars will be found tabulated in *Apparent Places of Fundamental Stars*. If the time is uncertain and *Polaris* is near the meridian, in which case the computed azimuth would be uncertain, it will be preferable to use *51 H. Cephei,* because this star would then be near its elongation, and comparatively large errors in the time would have little effect upon the computed azimuth.

The observations may be made with either repeating or direction instruments. The reader is referred to *Special Publication* 14, United States Coast and Geodetic Survey, for a discussion of precise azimuth determinations with the direction instrument. When a repeating theodolite

or ordinary transit is used the observations consist in repeating the angle between a mark and the star a certain number of times and then reversing the instrument and making another half-set containing the same number of repetitions. Since the star is continually changing its azimuth it is necessary to read and record the time of each pointing on the star with the vertical cross hair. The altitude of the star should be measured just before and again just after each half-set so that the altitude for any desired instant may be obtained by simple interpolation.

At all places in the United States the celestial pole is at such high altitudes that errors in the adjustment of the horizontal axis and of the line of sight will have a comparatively large effect upon the results. For this reason a striding level should be used for best results, provided that one is available for the instrument. The striding level permits the inclination of the horizontal axis to be measured while the telescope is pointing toward the star. Both ends of this level bubble are read, with the level first in the direct and then in the reversed position, and the inclination correction computed (see Art. 74). When the azimuth mark is much above or below the horizon a similar correction must be applied to the readings on the mark. When no striding level is available the plate level parallel to the horizontal axis should be sensitive and well adjusted, and it should be recentered before the second half-set is begun. The error caused by inclination of the axis may also be eliminated by taking half of the observations direct and half on the image of the star reflected in a basin of mercury.

In computing the results the azimuth of the star may be calculated for each of the observed times and the mean of these azimuths combined with the mean of the measured horizontal angles. The labor involved in this process is so great, however, that the common practice is first to compute the azimuth corresponding to the *mean of the observed times* and then to correct this result for the effect of the

curvature of the star's path, that is, by the difference between the mean azimuth and the azimuth at the mean of the times.

For observations made with the engineer's transit both the curvature correction and the inclination correction may be omitted provided that the instrument is in good adjustment and the time spent in making the pointings is limited to about 20 minutes. For the most precise results a correction must be applied for diurnal aberration in addition to those for inclination and curvature.

For a precise computation of the azimuth of the star Eq. 32 may be used (when the time is known with precision).

$$\tan Z_n = -\frac{\sin t}{\cos \phi \tan \delta - \sin \phi \cos t} \tag{32}$$

the minus sign being introduced in this form of the equation to give the azimuth from the north toward the east.

A second form may be obtained by dividing the numerator and denominator by $\cos \phi \tan \delta$, giving

$$\tan Z_n = -\frac{\cot \delta \sec \phi \sin t}{1 - \cot \delta \tan \phi \cos t}. \tag{83}$$

If $\cot \delta \tan \phi \cos t$ is denoted by a, then

$$\tan Z_n = -\cot \delta \sec \phi \sin t \frac{1}{1-a} \tag{84}$$

If values of $\log 1/(1 - a)$ are tabulated for different values of $\log a$ the use of this third form will be found more rapid than the others. Such tables will be found in *Special Publication* 14, United States Coast and Geodetic Survey.

For a less precise value of the azimuth we may use Eq. 31,

$$\sin Z = \sin t \cos \delta \sec h \tag{31}$$

For a close circumpolar such as *Polaris* we may substitute the arcs for the sines of small angles without appreciable error, obtaining

$$Z = p \sin t \sec h \tag{85}$$

where Z and p are both in seconds or both in minutes of angle. The precision of the computed azimuth depends largely upon the precision with which h can be measured. If the altitude of the star has been carefully determined and the latitude of the station is not well established, Eq. 85 should be used. If the vertical circle of the transit cannot be relied upon and the latitude is accurately known, Eqs. 32 or 84 will yield better results.

EXAMPLE 1. On February 10, 1947, it was desired to determine the azimuth of the line Middlesex-Suffolk by observation on *Polaris* at any hour angle. Instrument at station Middlesex; lat. 42° 21′ 00″ N, approximate long. 71° 06′ W. The longitude of this station had not been well established, making it necessary to observe for the local sidereal time instead of deducing it from radio time signals.

A time observation was accordingly made on *Regulus* (α *Leonis*): R.A. $10^h\ 05^m\ 33^s.83$; decl. +12° 13′ 35″.1, star in east.

Telescope	Vertical Angle	Watch
Direct	17° 05′	$07^h\ 17^m\ 53^s$ P.M., E.S.T.
	17 31	07 20 08
Reversed	17 49	07 21 44
	18 03	07 23 45

Mean	17 37	07 20 52.5
Refr.	− 02 58	
Alt.	17 34 02	

h	17° 34′ 02″
ϕ	42 21 00
p	77 46 25
$2s$	137 41 27
s	68 50 43
$s-h$	51 16 41
$s-p$	−08 55 42
$s-\phi$	26 29 43

$$\tan \tfrac{1}{2}t = \sqrt{\frac{\cos s \sin (s-h)}{\cos (s-p) \sin (s-\phi)}} \qquad (19)$$

$\log \cos s$	9.5573721
$\log \sin (s-h)$	9.8922009
$\log \sec (s-p)$	0.0052944
$\log \csc (s-\phi)$	0.3505443
	19.8054117
$\log \tan \frac{1}{2}t$	9.9027058
$\frac{1}{2}t$	38° 38′ 06″
t	= 77 16 12 E
	= $05^h\ 09^m\ 04^s.8$ E

H.A. *Regulus*	$18^h\ 50^m\ 55^s.2$
R.A. *Regulus*	10 05 33 .8
L.S.T.	04 56 29 .0
This corresponds to watch reading of	07 20 52 .5

The transit was then sighted at station Suffolk and the angle between the transit line and *Polaris* was repeated three times in both the direct and reversed positions of the telescope with the following results:

Polaris observation for azimuth. Line Middlesex to Suffolk. February 10, 1947. Lat. 42° 21′ 00″ N, long. 71° 06′ W.

Object Observed	Telescope	Repetitions	Horizontal Angle	Vernier *A*	Vernier *B*	Mean	Vertical Angle	Watch (E.S.T.)
Suffolk	*D*	0	00° 00′	00″	00″	00″		
Polaris		1	30 10	20			43° 01′ 00″	$07^h\ 26^m\ 10^s$ P.M.
		2						07 27 44
		3	90 29	30	30	30	43 00 30	07 29 12
Suffolk	*R*	0	00 00	00	00	00		
Polaris		1					43 00 00	07 31 09
		2						07 32 40
		3	90 24	00	00	00	42 58 30	07 34 23
	Means		30° 08′ 55″				43° 00′ 00″	$07^h\ 30^m\ 13^s$

Solar time of mean reading on *Polaris*	$07^h\ 30^m\ 13^s$
Solar time of mean reading on *Regulus*	07 20 52 .5
Difference in solar units	09 20 .5
Table III	1 .5
Difference in sidereal units	09 22 .0
L.S.T. of observation on *Regulus*	04 56 29 .0
L.S.T. of mean reading on *Polaris*	05 05 51 .0
R.A. of *Polaris*	01 46 04 .5
H.A. of *Polaris*	$03^h\ 19^m\ 46^s.5$
	49° 56′ 37″.5

Computing the azimuth of *Polaris* by Eq. 32, after determining from the *Ephemeris* that the star's declination was +89° 01′ 07″, we have

$\log \cos \phi$	9.8686700
$\log \tan \delta$	1.7662394
$\log \cos \phi \tan \delta$	1.6349094
$\cos \phi \tan \delta$	43.1429
$\log \sin \phi$	9.8284393
$\log \cos t$	9.8085752
$\log \sin \phi \cos t$	9.6370145
$\sin \phi \cos t$	0.4335
denominator	42.7094

$\log \sin t$	9.8838959
log denominator	1.6305235
$\log \tan Z$	8.2533724
Z	01° 01′ 36″ (west of north)

Azimuth of *Polaris*	178° 58′ 24″
Measured horizontal angle	30 08 55
Azimuth of line (Middlesex-Suffolk)	148° 49′ 29″

Solving by Eq. 85 and taking p as 3533″ and h, corrected for refraction, as 42° 59′ 00″, we have

$\log p$	3.5481436
$\log \sin t$	9.8838959
$\log \sec h$	0.1357548
$\log Z$	3.5677943
Z	3696″ = 01° 01′ 36″ west of north

This checks the result obtained by Eq. 32.

This same method may be applied in obtaining an approximate azimuth of *Polaris* with sufficient precision for checking the angles of a traverse through the use of tables published in the *Ephemeris* or the *Nautical Almanac* or by the use of Tables G and H of this volume.

As in the preceding example, the values recorded consist of the horizontal angle from the mark to *Polaris*, the measured altitude of the star, and the watch time at the instant of pointing on the star. The number of pointings may be reduced to one in each position of the telescope if desired.

The local sidereal time may be obtained as outlined above or may be deduced from the watch time of observation, provided that the error of the timepiece and the local longitude are known. The local sidereal time, combined with the right ascension of *Polaris*, will give the hour angle of the star. Table IV of the *Ephemeris* tabulates the azimuth of *Polaris* at all hour angles. The arguments in the table are the hour angle of the star and the latitude of the station. The azimuth of the star east or west of the north point is obtained by double interpolation. For the example given we would compute the hour angle as before and enter the table with the hour angle of $03^h\ 19^m\ 46^s.5$ and latitude 42° 21′ 00″. Table IV of the *Ephemeris* gives

H. A. \ Lat.	42°	44°
$3^h\ 10^m$	0° 59′.2	1° 01′.2
3 20	1 01 .4	1 03 .5

The interpolation gives the azimuth as $01° 01'.7 = 01° 01' 42''$ (west of north) which checks substantially the result obtained by Eqs. 32 and 85. A somewhat rougher interpolation may be made from Table IV of the *Nautical Almanac* which is similar except that the hour angle argument is given in units of arc rather than of time and the values of the azimuth are given only to the nearest one-tenth degree. If the *Ephemeris* is not at hand the azimuth may be found from Tables G and H. The watch time of the observation is corrected for the known error of the watch and then converted into local time. From Table V the local civil time of the upper culmination of *Polaris* may be found. The difference between these two times is the star's hour angle in mean solar units. This should be converted into sidereal units by applying a Table III correction, or the hour angle in sidereal units should be calculated as in the example given above. It should be observed that if the time of upper culmination is less than the observed time the difference is the hour angle measured toward the west, and the star is therefore west of the meridian if this hour angle is less than 12^h. If the time of upper culmination is greater than the observed time the difference is the hour angle measured toward the east (or 24^h − the true hour angle), and the star is east of the meridian if this angle is less than 12^h.

To obtain the azimuth from Tables G and H we use the formula

$$Z' = p' \sin t \sec h \tag{85}$$

Table G gives values of $p' \sin t$ for the years 1945, 1950, and 1955, and for every 4^m (or 1°) of hour angle. To multiply by sec h, enter Table H with the value of $p' \sin t$ at the top and the altitude h at the side. The number in the table is to be added to $p' \sin t$ to obtain the azimuth Z'.

Example 2. As an illustration of the use of these tables and of the application of Table V to the determination of the local hour angle of *Polaris*, we

will take the same *Polaris* observation as in Example 1. Instead of solving for the hour angle by the observation on *Regulus*, we will assume that the long. 71° 06′ W is correct and that the watch is known to be 54^s fast on E.S.T. From Table V we have

90° W L.C.T. of U.C. *Polaris* 2/10/47	$16^h\ 25^m\ 29^s$
Correction for long. (note *c*)	+ 11
	16 25 40
Correction to standard time (note *d*)	− 15 36
E.S.T. of U.C. for local place	16 10 04
Watch time of *Polaris* observation (see Example 1)	19 30 13
Watch fast	− 54
E.S.T. of *Polaris* observation	19 29 19
E.S.T. of U.C.	16 10 04
Difference (mean solar time units)	03 19 15
Table III	+ 32
L.H.A. of *Polaris* (star west of meridian)	$03^h\ 19^m\ 47^s$

Entering Table G we find by double interpolation, for this hour angle and for the year 1947, the value of $p' \sin t$ to be 45′.3. The sum to be added from Table H is found to be 16′.5, giving a total of 61′.8. The azimuth of *Polaris* is therefore 01° 01′ 48″ west of north. Combining this with the mean horizontal angle between the mark and the star (from Example 1), we obtain the azimuth of the line Middlesex-Suffolk as 148° 49′.3. This value checks closely the more precise value obtained from a solution by Eq. 32.

116 Azimuth by an Altitude of the Sun

The most commonly applied method of determining the azimuth of a line from solar observations consists of making a series of pointings on the sun for each of which pointings the time, the sun's altitude and the horizontal angle between the transit line and the sun are read. From the mean altitude, the known latitude, and the sun's declination at the instant corresponding to the mean of the observed times, the astronomical triangle is solved for the sun's azimuth by applying any one of Eqs. 22 to 29. This solar azimuth is then combined with the mean horizontal angle to give the desired azimuth of the line.

The instrument should be set up over one end of the transit line, carefully leveled, the plate vernier set at 0° and the vertical cross hair sighted on the point marking

TABLE G

Values of p' sin t for Polaris (in Minutes)

t	1945	1950	1955	t	t	1945	1950	1955	t
0h 00m	0.0	0.0	0.0	12h 00m	3h 00m	42.2	41.2	40.2	9h 00m
04	1.0	1.0	1.0	11 56	04	43.0	41.9	40.9	8 56
08	2.1	2.0	2.0	52	08	43.7	42.6	41.5	52
12	3.1	3.0	3.0	48	12	44.4	43.3	42.2	48
0 16	4.2	4.1	4.0	11 44	3 16	45.1	44.0	42.9	8 44
20	5.2	5.1	5.0	40	20	45.8	44.6	43.5	40
24	6.2	6.1	5.9	36	24	46.4	45.3	44.1	36
28	7.3	7.1	6.9	32	28	47.1	45.9	44.8	32
0 32	8.3	8.1	7.9	11 28	3 32	47.7	46.5	45.4	8 28
36	9.3	9.1	8.9	24	36	48.3	47.1	46.0	24
40	10.4	10.1	9.9	20	40	48.9	47.7	46.5	20
44	11.4	11.1	10.8	16	44	49.5	48.3	47.1	16
0 48	12.4	12.1	11.8	11 12	3 48	50.1	48.9	47.6	8 12
52	13.4	13.1	12.8	08	52	50.7	49.4	48.2	08
56	14.5	14.1	13.7	04	56	51.2	49.9	48.7	04
1 00	15.5	15.1	14.7	11 00	4 00	51.7	50.5	49.2	8 00
1 04	16.5	16.1	15.7	10 56	4 04	52.3	51.0	49.7	7 56
08	17.5	17.0	16.6	52	08	52.8	51.4	50.2	52
12	18.5	18.0	17.6	48	12	53.2	51.9	50.6	48
16	19.5	19.0	18.5	44	16	53.7	52.4	51.1	44
1 20	20.4	19.9	19.4	10 40	4 20	54.2	52.8	51.5	7 40
24	21.4	20.9	20.4	36	24	54.6	53.2	51.9	36
28	22.4	21.8	21.3	32	28	55.0	53.6	52.3	32
32	23.3	22.8	22.2	28	32	55.4	54.0	52.7	28
1 36	24.3	23.7	23.1	10 24	4 36	55.8	54.4	53.0	7 24
40	25.3	24.6	24.0	20	40	56.1	54.8	53.4	20
44	26.2	25.5	24.9	16	44	56.5	55.1	53.7	16
48	27.1	26.5	25.8	12	48	56.8	55.4	54.0	12
1 52	28.1	27.4	26.7	10 08	4 52	57.1	55.7	54.3	7 08
56	29.0	28.2	27.5	04	56	57.4	56.0	54.6	04
2 00	29.9	29.1	28.4	10 00	5 00	57.7	56.3	54.9	7 00
04	30.8	30.0	29.3	9 56	04	58.0	56.5	55.1	6 56
2 08	31.7	30.9	30.1	9 52	5 08	58.2	56.8	55.3	6 52
12	32.5	31.7	30.9	48	12	58.4	57.0	55.6	48
16	33.4	32.6	31.8	44	16	58.7	57.2	55.8	44
20	34.3	33.4	32.6	40	20	58.8	57.4	55.9	40
2 24	35.1	34.2	33.4	9 36	5 24	59.0	57.5	56.1	6 36
28	36.0	35.1	34.2	32	28	59.2	57.7	56.2	32
32	36.8	35.9	35.0	28	32	59.3	57.8	56.4	28
36	37.6	36.7	35.7	24	36	59.4	57.9	56.5	24
2 40	38.4	37.5	36.5	9 20	5 40	59.5	58.0	56.6	6 20
44	39.2	38.2	37.3	16	44	59.6	58.1	56.7	16
48	40.0	39.0	38.0	12	48	59.7	58.2	56.7	12
52	40.7	39.7	38.7	08	52	59.7	58.2	56.8	08
2 56	41.5	40.5	39.5	9 04	5 56	59.7	58.3	56.8	6 04
3 00	42.2	41.2	40.2	9 00	6 00	59.8	58.3	56.8	6 00

This table is based on the mean polar distance of *Polaris* for each year.

TABLE H

Correction for Altitude

Alt.	*p* sin t 10′	20′	30′	40′	50′	60′	Proportional Parts 1′	2′	3′	4′	5′	6′	7′	8′	9′
15°	0′.4	0′.7	1′.1	1′.4	1′.8	2′.1	0′.0	0′.1	0′.1	0′.1	0′.2	0′.2	0′.2	0′.3	0′.3
18	0.5	1.0	1.5	2.1	2.6	3.1	0.1	0.1	0.2	0.2	0.3	0.3	0.4	0.4	0.5
21	0.7	1.4	2.1	2.8	3.6	4.3	0.1	0.1	0.2	0.3	0.4	0.4	0.5	0.6	0.6
24	0.9	1.9	2.8	3.8	4.7	5.7	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
27	1.2	2.4	3.7	4.9	6.1	7.3	0.1	0.2	0.4	0.5	0.6	0.7	0.9	1.0	1.1
30	1.5	3.1	4.6	6.2	7.7	9.3	0.2	0.3	0.5	0.6	0.8	0.9	1.1	1.2	1.4
31	1.7	3.3	5.0	6.7	8.3	10.0	0.2	0.3	0.5	0.7	0.8	1.0	1.2	1.3	1.5
32	1.8	3.6	5.4	7.2	9.0	10.8	0.2	0.4	0.5	0.7	0.9	1.1	1.3	1.4	1.6
33	1.9	3.8	5.8	7.7	9.6	11.5	0.2	0.4	0.6	0.8	1.0	1.2	1.3	1.5	1.7
34	2.1	4.1	6.2	8.2	10.3	12.4	0.2	0.4	0.6	0.8	1.0	1.2	1.4	1.6	1.9
35	2.2	4.4	6.6	8.8	11.0	13.2	0.2	0.4	0.7	0.9	1.1	1.3	1.5	1.8	2.0
36	2.4	4.7	7.1	9.4	11.8	14.2	0.2	0.5	0.7	0.9	1.2	1.4	1.7	1.9	2.1
37	2.5	5.0	7.6	10.1	12.6	15.1	0.3	0.5	0.8	1.0	1.3	1.5	1.8	2.0	2.3
38	2.7	5.4	8.1	10.8	13.5	16.1	0.3	0.5	0.8	1.1	1.3	1.6	1.9	2.1	2.4
39	2.9	5.7	8.6	11.5	14.3	17.2	0.3	0.6	0.9	1.1	1.4	1.7	2.0	2.3	2.6
40	3.1	6.1	9.2	12.2	15.3	18.3	0.3	0.6	0.9	1.2	1.5	1.8	2.1	2.4	2.7
40–30	3.2	6.3	9.5	12.6	15.8	18.9	0.3	0.6	1.0	1.3	1.6	1.9	2.2	2.5	2.8
41	3.3	6.5	9.8	13.0	16.3	19.5	0.3	0.7	1.0	1.3	1.6	2.0	2.3	2.6	2.9
41–30	3.4	6.7	10.1	13.4	16.8	20.1	0.3	0.7	1.0	1.4	1.7	2.0	2.4	2.7	3.0
42	3.5	6.9	10.4	13.8	17.3	20.7	0.3	0.7	1.0	1.4	1.7	2.1	2.4	2.8	3.1
42–30	3.6	7.1	10.7	14.2	17.8	21.3	0.4	0.7	1.1	1.4	1.8	2.2	2.5	2.9	3.2
43	3.7	7.3	11.0	14.7	18.4	22.0	0.4	0.7	1.1	1.5	1.8	2.2	2.6	2.9	3.3
43–30	3.8	7.6	11.4	15.1	18.9	22.7	0.4	0.8	1.1	1.5	1.9	2.3	2.7	3.0	3.4
44	3.9	7.8	11.7	15.6	19.5	23.4	0.4	0.8	1.2	1.6	2.0	2.3	2.7	3.1	3.5
44–30	4.0	8.0	12.1	16.1	20.1	24.1	0.4	0.8	1.2	1.6	2.0	2.4	2.8	3.2	3.6
45	4.1	8.3	12.4	16.6	20.7	24.9	0.4	0.8	1.2	1.7	2 1	2.5	2.9	3.3	3.7
45–30	4.3	8.5	12.8	17.1	21.3	25.6	0.4	0.8	1.3	1.7	2.1	2.6	3.0	3.4	3.8
46	4.4	8.8	13.2	17.6	22.0	26.4	0.4	0.9	1.3	1.8	2.2	2.6	3.1	3.5	3.9
46–30	4.5	9.0	13.6	18.1	22.6	27.2	0.5	0.9	1.4	1.8	2.3	2.7	3.2	3.6	4.1
47	4.7	9.3	14.0	18.7	23.3	28.0	0.5	0.9	1.4	1.9	2.3	2.8	3.3	3.7	4.2
47–30	4.8	9.6	14.4	19.2	24.0	28.8	0.5	1.0	1.4	1.9	2.4	2.9	3.4	3.8	4.3
48	4.9	9.9	14.8	19.8	24.7	29.7	0.5	1.0	1.5	2.0	2.5	3.0	3.5	4.0	4.4
48–30	5.1	10.2	15.3	20.4	25.5	30.5	0.5	1.0	1.5	2.0	2.6	3.1	3.6	4.1	4.6
49	5.2	10.5	15.7	21.0	26.2	31.5	0.5	1.0	1.6	2.1	2.6	3.1	3.7	4.1	4.7
49–30	5.4	10.8	16.2	21.6	27.0	32.4	0.5	1.0	1.6	2.2	2.7	3.2	3.8	4.3	4.9
50	5.6	11.1	16.7	22.2	27.8	33.3	0.6	1.1	1.7	2.2	2.8	3.3	3.9	4.4	5.0
50–30	5.7	11.4	17.2	22.9	28.6	34.3	0.6	1.1	1.7	2.3	2.9	3.4	4.0	4.6	5.2
51	5.9	11.8	17.7	23.6	29.5	35.3	0.6	1.2	1.8	2.4	2.9	3.5	4.1	4.7	5.3
51–30	6.1	12.1	18.2	24.3	30.3	36.4	0.6	1.2	1.8	2.4	3.0	3.6	4.3	4.9	5.5
52	6.2	12.5	18.7	25.0	31.2	37.5	0.6	1.2	1.9	2.5	3.1	3.7	4.4	5.0	5.6
52–30	6.4	12.8	19.3	25.7	32.1	38.6	0.6	1.3	1.9	2.6	3.2	3.9	4.5	5.1	5.8
53	6.6	13.2	19.8	26.5	33.1	39.7	0.7	1.3	2.0	2.6	3.3	4.0	4.6	5.3	6.0
53–30	6.8	13.6	20.4	27.2	34.1	40.9	0.7	1.4	2.0	2.7	3.4	4.1	4.8	5.4	6.1
54	7.0	14.0	21.0	28.1	35.1	42.1	0.7	1.4	2.1	2.8	3.5	4.2	4.9	5.6	6.3
54–30	7.2	14.4	21.7	28.9	36.1	43.3	0.7	1.4	2.2	2.9	3.6	4.3	5.0	5.8	6.5
55	7.4	14.9	22.3	29.7	37.2	44.6	0.7	1.5	2.2	3.0	3.7	4.5	5.2	5.9	6.7

the other end of the line. The colored shade glass is then adjusted over the eyepiece, the upper clamp loosened, and the telescope turned toward the sun, the image of which is then brought into sharp focus. The sun's image should be made to traverse the field of view, by manipulation of the vertical tangent screw so that the error of mistaking one of the stadia hairs for the horizontal cross hair may be avoided. The apparent direction of motion of the sun's image should be observed carefully so that the proper quadrants may be selected for the pointings (see Art. 73). In the illustrative example given below the sun appears to be moving downward to the right so the tangency of the limbs to the cross hairs is perfected in the lower left and upper right quadrants.

Three or four pointings are normally made in each quadrant to insure accuracy and increase the precision. The telescope is reversed after the first half-set if the instrument has a full vertical circle, otherwise the index correction must be determined and applied.

In lieu of a colored eyepiece a card may be used to reflect the image of the sun as indicated in Art. 64. A solar shield, such as that described in Art. 65, is superior to either the colored eyepiece or the card in making this observation.

After the pointings on the sun have been completed the telescope should be turned to the mark again and the vernier reading checked to make certain that no movement of the lower plate has occurred. The differences in successive readings in each half-set should then be noted and their proportionality tested.

In computing the azimuth it is customary to neglect the curvature of the sun's path during the short interval between the first and last pointings, unless the series extends over a longer period than is usually required to make such observations. Near noon the curvature is greater than when the sun is observed near the prime vertical. With the observation made at least 3 hours before or after noon

and the pointings made within a 10-minute time interval the error introduced by neglecting curvature will be negligible for transit accuracy.

A major defect of this method is the difficulty of measuring the sun's altitude with precision with the transit. Near noon the sun's azimuth is changing much more rapidly than its altitude so that a small error in altitude will introduce a large error in azimuth. With the sun at least 3 hours away from the meridian the azimuth error resulting from an error in the measured altitude will be less than the latter value. The observation should not be made, however, with the sun less than 10° above the horizon because of uncertainty in the refraction correction.

The mean altitude corrected for refraction and parallax is the true altitude of the sun's center. The latitude of the observer must be known. The sun's declination must be computed for the instant corresponding to the mean of the observed times (the watch should not have a large error or else the error in determining the declination will affect the azimuth). These data suffice to permit the computation of the sun's azimuth which is then combined with the mean horizontal angle to give the azimuth of the mark.

If for any reason a series has been observed in one quadrant only, the mean vertical angle must be corrected by the angular value of the sun's semidiameter, and the mean horizontal angle from the mark to the observed limb must be reduced to the center of the sun by applying the correction $s \sec h$, where s is the semidiameter and h is the altitude of the center.

For observations made in the southern hemisphere the same formulae as used in the northern hemisphere may be applied either by considering the latitude ϕ as negative and employing the regular forms or by taking ϕ as positive and using the *south polar distance* instead of the north polar distance when employing Eq. 24; the resulting azimuth

in the second case will be that measured from the south point of the meridian. If Eq. 25 is used we may take ϕ as positive, reverse the sign of δ, and obtain the azimuth of the sun from the south point.

Example. To illustrate this method we shall compute the azimuth from the data of the observation on the sun for time and azimuth given as the example in Art. 96, p. 222. The field data are reproduced below.

Solar observation for time and azimuth at N.J.G.C.S., Monument 501. June 16, 1947. Lat. 40° 43′ 21″ N, long. 74° 17′ 49″ W.

Object Observed	Telescope	Horizontal Angle	Vertical Angle	Watch Time (E.D.S.T.)
Monument 500	D	000° 00′		
		150 57	52° 23′	03^h 38 47^s P.M.
		(15′)	(13′)	(71^s)
☉		151 12	52 10	03 39 58
		(19)	(16)	(86)
		151 31	51 54	03 41 24
	R	151 15	50 53	03 44 07
		(17)	(15)	(80)
☉		151 32	50 38	03 45 27
		(16)	(14)	(73)
		151 48	50 24	03 46 40
Monument 500		000 00		
Means		151° 22′ 30″	51° 23′ 40″	03^h 42^m $43^s.8$

The following values are taken from the time computation on pp. 222–223.

Corrected alt.	51° 23′ 00″	$s - p$	12° 43′ 30″
Corrected decl.	+23 20 40	$s - \phi$	38 39 29
s	79 22 50	$s - h$	27 59 50

Solving by

$$\tan \frac{Z}{2} = \sqrt{\frac{\sin (s - \phi) \sin (s - h)}{\cos s \cos (s - p)}} \tag{24}$$

log sin $(s - \phi)$	9.7956515
log sin $(s - h)$	9.6715697
log sec s	0.7345102
log sec $(s - p)$	0.0108001
	0.2125315
log tan $Z/2$	0.1062658
$Z/2$	51° 56′ 27″
Z	103 52 54
Angle, mark to sun	151 22 30

True bearing	S 75° 15′ 24″ E	(Line from Monument 501 to Monument 500)
Azimuth	284 44 36	(Clockwise from south)

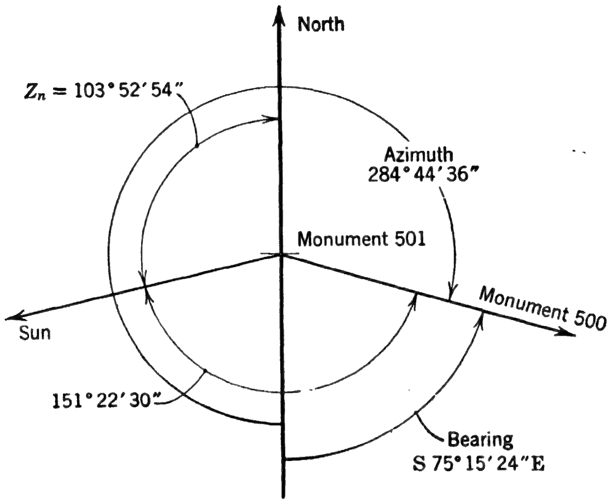


Fig. 83. Diagram for Example, Art. 116

117 Solar Azimuth When the Time Is Known

In Fig. 30, p. 48, the three sides and three angles of the astronomical triangle joining the pole, the zenith, and the sun are illustrated. In Art. 96 it was indicated that, when the sun's polar distance and zenith distance and the observer's colatitude are known or have been measured, the sun's hour angle can be calculated and the watch correction determined, provided that the observer's longitude is known. In Art. 116 it was shown that, with the same three parts of the astronomical triangle known, the sun's azimuth could be calculated and combined with a measured horizontal angle between a mark and the sun to give the azimuth of the mark. It was further pointed out that a major defect in this latter method lay in the difficulty of measuring the sun's altitude (and hence the zenith distance)

precisely with the engineer's transit. It is clear that the sun's azimuth may also be determined from a solution of this triangle if we know the colatitude, the sun's local hour angle, and its polar distance.

The present widespread availability of radio time signals makes possible the close determination of the sun's local hour angle, provided that the observer's longitude is known, and hence eliminates the necessity of measuring the sun's altitude. Provided that the longitude is accurately known and the error of the watch can be determined from radio signals to the nearest second, the error of the resulting azimuth should be less than 15 seconds of arc. Furthermore, the time method permits greater flexibility than the altitude method with respect to the hours during which the observation may be made. In the latitude of Newark and with the sun south of the equator the observation may be carried out even at noon without the error in azimuth exceeding the error in determining the hour angle. For all declinations of the sun the observation may be made when the sun is at least 2 hours on either side of noon without appreciable azimuth error.

The procedure in making the observation is similar to that described in Art. 116 with the exception that the vertical circle reading may be omitted. The sun's image may be sighted in opposite quadrants as was done in the example in Art. 116, or the time may be recorded at the instant that a small segment of the sun's image, placed above the horizontal cross hair, is accurately bisected by the vertical wire as the sun moves westward. Some observers prefer to record the time when the east limb of the image leaves (becomes tangent to) the vertical wire, with the point of tangency approximately on the horizontal cross hair. In this case the altitude of the sun must be measured, or computed from Eq. 30 so that the measured horizontal angle from the mark to the limb of the sun may be reduced to the sun's center by multiplying the semidiameter by secant

h and adding this correction to the measured (clockwise) horizontal angle. The sun's azimuth may be computed by Eq. 32, or Napier's Analogies may be used to derive Eqs. 86 and 87, from which Z may be determined readily by logarithmic computation.

$$\tan\frac{(Z+S)}{2} = \frac{\cos\dfrac{(\phi-\delta)}{2}\cot\dfrac{t}{2}}{\sin\dfrac{(\phi+\delta)}{2}} \qquad (86)$$

$$\tan\frac{(Z-S)}{2} = \frac{\sin\dfrac{(\phi-\delta)}{2}\cot\dfrac{t}{2}}{\cos\dfrac{(\phi+\delta)}{2}} \qquad (87)$$

Example. Compute the azimuth of the line Tacna-Tacora from the following solar observation. Date, September 25, 1947. Instrument at station Tacna, lat. 17° 42′ 08″ S; long. 70° 08′ 15″ W. Watch is 03ˢ fast on E.D.S.T.

Object Observed	Telescope	Horizontal Angle		Watch	
Tacora	*D*	00° 00′ 00″			
		78 15 30		$10^h\ 56^m\ 31^s$ A.M.	
$\overline{\odot}\vert$			(15′.5)		(61^s)
		78 00 00		10 57 32	
			(16)		(63)
		77 44 00		10 58 35	
	R	76 20 30		10 59 53	
			(16.5)		(65)
		76 04 00		11 00 58	
$\underline{\vert\odot}$			(14)		(55)
		75 50 00		11 01 53	
Tacora		00 00 00			
Means		77° 02′ 20″		$10^h\ 59^m\ 13^s.7$	

Watch	$10^h\ 59^m\ 13^s.7$ A.M.
Watch fast	03
E.D.S.T.	10 59 $10^s.7$
Long. W	04
G.C.T.	14 59 10 .7

G.C.T.	$14^h\ 59^m\ 10^s.7$
Eq. T.	$+\quad 08\ \ 07.7$
G.A.T.	15 07 18 .4
Long. W	04 40 33 .0
L.A.T.	10 26 45 .4 A.M.
M.A. (H.A. east)	$01^h\ 33^m\ 14^s.6$
	$=23°\ 18'\ 39''$
Decl. (from *Ephemeris*)	−00 40 27
Lat.	−17 42 08

Since both the sun and the observer are south of the equator it will be convenient to consider δ and ϕ as positive and solve Eq. 32 for the azimuth of the sun east of the *south* point. Using natural functions and a calculating machine we have

$$\tan Z = \frac{\sin t}{\cos\phi \tan\delta - \sin\phi \cos t} \tag{32}$$

$\sin 23°\,18'\,39'' = 0.395719 =$ numerator
$\cos 17°\,42'\,08'' \times \tan 00°\,40'\,27'' = 0.952650 \times 0.011766 = 0.011209$
$\sin 17\ \ 42\ \ 08 \times \cos 23\ \ 18\ \ 39 = 0.304070 \times 0.918372 = 0.279249$
$0.011209 - 0.279249 = -0.268040 =$ denominator

$$\tan Z = \frac{0.395719}{-0.268040} = 1.476343 \text{ (negative)}$$

Z	124° 06′ 42″ (east of south)
Horizontal angle	77 02 20

From Fig. 84

True bearing, Tacna-Tacora	N 21° 09′ 02″ W
Azimuth, Tacna-Tacora	158 50 58

118 Azimuth by the Use of the Solar Attachment

The several types of solar attachment have been described in Art. 66. This device, regardless of type, has been designed to permit the sides of the astronomical triangle joining the sun, pole, and zenith to be set off instrumentally in accordance with their angular values at the time and place of observation.

The manner of making the two or three arc settings required prior to the positioning of the instrument differs slightly with the several types. The settings for the Smith attachment, currently most widely used, will be considered here.

The transit, with solar telescope attached, is set up over the desired station, and the latitude and the sun's corrected declination are carefully set off on the latitude and declination arcs. The solar telescope is then revolved in its collar bearings until the hour circle index indicates, as closely as possible, the local apparent time of observation. The transit is carefully leveled, the horizontal circle set at zero,

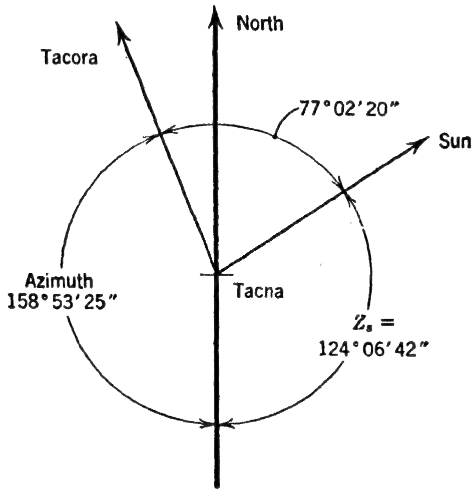


Fig. 84. Diagram for Example, Art. 117

and, with the lower motion free, the sun's image is brought into the field of the solar telescope. The lower motion is then clamped and the sun's image placed accurately between the equatorial wires by manipulating the lower tangent screw and, if necessary, by rotating the solar telescope slightly in its collar bearings.

In this position the solar telescope points toward the celestial pole, the mirror attached to the declination arm is directed toward the sun, the vertical axis of the transit points toward the zenith, and the transit telescope rotates in the plane of the meridian. The upper motion is then

loosened and the vertical cross hair set on the next survey station, the resulting horizontal angle being thus a measure of the azimuth of the transit line.

If the time of observation is predetermined and the latitude of the station to be occupied closely determined and its longitude approximately known, all calculations necessary for the arc settings may be completed before going into the field. The refraction correction to be applied to the declination is that corresponding to the observer's latitude, the sun's local hour angle (east or west of the meridian), and the sun's declination. This correction may be taken from *Standard Field Tables,* published by the General Land Office; from the ephemerides issued by the several instrument makers; or from the tables of many surveying textbooks. Tables I and VIII of this text are *not* applicable. When the sun and the observer are on the same side of the equator the correction is to be added algebraically to the declination; when they are on opposite sides of the equator it should be subtracted algebraically.

EXAMPLE. Compute the settings for a solar attachment azimuth observation at 9^h A.M., E.S.T., April 21, 1947, in lat. 42° 30′ N, long. 72° 40′ W.

E.S.T.	$09^h\ 00^m\ 00^s$	Decl. at Greenwich 14^h	+11° 41′ 42″
Long. W	05 00 00	Refr. correction	+ 47
G.C.T.	14 00 00	(Taken from Table II, Berger *Ephemeris* for 1947)	
Eq. T.	+01 10		
G.A.T.	14 01 10		
Long. W	04 50 40	Corrected decl.	+11° 42′ 29″
L.A.T.	09 10 30		

The settings then, to the nearest ½ minute, are

Lat.	42° 30′ .0
Decl.	+11 42 .5
H.A.	$02^h\ 49^m.5$

119 Azimuth by an Altitude of a Star near the Prime Vertical

The method described in Art. 116 applies equally well to an observation on a star, except that the star's image is

bisected with both cross hairs, and the parallax and semidiameter corrections become zero. The declination of the star changes so little during one day that it may be regarded as constant, and consequently the time of observation is not required. Errors in the altitude and the latitude may be partially eliminated by combining the results of two observations, one on a star about due east and the other on one about due west.

EXAMPLE. Determine the true bearing of the mark from the following observation on *Pollux* (*β Geminorum*), February 1, 1947, star in east. Lat. 40° 43′ 40″ N, long. $4^h\ 56^m\ 15^s$ W; obs. alt. 28° 01′ 30″; I.C. −30″; horizontal angle (clockwise), mark to star, 46° 02′ 30″. From the *Nautical Almanac* we find the declination of *Pollux* on this date to be +28° 09′ 24″.

$$\tan \tfrac{1}{2}Z = \sqrt{\frac{\sin (s - \phi) \sin (s - h)}{\cos s \cos (s - p)}} \qquad (24)$$

Obs. alt.	28° 01′ 30″		
I.C.	− 30		
	28 01 00		
Refraction	01 47	log sin $(s - \phi)$	9.6185760
h	27 59 13	log sin $(s - h)$	9.7823842
ϕ	40 43 40	log sec s	0.3786141
p	61 50 36	log sec $(s - p)$	0.0007812
$2s$	130 33 29		19.7803555
s	65 16 44	log tan $\frac{1}{2}Z$	9.8901777
$s - \phi$	24 33 04	$\frac{1}{2}Z$	37° 49′ 54″
$s - h$	37 17 31	Z	75 39 58
$s - p$	03 26 08	Horizontal angle	46 02 30
		Bearing of mark	N 29° 37′ 28″ E

120 Favorable Conditions for Non-Meridian Altitude Azimuth Observations

From an inspection of the astronomical triangle it may be inferred that the nearer the star, sun, or other observed body is to the observer's meridian the less favorable are the conditions for an accurate determination of azimuth from a measured altitude. When the body is on the meridian the azimuth becomes indeterminate. Also, as the observer approaches the pole the accuracy diminishes, and when he is at the pole the azimuth is indeterminate.

To find from the equations the error in Z caused by an

error in h, differentiate Eq. 13, regarding h as the independent variable; the result is

$$0 = \sin\phi \cos h + \cos\phi\left(-\cos h \sin Z \frac{dZ}{dh} - \cos Z \sin h\right)$$

or

$$\cos\phi \cos h \sin Z \frac{dZ}{dh} = \sin\phi \cos h - \cos\phi \cos Z \sin h$$

$$= \cos\delta \cos S \text{ (by Eq. 14)}$$

therefore

$$\frac{dZ}{dh} = \frac{\cos\delta \cos S}{\cos\phi \cos h \sin Z}$$

which by Eq. 15

$$= \frac{\cos S}{\sin S \cos h}$$

$$= \frac{1}{\cos h \tan S} \tag{88}$$

If the declination of the body is greater than the latitude and of the same sign there will be an elongation, and at this point the angle S (at the sun or star) will be 90°; the error dZ will therefore be zero. For objects whose declinations are such that an elongation is possible it is clear that this is the most favorable position for an accurate determination of azimuth since an error in altitude has no effect upon Z.

If the declination is less than the latitude, or is of opposite sign, the most favorable position will depend partly upon S and partly upon h. From Eq. 15 it is seen that the maximum value of S occurs simultaneously with the maximum value of Z, that is, when the body is on the prime vertical ($Z = 90°$ or $270°$). To determine the influence of h suppose that there are two positions of the object, one north of the prime vertical and one south of it, such that the angle

S is the same for the two. The minimum error (dZ) will then occur where $\cos h$ is greater; this corresponds to the value of h which is the lesser and therefore on the side of the prime vertical toward the pole. The exact position of the body for greatest accuracy could be found for any particular case by differentiating the above expression and placing it equal to zero.

To find the error in the azimuth caused by an error in latitude differentiate Eq. 13 with respect to $d\phi$. This gives

$$0 = \sin h \cos \phi + \cos h \left(- \cos \phi \sin Z \frac{dZ}{d\phi} - \cos Z \sin \phi \right)$$

or

$$\cos h \cos \phi \sin Z \frac{dZ}{d\phi} = \sin h \cos \phi - \cos h \cos Z \sin \phi$$

$$= \cos \delta \cos t \text{ (by Eq. 16)}$$

therefore

$$\frac{dZ}{d\phi} = \frac{\cos \delta \cos t}{\cos h \cos \phi \sin Z}$$

$$= \frac{\sin Z \cos t}{\sin t \cos \phi \sin Z} \text{ (by Eq. 12)}$$

$$= \frac{1}{\tan t \cos \phi} \tag{89}$$

From this equation it is evident that the least error in Z resulting from an error in ϕ will occur when the object is on the 6-hour circle ($t = 90°$).

Combining the two results it is clear that observations on an object which is in the region between the 6-hour circle and the prime vertical will give results slightly better than elsewhere; observations on the body when on the other side of the prime vertical will, however, be almost as accurate. The most important matter so far as the spherical triangle is concerned is to avoid observations when the body is near the meridian.

The above discussion refers to the trigonometric conditions only. Another factor of great importance is the atmospheric refraction near the horizon. An altitude observed when the body is within 10° of the horizon is subject to large uncertainties in the refraction correction because this correction varies with temperature and pressure, and the observer often does not know what the actual conditions are. This error may be greater than the error of the spherical triangle. When the two requirements are in conflict it will often be better to observe the body nearer to the meridian, than would ordinarily be advisable, rather than to take the observation when the body is too low for good observing. In the winter in high latitudes the interval of time during which a solar observation may be made is rather limited so that it is not possible to observe very near the prime vertical. The only remedy is to obtain the altitude and the latitude with greater accuracy if this is possible.

121 Azimuth by Equal Altitudes of a Star

The meridian may be found in a very simple manner by means of two equal altitudes of a star, one east of the meridian and one west. This method has the advantage that the coordinates of the star are not required so that the *Almanac* or other table is not necessary. The method is somewhat inconvenient because it requires two observations at night several hours apart. It is of special value to surveyors in the southern hemisphere where there is no bright star near the pole. The star to be used should be approaching the meridian (in the evening) and about 3^h or 4^h from it. The altitude should be a convenient one for measuring with the transit, and the star should be one which can be identified with certainty 6^h or 8^h later. Care should be taken to use a star which will reach the same altitude on the opposite side of the meridian before daylight interferes with the observation. In the northern hemisphere one of the bright stars in *Cassiopeia* might be used while a star in

the *Southern Cross* might serve an observer south of the equator. The actual star or stars selected will of course depend upon which constellations are favorably positioned at the date and time of observation.

When several hours east of the meridian (west in the case of a circumpolar star approaching lower culmination) the image of the star should be bisected with both cross hairs and the altitude read and recorded. A note or sketch should be made showing which star is used. The direction of the star should be marked on the ground by setting a tack in a stake or else the horizontal angle should be measured from some reference mark to the position of the star at the time of observation. When the star is approaching the same altitude on the opposite side of the meridian, the telescope should be set at exactly the same altitude as was read at the first observation. When the star comes into the field of view it is bisected with the vertical cross hair and followed in azimuth until it reaches the horizontal hair. The motion in azimuth should be stopped at this instant. Another point is then set on the ground (at the same distance from the transit as the first) or else another angle is turned from the same reference mark. The bisector of the angle between the two directions is the meridian line through the transit. It will usually be found more practicable to turn angles from a fixed mark to the star than to set stakes. The accuracy of the result may be increased by observing the star at several different altitudes and using the mean of the horizontal angles.

In this method the index correction (or that part of it caused by nonadjustment) is eliminated since it is the same for both observations. For all practical purposes the refraction error is also eliminated. Error in the adjustment of the horizontal axis and the line of sight will be eliminated if the first half of the set is taken with the telescope direct and the second half with the telescope reversed (applicable only when transit has full vertical circle). Care

should be taken to relevel the plates just before the observation is begun; the leveling should not, of course, be done between the pointing on the mark and the pointing on the star but may be done whenever the lower clamp is loose.

122 Observation for Meridian by Equal Altitudes of the Sun in the Forenoon and in the Afternoon

The general method of Art. 121 may be applied to the sun by measuring the horizontal angle between a mark and the sun when it has a certain altitude in the forenoon and measuring the angle again to the sun when it has an equal altitude in the afternoon. Since the sun's declination will change during the interval, the mean of the two angles will not be the true angle between the meridian and the mark but will require a small correction. The angle between the upper branch of the meridian and the point midway between the two directions of the sun is given by the equation

$$\text{Correction} = \frac{\frac{1}{2}d}{\cos\phi \sin t} \tag{90}$$

where d is the hourly change in declination multiplied by the number of hours elapsed between the two observations, ϕ is the latitude, and t is the hour angle of the sun, or approximately half the elapsed interval of time. The correction depends upon the change in the declination, not upon its absolute value, so that the hourly change may be taken with sufficient accuracy from the *Almanac* for any year for the corresponding date.

In making the observation the instrument is set up at one end of the line whose azimuth is to be determined and the plate vernier set at 0°. The point at the other end of the line is bisected by the vertical cross hair, using the lower motion. The card or sun glass is then put in position, the upper clamp loosened, and the telescope pointed at the sun. It is not necessary to observe on both edges

TABLE J

VARIATION PER HOUR IN SUN'S DECLINATION (1947)

Day of Month	January	February	March	April	May	June	July	August	September	October	November	December
1	+11″	+42″	+57″	+58″	+46″	+21″	−09″	−37″	−54″	−58″	−48″	−24″
5	16	45	58	57	44	18	13	40	55	58	46	20
10	21	48	59	56	40	13	18	43	57	57	43	15
15	27	51	59	54	36	08	23	46	58	56	39	09
20	32	53	59	52	32	+02	27	49	58	54	35	−03
25	36	+55	59	50	28	−03	31	51	59	52	30	+03
30	+40		+58	+47	+23	−08	−35	−53	−58	−50	−26	+09

of the sun but is sufficient to sight, say, the upper limb at both observations and to sight the vertical cross hair on the opposite limb in the afternoon from that used in the forenoon. The horizontal hair is therefore set on the *upper* limb and the vertical hair on the *left* limb. When the instrument is in this position the time should be noted as accurately as possible. The altitude and the horizontal angle are both read. In the afternoon the instrument is set up at the same point, and the same observation is made, except that the vertical hair is now sighted on the *right* limb; the horizontal hair is set on the *upper* limb as before. A few minutes before the sun reaches an altitude equal to that observed in the morning the vertical circle is set to read exactly the same altitude as was read at the first observation. As the sun's altitude decreases the vertical hair is kept tangent to the right limb until the upper edge of the sun is in contact with the horizontal hair. At this instant the time is again noted accurately; the horizontal angle is then read. The mean of the two circle readings, corrected for the change in declination, is the angle from the mark to the south point of the horizon. The algebraic sign of the correction is determined from the fact that if, for an observer in the northern hemisphere, the sun is increasing in declination the mean of the two vernier readings lies to the west of the south point, and vice versa. The precision of the result may be increased

124 Convergence of the Meridians

When observations for azimuth are made at two different points of a survey for the purpose of verifying the angular measurements, the convergence of the meridians at the two places is likely to be appreciable whenever the difference in their longitudes exceeds a few thousand feet. At the equator the two meridians are parallel, regardless of their difference in longitude; at the poles the convergence is equal to the difference in longitude. For surveys of small areas, extending not over 30 miles in latitude or longitude, it may be shown that the convergence always equals the product of the difference in longitude and the sine of the mean or middle latitude between the two places (for surveys of larger areas small correction factors must be introduced). Table VII has been computed in accordance with this formula, the convergence in seconds of arc being given for each degree of latitude and for each 1000 feet of distance along the parallel of latitude.

Whenever it is desired to check the measured angles of a traverse between two stations at which azimuths have been observed the differences in latitude and departure (longitude) should be computed for each line and the total differences in latitude and departure of the two azimuth stations obtained. Then, in the column containing the number of thousands of feet in this departure and on the line corresponding to the middle latitude, the angular convergence of the meridians will be found. The convergence for numbers not in the table may be determined by combining those that are given. For instance, that for 66,500 feet, in latitude 40° N, may be found by adding together 10 times the angle for 6000 feet, the angle for 6000 feet, and one-tenth the angle for 5000 feet. The result is 549″.3, the correction to be applied to the second observed azimuth to refer the line to the first meridian.

EXAMPLE. At station 1 (lat. 40° 35′ N) the azimuth of the line 1 to 2 was found to be 82° 15′ 20″. The survey proceded in a general southwesterly

direction to station 21 in lat. 40° 25′ N (new latitude computed from survey and checked by observation) at which point the azimuth of the line 21 to 22 was found by observation to be 89° 10′ 40″. Calculation of the departures from the distances and bearings of the survey indicated that station 21 was 45,670 feet westerly of station 1. From Table VII the convergence for 45,670 feet and for the middle latitude of 40° 30′ is found to be 06′ 24″.0. The azimuth of line 21 to 22 as calculated from the azimuth of line 1 to 2, using

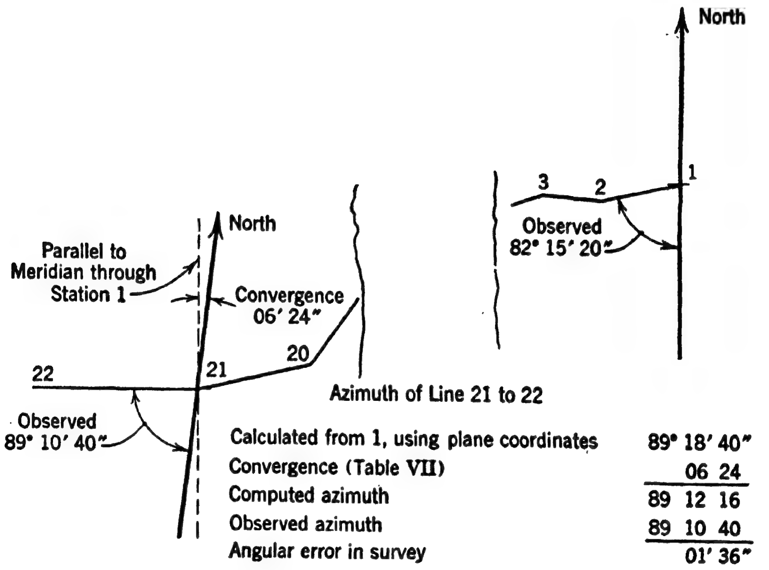


Fig. 85

the measured deflection angles of the survey, was 89° 18′ 40″. Applying the convergence correction of 06′ 24″ to this, we obtain 89° 12′ 16″ as the calculated true azimuth of line 21 to 22. Comparing this with the azimuth of 89° 10′ 40″, as obtained astronomically, we obtain 01′ 36″ as the value of the accumulated angular error of the survey between stations 1 and 21. The relationship of the azimuths is evident from a study of Fig. 85.

125 Grid Azimuths and True Azimuths

In executing surveys in those states where state-wide plane coordinate systems are in use, or in utilizing military fire-control and tactical maps based on the progressive military grid systems of the United States or other nations, it is of

fundamental importance to distinguish between azimuths referred to the true meridian at a survey point and azimuths referred to the grid north of the plane coordinate system.

Strictly speaking, we should also distinguish between the true geodetic azimuth of a survey line, as computed from the geodetic latitudes and longitudes of the end stations and the observed astronomic azimuth of this same line. This difference, resulting from the local deflection of the vertical at the station, is too slight, however, to be of significance with respect to observations made with the engineer's transit. For our present purposes therefore, we can consider that either a computed geodetic azimuth or an observed astronomic azimuth will refer to the same true meridian through the station within the limits of precision of the observations.

The grid azimuth of a line, on the other hand, is the azimuth referred to the grid north of the plane coordinate system. At any survey point grid north will be a line parallel to the central meridian of the system and will therefore depart from the true meridian through the station unless the station lies along the central meridian of the system. This difference in direction between true north and grid north is called the *grid declination* (from the analogy to magnetic declination) or the *gisement*. Since its magnitude, for a given coordinate system, is dependent on the difference in longitude between the central meridian and the station occupied, the gisement is essentially similar in character to the convergence of true meridians previously discussed in Art. 124. The actual value of the gisement is, however, also governed by the properties of the projection on which the given plane coordinate system is based and must be taken from a special table for that system rather than from Table VII.

The two systems currently used for state-wide plane coordinates are based upon either the Lambert conformal (conic) projection or the transverse Mercator (cylindrical)

projection. The former is widely used in those states having their greatest dimension in an east-west direction; the latter is used where the state extends chiefly north and south. Certain states, such as New York, may use the Lambert conformal projection for one part of the state (Long Island) and one or more transverse Mercator systems (three in this instance) for other portions of the state. The rectangular grid system for progressive military maps in the United States is based upon the polyconic projection. In all these plane coordinate systems the means of reconciling true and grid azimuths is essentially the same, differing only in detail in the computation of the gisement.

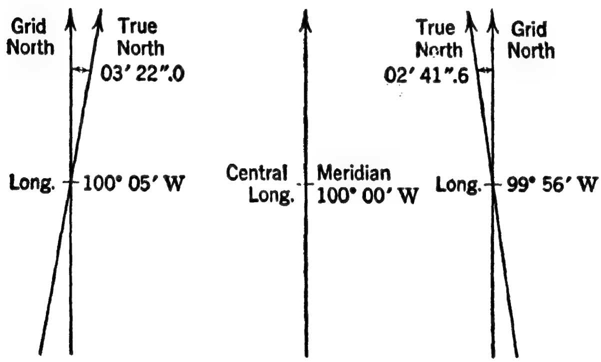


Fig. 86

Projection tables have been computed for each state or portion thereof using the Lambert system. A single grid declination table suffices for all states using the transverse Mercator projection. The above tables are supplied in pamphlet form by the United States Coast and Geodetic Survey. Other gisement tables (See *Technical Manuals* 7 and 19, Army Map Service) provide similar data for the military maps of the United States.

Table K shows a small portion of the Lambert projection table for the Nebraska (north) plane coordinate system.

Here the convergence angle θ is taken out directly opposite the observer's longitude. Figure 86 shows the difference between true and grid north for points in northern Nebraska

TABLE K

EXCERPT FROM LAMBERT PROJECTION TABLE FOR NEBRASKA (NORTH)

$1''$ of long. $= 0''.67345079$ of θ

Longitude	θ
99° 56′	+00° 02′ 41″.6282
57	+00 02 01 .2211
58	+00 01 20 .8141
59	+00 00 40 .4070
100 00	00 00 00 .0000
100° 01′	−00° 00′ 40″.4070
02	−00 01 20 .8141
03	−00 02 01 .2211
04	−00 02 41 .6282
05	−00 03 22 .0352

in longitudes 99° 56′ W and 100° 05′ W respectively, the central meridian being that of longitude 100° 00′ W. Thus, if an observer in northern Nebraska, in longitude 99° 56′ W, determined from star observations the astronomic azimuth of a survey line and desired to compare this azimuth with the corresponding grid azimuth of the same line, it would be necessary to subtract 00° 02′ 41″.6 from the true azimuth before a correct comparison could be made.

Table L shows a portion of the grid declination table for those states using the transverse Mercator projection for their plane coordinate systems. In this instance, in addition to knowing the difference in longitude between the central meridian of the particular state system and the meridian of the observer, the observer's latitude must be known as well.

When a local survey, oriented on an observed azimuth

TABLE L

TABLE OF $\Delta\alpha$—TRANSVERSE MERCATOR SYSTEMS

Latitude	$\Delta\lambda = \lambda$ Central Meridian $- \lambda$ of Station						Tabular Difference	
	0° 05′	0° 10′	0° 15′	0° 20′	0° 25′	0° 30′	$\Delta\lambda = 1'$	$\Delta\lambda = 1''$
40°00′	03′12″.8	06′25″.7	09′38″.5	12′51″.4	16′04″.2	19′17″.0	−38.57	−0.643
10	13 .5	27 .0	40 .5	54 .0	07 .5	21 .0	−38.71	−0.645
20	14 .2	28 .3	42 .5	56 .7	10 .9	25 .0	−38.84	−0.647
30	14 .8	29 .7	44. 5	12 59 .3	14 .2	29 .0	−38.97	−0.650
40	15 .5	31 .0	46 .5	13 02 .0	17 .5	33 .0	−39.10	−0.652
50	16 .2	32 .3	48 .5	04 .6	20 .8	37 .0	−39.23	−0.654

at the initial point, is extended to and tied in with a monumented line for which the grid azimuth on the state plane coordinate system has been published, the surveyor has a dual problem to solve in checking the accuracy of his angular measurements: (1) the determination of the meridian convergence between his initial local survey point and the end of the monumented line; and (2) the reconciliation between the true and grid azimuths at the monumented point.

EXAMPLE. To illustrate this problem, assume that a local survey in the state of New Jersey (which has a state plane coordinate system on the transverse Mercator projection with the central meridian in long. 74° 40′ 00″ W) started at point *A* in lat. 40° 43′ N, long. 74° 17′ W. At this point the astronomic azimuth of the first line of the survey was determined to be 246° 10′.5. The survey was extended in a general northeasterly direction and tied-in to the monumented line M105–M106 which had a published grid azimuth of 271° 43′.6. The position of M105 was lat. 40° 49′ N, long. 74° 08′ W. The computed algebraic sum of the departures of the traverse courses was 41,553 feet and the algebraic sum of the traverse deflection angles was 25° 42′ right. Thus the computed azimuth of the line M105–M106, based on the starting astronomic azimuth and the deflection angles of the survey would be as follows:

Starting astronomic azimuth, line *A–B*		246° 10′.5	
Algebraic sum of deflection angles		25 42	Right
Total		271 52 .5	
Table VII convergence for 41,553 feet (ϕ = 40° 46′)	+	05 .9	
Computed true azimuth of line M105–M106		271° 58′.4	

To compare this latter azimuth with the published grid azimuth of the line we must apply a convergence correction for the transverse Mercator projection (Table L). The longitude difference between M105 and the central meridian of the system is 00° 32′. The correction for this longitude difference and for latitude 40° 49′ amounts to 20′.9. Since the grid azimuth will be less than the true azimuth for a station east of the central meridian we subtract this correction from the computed true azimuth found above. This gives a computed grid azimuth of the line M105–M106 of 271° 37′.5 compared with the published value of 271° 43′.6. This indicates an accumulated angular error in the survey of 06′.1.

Problems

1. Compute the approximate E.S.T. of the eastern elongation of *Polaris* on September 10 of the current year. Observer in lat. 40° 43′ N, long. 74° 11′ W. Use Table V of this text and check by Table VII of *Ephemeris*.

2. Compute the exact E.S.T. of the western elongation of *Polaris* on March 7 of the current year. Observer in lat. 42° 21′.5 N, long. 71° 06′ W. Refer to *Ephemeris* or *Almanac* for necessary solar and star data.

3. Compute the azimuth of *Polaris* at elongation from the data of Problem 2.

4. On March 10 in a certain year a transit was set up over triangulation station Dana and sighted on a light shown from station Howe. A horizontal angle of 65° 34′ 30″ was then turned clockwise to *Polaris* when the star was at western elongation. Position of Dana 40° 45′ 35″ N, 74° 16′ 40″ W; decl. of *Polaris* +88° 59′ 47″. Compute the true bearing of Howe from Dana.

5. On April 12 in a certain year it was intended to make an observation on *Polaris* at western elongation. At the instant of elongation the star was obscured by clouds. Twelve minutes later a horizontal angle of 163° 23′ 40″ was measured clockwise from mark to star. Observer in lat. 44° 46′ N, long. 10° 50′ E; decl. of *Polaris* +88° 59′ 37″. Compute the azimuth of the mark.

6. An observer in Johannesburg, South Africa, lat. 26° 11′ 14″ S, long. $01^h\ 52^m\ 07^s$ E, desired to make an azimuth observation at elongation on July 13 of the current year. Determine whether *β Carinae* (*β Argus* in *Almanac*) or *α Trianguli Australis* would be most favorably situated for such an observation at a convenient time in the evening. State whether the observation should be made at eastern or western elongation. Compute the -2^h zone time of elongation and the star's altitude and azimuth at this instant.

7. The following observation for azimuth was taken January 1 in Montevideo, Uruguay, lat. 34° 54′ S, long. $03^h\ 44^m\ 50^s$ W. Observation on *Miaplacidus* (*β Carinae*) at eastern elongation. Horizontal angle, clockwise, mark to star, 121° 12′. Determine the true bearing of the mark. Compute the $+4^h$ zone time of elongation and the approximate altitude which was laid off to bring the star into the field of view. Use *Ephemeris* data for the current year.

8. On September 23 of a certain year an observer in long. 71° 07′ 30″ W turned a clockwise angle of 155° 41′ from a mark to *Polaris*. The vertical

angle to the star was 42° 25′ with no index error. The corrected $+4^h$ zone time of observation was $20^h\ 23^m\ 22^s$. The right ascension and declination of *Polaris* for the instant of observation were $01^h\ 46^m\ 03^s.8$ and +88° 59′ 36″ .1 respectively. Compute the latitude of the observer and the true bearing of the mark. R.A.M.S. $+12^h$ at 0^h G.C.T. September 24 was $0^h\ 07^m\ 49^s.016$.

9. Compute the azimuth of the mark from the following observation on the sun, May 18, 1944.

Object Observed	Horizontal Angle	Vertical Angle	Watch Time
Mark	00° 00′		
$\underline{\odot}\vert$	50 35	46° 51′	$03^h\ 52^m\ 06^s$ P.M.
	50 46	46 41	03 52 57
	50 58	46 31	03 53 52
$\vert\overline{\odot}$	50 35	45 41	03 55 32
	50 48	45 29	03 56 33
	50 59	45 19	03 57 29
Mark	00 00		

Lat. 40° 44′ 30″ N, long. 74° 11′ 00″ W; I.C. 00′; sun's decl. at 0^h G.C.T. May 18 +19° 28′ 24″.1 and at 0^h G.C.T. May 19 +19° 41′ 32″.8; Eq. T. at 0^h G.C.T. May 18 $+03^m\ 43^s.02$ and at 0^h G.C.T. May 19 $+03^m\ 40^s.72$. Watch was 27^s fast of E.D.S.T.

10. Compute the azimuth of the line *A-B* from the following solar observation. Date, May 18, 1944. Instrument at station *A*, lat. 40° 44′ 30″ N, long. 74° 11′ 00″ W.

Object Observed	Telescope	Horizontal Angle	Eastern Daylight Saving Time
B	*D*	000° 00′ 00″	
$\vert\overline{\odot}$		152 04 00	$04^h\ 02^m\ 10^s$ P.M.
		152 17 30	04 03 14
		152 30 00	04 04 16
$\overline{\odot}\vert$	*R*	153 41 00	04 06 17
		153 50 00	04 07 02
		154 01 00	04 08 01
B		000 00 00	

See problem 9 for sun's declination and the equation of time.

11. Compute the settings for a Smith solar attachment azimuth observation to be made at 09^h A.M., $+5^h$ zone time, on September 10 of the current year, in lat. 37° 42′ S, long. 69° 20′ W.

12. Determine the azimuth of the mark from the following observation on *Regulus* (α *Leonis*), February 5, 1945, star in east. Lat. 40° 43′ 35″ N, long. 74° 17′ 35″ W; obs. alt. 36° 02′; horizontal angle (clockwise), mark to star 187° 23′ 30″; decl. of *Regulus* on Greenwich date of observation +12° 14′ 08″.

13. On April 19 of a certain year, in lat. 42° 18′ N, long. 71° 09′ W, a transit was set over station Dorchester, and the following observation on the sun at equal altitudes was made to determine the azimuth of the line Dorchester-Atlantic.

Observation	Object Observed	Horizontal Angle	Vertical Angle	Time
A.M.	Atlantic	000° 00′ 00″		
	☉	337 03 30	24° 58′	$08^h\ 19^m\ 30^s$
P.M.	Atlantic	000 00 00		
	☉	142 17 15	24 58	05 12 15

The watch was regulated to E.D.S.T. The increase in the sun's declination for the date may be taken from Table J. Compute the azimuth of the line.

14. Compute the azimuth of the mark from the following observation on the sun near noon. Use *Ephemeris* data for the current year, assuming the sun to have the altitude shown at the corresponding instant of this year. Date, March 3. Lat 42° 01′ N., long. 71° 07′ W. The watch is three seconds fast of E.S.T.

Object Observed	Horizontal Angle	Vertical Angle	Watch Time
Mark	00° 00′		
☉	36 56	40° 31′	$11^h\ 58^m\ 50^s$
☉	38 09	41 02	12 00 20
Mark	00 00		

15. The observed altitude of *Arcturus* (α *Bootis*) was 35° 10′ 30″ at a certain instant on August 14, the star being in the west; horizontal angle (clockwise) from a mark to the star 156° 30′ 30″; lat. 40° 44′ 30″ N, long. $07^h\ 05^m\ 20^s$ W. Using *Ephemeris* data for the current year, compute the true bearing of the mark.

16. A transit and tape traverse was run from Hub *A* to Hub *H*. The bearing of the line *AB* was known to be true west. The deflection angles were measured as follows: at *B*, 08° 41′ *L*; at *C*, 11° 28′ *R*; at *D*, 06° 29′ *L*; at *E*, 03° 47′ *R*; at *F*, 16° 20′ *L*; and at *G*, 88° 14′ *L*. With the transit over Hub *G*, an azimuth observation was taken on *Polaris* at eastern elongation. The horizontal angle, clockwise, from the line *GH* to *Polaris* was 195° 52′. Hub *G* was in lat. 40° 44′ 30″ N, long. 74° 11′ 10″ W. Hub *G* lay to the west

of hub A a distance of 14,200 feet, as determined by computation of the traverse. Decl. of *Polaris* on the date of observation $+ 88° 59' 05''$. Determine the discrepancy between the astronomic azimuth of GH and its azimuth as computed from the traverse notes.

17. Prove that the horizontal angle between the center of the sun and the right or left limb is $s \sec h$ where s is the apparent angular semidiameter and h is the apparent altitude. Show a sketch.

18. On a certain date, in lat. 17° 42′ S, long. 70° 08′ W, a transit was set over station *Tacna* and, with vernier reading zero, was sighted at station *Caplina*. A series of six pointings were made on the sun, three with telescope direct and the sun sighted in the second quadrant and three with telescope reversed and with the sun in the fourth quadrant. Horizontal angles were read clockwise from *Caplina* to the sun. The means of the horizontal angles, vertical angles, and times were 25° 43′ 50″, 61° 31′, and $09^h\ 58^m\ 44^s$ respectively. Decl. of the sun at the instant of observation $-00° 51' 05''$. Compute the azimuth of the line *Tacna-Caplina*.

19. Station 0 + 00 of a highway relocation survey was established at Hub A in lat. 40° 45′ N, long. 74° 19′ W. The astronomic bearing of the line AB of this survey was observed and computed as S 65° 36′ 20″ E. It was desired to compute the survey on New Jersey grid bearings. Determine the New Jersey grid bearing of the line AB. The central meridian of the New Jersey Transverse Mercator system is in long. 74° 40′ 00″ W. The survey was extended in an easterly direction and at station 89 + 53.40 was tied in on Monument 698 of the N.J.G.C.S. The grid bearing of the line Monument 698-Monument 699 was known to be S 73° 16′ 37″ E. The algebraic sum of the deflection angles from Hub B to and including that measured at Monument 698 was 07° 25′ 00″ left. Determine the angular error in the highway survey.

20. Compute the bearing of the mark from the following observation on the sun, taken on February 26. Use *Ephemeris* data for the current year. Lat. 40° 44′ 30″ N, long. 74° 11′ 10″ W.

Object Observed	Horizontal Angle	Vertical Angle	Eastern Standard Time
Mark	000° 00′		
$\underline{\odot}\vert$	175 16	24° 03′	$03^h\ 22^m\ 35^s$ P.M.
	175 27	23 56	03 23 24
	175 36	23 50	03 24 08
$\vert\overline{\odot}$	175 15	23 07	03 25 16
	175 25	23 00	03 26 04
	175 33	22 55	03 26 39
Mark	000 00		

TABLES

TABLE I. MEAN REFRACTION.

Barometer, 29.5 inches. Thermometer, 50° F.

App. Alt.	Refr.	App. Alt.	Refr.	App. Alt.	Refr.	App. Alt.	Refr.
0° 00′	33′ 51″	10° 00′	5′ 13″	20° 00′	2′ 36″	35° 00′	1′ 21″
30	28 11	30	4 59	30	2 32	36 00	1 18
1 00	23 51	11 00	4 46	21 00	2 28	37 00	1 16
30	20 33	30	4 34	30	2 24	38 00	1 13
2 00	17 55	12 00	4 22	22 00	2 20	40 00	1 08
30	15 49	30	4 12	30	2 17	42 00	1 03
3 00	14 07	13 00	4 02	23 00	2 14	44 00	0 59
30	12 42	30	3 54	30	2 11	46 00	0 55
4 00	11 31	14 00	3 45	24 00	2 08	48 00	0 51
30	10 32	30	3 37	30	2 05	50 00	0 48
5 00	9 40	15 00	3 30	25 00	2 02	52 00	0 45
30	8 56	30	3 23	26 00	1 57	54 00	0 41
6 00	8 19	16 00	3 17	27 00	1 52	56 00	0 38
30	7 45	30	3 10	28 00	1 47	58 00	0 36
7 00	7 15	17 00	3 05	29 00	1 43	60 00	0 33
30	6 49	30	2 59	30 00	1 39	65 00	0 27
8 00	6 26	18 00	2 54	31 00	1 35	70 00	0 21
30	6 05	30	2 49	32 00	1 31	75 00	0 15
9 00	5 46	19 00	2 44	33 00	1 28	80 00	0 10
30	5 29	30	2 40	34 00	1 24	85 00	0 05
10 00	5 13	20 00	2 36	35 00	1 21	90 00	0 00

Correction for Air Temperature

App. Alt.	Height of Thermometer (Fahr.)			
	+20°	+40°	+60°	+80°
5°	+48″	+16″	−14″	−40″
10°	+26	+ 8	− 7	−22
15°	+17	+ 6	− 5	−15
20°	+13	+ 4	− 4	−11

TABLE II. FOR CONVERTING SIDEREAL INTO MEAN SOLAR TIME.

(Increase in Sun's Right Ascension for Sidereal h. m. s.)

Mean Time = Sidereal Time − C'.

Sid. Hrs.	Corr.	Sid. Min.	Corr.	Sid. Min.	Corr.	Sid. Sec.	Corr.	Sid. Sec.	Corr.
	m s		s		s		s		s
1	0 9.830	1	0.164	31	5.079	1	0.003	31	0.085
2	0 19.659	2	0.328	32	5.242	2	0.005	32	0.087
3	0 29.489	3	0.491	33	5.406	3	0.008	33	0.090
4	0 39.318	4	0.655	34	5.570	4	0.011	34	0.093
5	0 49.148	5	0.819	35	5.734	5	0.014	35	0.096
6	0 58.977	6	0.983	36	5.898	6	0.016	36	0.098
7	1 8.807	7	1.147	37	6.062	7	0.019	37	0.101
8	1 18.636	8	1.311	38	6.225	8	0.022	38	0.104
9	1 28.466	9	1.474	39	6.389	9	0.025	39	0.106
10	1 38.296	10	1.638	40	6.553	10	0.027	40	0.109
11	1 48.125	11	1.802	41	6.717	11	0.030	41	0.112
12	1 57.955	12	1.966	42	6.881	12	0.033	42	0.115
13	2 7.784	13	2.130	43	7.045	13	0.035	43	0.117
14	2 17.614	14	2.294	44	7.208	14	0.038	44	0.120
15	2 27.443	15	2.457	45	7.372	15	0.041	45	0.123
16	2 37.273	16	2.621	46	7.536	16	0.044	46	0.126
17	2 47.102	17	2.785	47	7.700	17	0.046	47	0.128
18	2 56.932	18	2.949	48	7.864	18	0.049	48	0.131
19	3 6.762	19	3.113	49	8.027	19	0.052	49	0.134
20	3 16.591	20	3.277	50	8.191	20	0.055	50	0.137
21	3 26.421	21	3.440	51	8.355	21	0.057	51	0.139
22	3 36.250	22	3.604	52	8.519	22	0.060	52	0.142
23	3 46.080	23	3.768	53	8.683	23	0.063	53	0.145
24	3 55.909	24	3.932	54	8.847	24	0.066	54	0.147
		25	4.096	55	9.010	25	0.068	55	0.150
		26	4.259	56	9.174	26	0.071	56	0.153
		27	4.423	57	9.338	27	0.074	57	0.156
		28	4.587	58	9.502	28	0.076	58	0.158
		29	4.751	59	9.666	29	0.079	59	0.161
		30	4.915	60	9.830	30	0.082	60	0.164

TABLE III. FOR CONVERTING MEAN SOLAR INTO SIDEREAL TIME.

(Increase in Sun's Right Ascension for Solar h. m. s.)

Sidereal Time = Mean Time + C.

Mean Hrs.	Corr.	Mean Min.	Corr.	Mean Min.	Corr.	Mean Sec.	Corr.	Mean Sec.	Corr.
	m s		s		s		s		s
1	0 9.856	1	0.164	31	5.093	1	0.003	31	0.085
2	0 19.713	2	0.329	32	5.257	2	0.005	32	0.088
3	0 29.569	3	0.493	33	5.421	3	0.008	33	0.090
4	0 39.426	4	0.657	34	5.585	4	0.011	34	0.093
5	0 49.282	5	0.821	35	5.750	5	0.014	35	0.096
6	0 59.139	6	0.986	36	5.914	6	0.016	36	0.099
7	1 8.995	7	1.150	37	6.078	7	0.019	37	0.101
8	1 18.852	8	1.314	38	6.242	8	0.022	38	0.104
9	1 28.708	9	1.478	39	6.407	9	0.025	39	0.107
10	1 38.565	10	1.643	40	6.571	10	0.027	40	0.110
11	1 48.421	11	1.807	41	6.735	11	0.030	41	0.112
12	1 58.278	12	1.971	42	6.900	12	0.033	42	0.115
13	2 8.134	13	2.136	43	7.064	13	0.036	43	0.118
14	2 17.991	14	2.300	44	7.228	14	0.038	44	0.120
15	2 27.847	15	2.464	45	7.392	15	0.041	45	0.123
16	2 37.704	16	2.628	46	7.557	16	0.044	46	0.126
17	2 47.560	17	2.793	47	7.721	17	0.047	47	0.129
18	2 57.417	18	2.957	48	7.885	18	0.049	48	0.131
19	3 7.273	19	3.121	49	8.049	19	0.052	49	0.134
20	3 17.129	20	3.285	50	8.214	20	0.055	50	0.137
21	3 26.986	21	3.450	51	8.378	21	0.057	51	0.140
22	3 36.842	22	3.614	52	8.542	22	0.060	52	0.142
23	3 46.699	23	3.778	53	8.707	23	0.063	53	0.145
24	3 56.555	24	3.943	54	8.871	24	0.066	54	0.148
		25	4.107	55	9.035	25	0.068	55	0.151
		26	4.271	56	9.199	26	0.071	56	0.153
		27	4.435	57	9.364	27	0.074	57	0.156
		28	4.600	58	9.528	28	0.077	58	0.160
		29	4.764	59	9.692	29	0.079	59	0.162
		30	4.928	60	9.856	30	0.082	60	0.164

TABLE IV.

PARALLAX — SEMIDIAMETER — DIP.

(A) Sun's parallax.

Sun's altitude.	Sun's parallax.
0°	9″
10	9
20	8
30	8
40	7
50	6
60	4
70	3
80	2
90	0

(B) Sun's semidiameter.

Date.	Semidiameter.
Jan. 1	16′ 18″
Feb. 1	16 16
Mar. 1	16 10
Apr. 1	16 02
May 1	15 54
June 1	15 48
July 1	15 46
Aug. 1	15 47
Sept. 1	15 53
Oct. 1	16 01
Nov. 1	16 09
Dec. 1	16 15

(C) Dip of the sea horizon.

Height of eye in feet.	Dip of sea horizon.
1	0′ 59″
2	1 23
3	1 42
4	1 58
5	2 11
6	2 24
7	2 36
8	2 46
9	2 56
10	3 06
11	3 15
12	3 24
13	3 32
14	3 40
15	3 48
16	3 55
17	4 02
18	4 09
19	4 16
20	4 23
21	4 29
22	4 36
23	4 42
24	4 48
25	4 54
26	5 00
27	5 06
28	5 11
29	5 17
30	5 22
35	5 48
40	6 12
45	6 36
50	6 56
55	7 16
60	7 35
65	7 54
70	8 12
75	8 29
80	8 46
85	9 02
90	9 18
95	9 33
100	9 48

TABLE V

LOCAL CIVIL TIME OF UPPER CULMINATION OF POLARIS IN THE YEAR 1947

(*Computed for* 90°, *or* 6 *hours west of Greenwich*)

Date, 1947		Civil time of upper culmination	Variation per day	Date, 1947		Civil time of upper culmination	Variation per day
January	1	$19^h\ 03^m\ 35^s$	$-3^m\ 57^s$	July	20	$05^h\ 57^m\ 04^s$	$-3^m\ 55^s$
January	11	18 24 04	−3 57	July	30	05 17 58	−3 55
January	21	17 44 32	−3 57	August	9	04 38 51	−3 55
January	31	17 05 01	−3 57	August	19	03 59 43	−3 55
February	10	16 25 29	−3 57	August	29	03 20 34	−3 55
February	20	15 45 58	−3 57	September	8	02 41 25	−3 55
March	2	15 06 29	−3 57	September	18	02 02 14	−3 55
March	12	14 27 01	−3 57	September	28	01 23 02	−3 55
March	22	13 47 36	−3 56	October	8	00 43 49	−3 56
April	1	13 08 13	−3 56	October	18	00 04 32	−3 56
April	11	12 28 52	−3 56	October	19*	00 00 36	−3 56
April	21	11 49 33	−3 56	October	19*	23 56 40	−3 56
May	1	11 10 18	−3 55	October	28	23 21 18	−3 56
May	11	10 31 03	−3 55	November	7	22 41 59	−3 56
May	21	09 51 51	−3 55	November	17	22 02 36	−3 56
May	31	09 12 40	−3 55	November	27	21 23 12	−3 57
June	10	08 33 31	−3 55	December	7	20 43 46	−3 57
June	20	07 54 24	−3 55	December	17	20 04 18	−3 57
June	30	07 15 17	−3 55	December	27	19 24 49	−3 57
July	10	06 36 11	−3 55	January	6 '48	18 43 44	−3 57

* Two upper culminations on this date. For the period for which Table V*a* has been prepared there will be two upper culminations on or about this date.

TABLE V*a*

MEAN TIME INTERVAL BETWEEN UPPER CULMINATION AND ELONGATION

Latitude	Time interval	Latitude	Time interval	Latitude	Time interval	Latitude	Time interval
10°	$5^h\ 58^m.3$	24°	$5^h\ 57^m.3$	38°	$5^h\ 56^m.0$	52°	$5^h\ 54^m.0$
12	5 58 .2	26	5 57 .1	40	5 55 .7	54	5 53 .6
14	5 58 .0	28	5 56 .9	42	5 55 .5	56	5 53 .2
16	5 57 .9	30	5 56 .8	44	5 55 .2	58	5 52 .7
18	5 57 .7	32	5 56 .6	46	5 55 .0	60	5 52 .2
20	5 57 .6	34	5 56 .4	48	5 54 .7	62	5 51 .6
22	5 57 .4	36	5 56 .2	50	5 54 .3	64	5 51 .0

Eastern elongation precedes and western elongation follows upper culmination by the time interval given in Table V*a*. The time interval between upper and lower culmination is 12^h diminished by one-half the numerical value of the variation per day as given in Table V.

NOTES

a. To refer the times in Table V to other years:

For Year		
1948 before March 1	add	$1^m.5$
1948 on and after March 1	subtract	2 .5
1949	subtract	0 .9
1950	add	0 .8
1951	add	2 .5
1952 before March 1	add	4 .3
1952 on and after March 1	add	0 .4
1953	add	2 .2
1954	add	4 .0
1955	add	5 .9
1956 before March 1	add	7 .7
1956 on and after March 1	add	3 .8
1957	add	5 .6
1958	add	7 .3
1959	add	8 .9
1960 before March 1	add	9 .3
1960 on and after March 1	add	6 .5
1961	add	8 .1

b. To refer to other than the tabular days: SUBTRACT from the time for the preceding tabular day the product of the variation per day and the days elapsed, as given below:

Days elapsed	Variation per day						Days elapsed	Variation per day					
	3^m	57^s	3^m	56^s	3^m	55^s		3^m	57^s	3^m	56^s	3^m	55^s
	m	*s*	*m*	*s*	*m*	*s*		*m*	*s*	*m*	*s*	*m*	*s*
1............	3	57	3	56	3	55	6............	23	42	23	36	23	30
2............	7	54	7	52	7	50	7............	27	39	27	32	27	25
3............	11	51	11	48	11	45	8............	31	36	31	28	31	20
4............	15	48	15	44	15	40	9............	35	33	35	24	35	15
5............	19	45	19	40	19	35							

c. To refer to any other than the tabular longitude (90°): ADD 0.1^m for each 10° east of the ninetieth meridian or SUBTRACT 0.1^m for each 10° west of the ninetieth meridian.

d. To refer to standard time: ADD to the quantities in Table V four minutes for every degree of longitude the place of observation is west of the standard meridian (60°, 75°, 90°, etc.). SUBTRACT when the place is east of the standard meridian.

TABLE VI

FOR REDUCING TO ELONGATION OBSERVATIONS MADE NEAR ELONGATION

Azimuth at Elon / Time*	1° 0′	1° 10′	1° 20	1° 30′	1° 40′	1° 50′	2° 0′	2° 10′	Azimuth at Elon / Time
m	$''$	$''$	$''$	$''$	$''$	$''$	$''$	$''$	*m*
0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0
1	0.0	0.0	0.0	+ 0.1	+ 0.1	+ 0.1	+0.1	+ 0.1	1
2	+ 0.1	+ 0.2	+ 0.2	0.2	0.2	0.3	0.3	0.3	2
3	0.3	0.4	0.4	0.5	0.5	0.6	0.6	0.7	3
4	0.5	0.6	0.7	0.8	0.9	1.0	1.1	1.2	4
5	+ 0.9	+ 1.0	+ 1.1	+ 1.3	+ 1.4	+ 1.6	+ 1.7	+ 1.9	5
6	1.2	1.4	1.6	1.8	2.1	2.3	2.5	2.7	6
7	1.7	2.0	2.2	2.5	2.8	3.1	3.4	3.7	7
8	2.2	2.6	2.9	3.3	3.7	4.0	4.4	4.8	8
9	2.8	3.2	3.7	4.2	4.6	5.1	5.6	6.0	9
10	+ 3.4	+ 4.0	+ 4.6	+ 5.1	+ 5.7	+ 6.3	+ 6.9	+ 7.4	10
11	4.1	4.8	5.5	6.2	6.9	7.6	8.3	9.0	11
12	4.9	5.8	6.6	7.4	8.2	9.0	9.9	10.7	12
13	5.8	6.8	7.7	8.7	9.7	10.6	11.6	12.6	13
14	6.7	7.8	9.0	10.1	11.2	12.3	13.4	14.6	14
15	+ 7.7	+ 9.0	+10.3	+11.6	+12.8	+14.1	+15.4	+16.7	15
16	8.8	10.2	11.7	13.2	14.6	16.1	17.5	19.0	16
17	9.9	11.5	13.2	14.9	16.5	18.2	19.8	21.5	17
18	11.1	12.9	14.8	16.7	18.5	20.4	22.2	24.1	18
19	12.4	14.4	16.5	18.6	20.6	22.7	24.7	26.8	19

* Sidereal time from elongation.

TABLE VII

Convergence of the Meridians in Seconds for Each 1000 Feet on the Parallel

Lat.	Distance (East or West)								
φ	1000	2000	3000	4000	5000	6000	7000	8000	9000
°	″	″	″	″	″	″	″	″	″
20	3.51	7.01	10.51	14.02	17.52	21.03	24.53	28.04	31.54
21	3.78	7.57	11.35	15.13	18.91	22.69	26.48	30.26	34.04
22	3.98	7.96	11.94	15.92	19.90	23.88	27.86	31.84	35.83
23	4.18	8.36	12.55	16.73	20.91	25.09	29.27	33.46	37.64
24	4.39	8.77	13.16	17.54	21.93	26.32	30.70	35.09	39.47
25	4.59	9.19	13.78	18.37	22.97	27.56	32.15	36.75	41.34
26	4.80	9.61	14.42	19.22	24.02	28.83	33.63	38.44	43.24
27	5.02	10.04	15.06	20.08	25.10	30.11	35.13	40.15	45.17
28	5.24	10.48	15.71	20.95	26.19	31.42	36.66	41.90	47.13
29	5.46	10.92	16.38	21.84	27.30	32.76	38.22	43.68	49.14
30	5.69	11.37	17.06	22.74	28.43	34.12	39.80	45.49	51.17
31	5.92	11.83	17.75	23.67	29.59	35.51	41.42	47.34	53.26
32	6.16	12.31	18.46	24.62	30.77	36.92	43.08	49.23	55.38
33	6.39	12.78	19.17	25.57	31.96	38.36	44.75	51.15	57.54
34	6.64	13.29	19.92	26.57	33.21	39.85	46.49	53.13	59.77
35	6.89	13.79	20.68	27.58	34.47	41.37	48.26	55.15	62.05
36	7.15	14.31	21.46	28.61	35.77	42.92	50.07	57.22	64.38
37	7.42	14.84	22.26	29.67	37.09	44.51	51.93	59.35	66.77
38	7.69	15.38	23.08	30.77	38.46	46.15	53.84	61.53	69.22
39	7.97	15.95	23.92	31.89	39.86	47.83	55.80	63.77	71.74
40	8.26	16.52	24.78	33.04	41.30	49.56	57.82	66.08	74.34
41	8.55	17.11	25.67	34.22	42.78	51.33	59.89	68.45	77.00
42	8.86	17.72	26.58	35.45	44.31	53.17	62.03	70.89	79.76
43	9.18	18.36	27.53	36.71	45.89	55.06	64.24	73.42	82.60
44	9.50	19.01	28.51	38.01	47.52	57.02	66.52	76.02	85.53
45	9.84	19.68	29.52	39.36	49.20	59.04	68.88	78.72	88.56
46	10.19	20.38	30.57	40.76	50.95	61.13	71.32	81.51	91.70
47	10.55	21.10	31.65	42.20	52.76	63.31	73.86	84.41	94.96
48	10.93	21.85	32.78	43.71	54.63	65.56	76.49	87.41	98.34
49	11.32	22.63	33.95	45.27	56.59	67.90	79.22	90.54	101.85
50	11.72	23.45	35.17	46.89	58.62	70.34	82.06	93.78	105.51

TABLE VIII

Correction for Parallax and Refraction to be Subtracted from Observed Altitude of the Sun

Apparent altitude	Temperature, centigrade							Apparent altitude
	−10°	0°	+10°	+20°	+30°	+40°	+50°	
	Temperature, Fahrenheit							
	14°	32°	50°	68°	86°	104°	122°	
°	′ ″	′ ″	′ ″	′ ″	′ ″	′ ″	′ ″	°
0	36 36	35 15	34 00	32 50	31 45	30 45	29 50	0
1	26 10	25 12	24 18	23 28	22 42	22 00	21 20	1
2	19 35	18 51	18 11	17 34	16 59	16 26	15 57	2
3	15 20	14 46	14 15	13 46	13 18	12 53	12 30	3
4	12 31	12 03	11 37	11 13	10 50	10 30	10 11	4
5	10 29	10 05	9 44	9 24	9 05	8 48	8 32	5
6	8 59	8 38	8 20	8 03	7 47	7 32	7 18	6
7	7 49	7 31	7 15	7 00	6 46	6 33	6 21	7
8	6 55	6 39	6 25	6 12	5 59	5 48	5 37	8
9	6 11	5 57	5 44	5 32	5 21	5 11	5 01	9
10	5 34	5 22	5 10	4 59	4 49	4 39	4 30	10
11	5 04	4 52	4 42	4 32	4 23	4 14	4 06	11
12	4 39	4 29	4 19	4 10	4 01	3 53	3 46	12
13	4 17	4 07	3 58	3 50	3 42	3 35	3 28	13
14	3 58	3 49	3 41	3 33	3 26	3 19	3 13	14
15	3 42	3 34	3 26	3 1[illegible]	3 12	3 06	3 00	15
16	3 27	3 19	3 12	3 05	2 59	2 53	2 47	16
17	3 14	3 07	3 00	2 54	2 48	2 42	2 37	17
18	3 02	2 55	2 49	2 43	2 37	2 32	2 27	18
19	2 52	2 45	2 39	2 33	2 28	2 23	2 19	19
20	2 42	2 36	2 30	2 25	2 20	2 15	2 11	20
21	2 33	2 27	2 22	2 17	2 12	2 08	2 04	21
22	2 26	2 20	2 15	2 10	2 06	2 02	1 58	22
23	2 18	2 13	2 08	2 03	1 59	1 55	1 51	23
24	2 12	2 07	2 02	1 58	1 54	1 50	1 46	24
25	2 05	2 00	1 56	1 52	1 48	1 44	1 41	25
26	2 00	1 55	1 51	1 47	1 43	1 39	1 36	26
27	1 55	1 50	1 46	1 42	1 38	1 35	1 32	27
28	1 49	1 45	1 41	1 37	1 34	1 31	1 28	28
29	1 45	1 41	1 37	1 33	1 30	1 27	1 24	29
30	1 41	1 37	1 33	1 30	1 26	1 23	1 21	30
32	1 33	1 29	1 26	1 23	1 19	1 17	1 15	32
34	1 26	1 22	1 19	1 16	1 13	1 11	1 09	34
36	1 19	1 16	1 13	1 10	1 08	1 05	1 03	36
38	1 13	1 10	1 07	1 04	1 02	1 00	0 58	38
40	1 08	1 05	1 02	1 00	0 58	0 56	0 54	40
42	1 03	1 00	0 58	0 56	0 54	0 52	0 50	42
44	0 59	0 56	0 54	0 52	0 50	0 48	0 47	44
46	0 54	0 52	0 50	0 48	0 46	0 45	0 43	46
48	0 51	0 49	0 47	0 45	0 44	0 42	0 41	48
50	0 47	0 45	0 43	0 41	0 40	0 38	0 37	50
55	0 39	0 37	0 36	0 35	0 33	0 32	0 31	55
60	0 32	0 30	0 29	0 28	0 27	0 26	0 25	60
65	0 25	0 24	0 23	0 22	0 21	0 20	0 20	65
70	0 20	0 19	0 18	0 17	0 17	0 16	0 15	70
75	0 14	0 14	0 13	0 12	0 12	0 12	0 11	75
80	0 10	0 09	0 09	0 09	0 08	0 08	0 08	80
85	0 04	0 04	0 04	0 04	0 04	0 04	0 03	85
90	0 00	0 00	0 00	0 00	0 00	0 00	0 00	90

TABLE IX

Conversion of Arc to Time

°	h	m	°	h	m	°	h	m	°	h	m	°	h	m	°	h	m	′	m	s	″	s
0	0	0	60	4	0	120	8	0	180	12	0	240	16	0	300	20	0	0	0	0	0	0.00
1	0	4	61	4	4	121	8	4	181	12	4	241	16	4	301	20	4	1	0	4	1	0.07
2	0	8	62	4	8	122	8	8	182	12	8	242	16	8	302	20	8	2	0	8	2	0.13
3	0	12	63	4	12	123	8	12	183	12	12	243	16	12	303	20	12	3	0	12	3	0.20
4	0	16	64	4	16	124	8	16	184	12	16	244	16	16	304	20	16	4	0	16	4	0.27
5	0	20	65	4	20	125	8	20	185	12	20	245	16	20	305	20	20	5	0	20	5	0.33
6	0	24	66	4	24	126	8	24	186	12	24	246	16	24	306	20	24	6	0	24	6	0.40
7	0	28	67	4	28	127	8	28	187	12	28	247	16	28	307	20	28	7	0	28	7	0.47
8	0	32	68	4	32	128	8	32	188	12	32	248	16	32	308	20	32	8	0	32	8	0.53
9	0	36	69	4	36	129	8	36	189	12	36	249	16	36	309	20	36	9	0	36	9	0.60
10	0	40	70	4	40	130	8	40	190	12	40	250	16	40	310	20	40	10	0	40	10	0.67
11	0	44	71	4	44	131	8	44	191	12	44	251	16	44	311	20	44	11	0	44	11	0.73
12	0	48	72	4	48	132	8	48	192	12	48	252	16	48	312	20	48	12	0	48	12	0.80
13	0	52	73	4	52	133	8	52	193	12	52	253	16	52	313	20	52	13	0	52	13	0.87
14	0	56	74	4	56	134	8	56	194	12	56	254	16	56	314	20	56	14	0	56	14	0.93
15	1	0	75	5	0	135	9	0	195	13	0	255	17	0	315	21	0	15	1	0	15	1.00
16	1	4	76	5	4	136	9	4	196	13	4	256	17	4	316	21	4	16	1	4	16	1.07
17	1	8	77	5	8	137	9	8	197	13	8	257	17	8	317	21	8	17	1	8	17	1.13
18	1	12	78	5	12	138	9	12	198	13	12	258	17	12	318	21	12	18	1	12	18	1.20
19	1	16	79	5	16	139	9	16	199	13	16	259	17	16	319	21	16	19	1	16	19	1.27
20	1	20	80	5	20	140	9	20	200	13	20	260	17	20	320	21	20	20	1	20	20	1.33
21	1	24	81	5	24	141	9	24	201	13	24	261	17	24	321	21	24	21	1	24	21	1.40
22	1	28	82	5	28	142	9	28	202	13	28	262	17	28	322	21	28	22	1	28	22	1.47
23	1	32	83	5	32	143	9	32	203	13	32	263	17	32	323	21	32	23	1	32	23	1.53
24	1	36	84	5	36	144	9	36	204	13	36	264	17	36	324	21	36	24	1	36	24	1.60
25	1	40	85	5	40	145	9	40	205	13	40	265	17	40	325	21	40	25	1	40	25	1.67
26	1	44	86	5	44	146	9	44	206	13	44	266	17	44	326	21	44	26	1	44	26	1.73
27	1	48	87	5	48	147	9	48	207	13	48	267	17	48	327	21	48	27	1	48	27	1.80
28	1	52	88	5	52	148	9	52	208	13	52	268	17	52	328	21	52	28	1	52	28	1.87
29	1	56	89	5	56	149	9	56	209	13	56	269	17	56	329	21	56	29	1	56	29	1.93
30	2	0	90	6	0	150	10	0	210	14	0	270	18	0	330	22	0	30	2	0	30	2.00
31	2	4	91	6	4	151	10	4	211	14	4	271	18	4	331	22	4	31	2	4	31	2.07
32	2	8	92	6	8	152	10	8	212	14	8	272	18	8	332	22	8	32	2	8	32	2.13
33	2	12	93	6	12	153	10	12	213	14	12	273	18	12	333	22	12	33	2	12	33	2.20
34	2	16	94	6	16	154	10	16	214	14	16	274	18	16	334	22	16	34	2	16	34	2.27
35	2	20	95	6	20	155	10	20	215	14	20	275	18	20	335	22	20	35	2	20	35	2.33
36	2	24	96	6	24	156	10	24	216	14	24	276	18	24	336	22	24	36	2	24	36	2.40
37	2	28	97	6	28	157	10	28	217	14	28	277	18	28	337	22	28	37	2	28	37	2.47
38	2	32	98	6	32	158	10	32	218	14	32	278	18	32	338	22	32	38	2	32	38	2.53
39	2	36	99	6	36	159	10	36	219	14	36	279	18	36	339	22	36	39	2	36	39	2.60
40	2	40	100	6	40	160	10	40	220	14	40	280	18	40	340	22	40	40	2	40	40	2.67
41	2	44	101	6	44	161	10	44	221	14	44	281	18	44	341	22	44	41	2	44	41	2.73
42	2	48	102	6	48	162	10	48	222	14	48	282	18	48	342	22	48	42	2	48	42	2.80
43	2	52	103	6	52	163	10	52	223	14	52	283	18	52	343	22	52	43	2	52	43	2.87
44	2	56	104	6	56	164	10	56	224	14	56	284	18	56	344	22	56	44	2	56	44	2.93
45	3	0	105	7	0	165	11	0	225	15	0	285	19	0	345	23	0	45	3	0	45	3.00
46	3	4	106	7	4	166	11	4	226	15	4	286	19	4	346	23	4	46	3	4	46	3.07
47	3	8	107	7	8	167	11	8	227	15	8	287	19	8	347	23	8	47	3	8	47	3.13
48	3	12	108	7	12	168	11	12	228	15	12	288	19	12	348	23	12	48	3	12	48	3.20
49	3	16	109	7	16	169	11	16	229	15	16	289	19	16	349	23	16	49	3	16	49	3.27
50	3	20	110	7	20	170	11	20	230	15	20	290	19	20	350	23	20	50	3	20	50	3.33
51	3	24	111	7	24	171	11	24	231	15	24	291	19	24	351	23	24	51	3	24	51	3.40
52	3	28	112	7	28	172	11	28	232	15	28	292	19	28	352	23	28	52	3	28	52	3.47
53	3	32	113	7	32	173	11	32	233	15	32	293	19	32	353	23	32	53	3	32	53	3.53
54	3	36	114	7	36	174	11	36	234	15	36	294	19	36	354	23	36	54	3	36	54	3.60
55	3	40	115	7	40	175	11	40	235	15	40	295	19	40	355	23	40	55	3	40	55	3.67
56	3	44	116	7	44	176	11	44	236	15	44	296	19	44	356	23	44	56	3	44	56	3.73
57	3	48	117	7	48	177	11	48	237	15	48	297	19	48	357	23	48	57	3	48	57	3.80
58	3	52	118	7	52	178	11	52	238	15	52	298	19	52	358	23	52	58	3	52	58	3.87
59	3	56	119	7	56	179	11	56	239	15	56	299	19	56	359	23	56	59	3	59	59	3.93
60	4	0	120	8	0	180	12	0	240	16	0	300	20	0	360	24	0	60	4	0	60	4.00

TABLE X

Values of $m = \dfrac{2 \sin^2 \frac{1}{2} t}{\sin 1''}$

t	0^m	1^m	2^m	3^m	4^m	5^m	6^m	7^m	8^m
s	"	"	"	"	"	"	"	"	"
0	0.00	1.96	7.85	17.67	31.42	49.09	70.68	96.20	125.65
1	0.00	2.03	7.98	17.87	31.68	49.41	71.07	96.66	126.17
2	0.00	2.10	8.12	18.07	31.94	49.74	71.47	97.12	126.70
3	0.00	2.16	8.25	18.27	32.20	50.07	71.86	97.58	127.22
4	0.01	2.23	8.39	18.47	32.47	50.40	72.26	98.04	127.75
5	0.01	2.31	8.52	18.67	32.74	50.73	72.66	98.50	128.28
6	0.02	2.38	8.66	18.87	33.01	51.07	73.06	98.97	128.81
7	0.02	2.45	8.80	19.07	33.27	51.40	73.46	99.43	129.34
8	0.03	2.52	8.94	19.28	33.54	51.74	73.86	99.90	129.87
9	0.04	2.60	9.08	19.48	33.81	52.07	74.26	100.37	130.40
10	0.05	2.67	9.22	19.69	34.09	52.41	74.66	100.84	130.94
11	0.06	2.75	9.36	19.90	34.36	52.75	75.06	101.31	131.47
12	0.08	2.83	9.50	20.11	34.64	53.09	75.47	101.78	132.01
13	0.09	2.91	9.64	20.32	34.91	53.43	75.88	102.25	132.55
14	0.11	2.99	9.79	20.53	35.19	53.77	76.29	102.72	133.09
15	0.12	3.07	9.94	20.74	35.46	54.11	76.69	103.20	133.63
16	0.14	3.15	10.09	20.95	35.74	54.46	77.10	103.67	134.17
17	0.16	3.23	10.24	21.16	36.02	54.80	77.51	104.15	134.71
18	0.18	3.32	10.39	21.38	36.30	55.15	77.93	104.63	135.25
19	0.21	3.40	10.54	21.60	36.58	55.50	78.34	105.10	135.80
20	0.22	3.49	10.69	21.82	36.87	55.84	78.75	105.58	136.34
21	0.24	3.58	10.84	22.03	37.15	56.19	79.16	106.06	136.88
22	0.26	3.67	11.00	22.25	37.44	56.55	79.58	106.55	137.43
23	0.28	3.76	11.15	22.47	37.72	56.90	80.00	107.03	137.98
24	0.32	3.85	11.31	22.70	38.01	57.25	80.42	107.51	138.53
25	0.34	3.94	11.47	22.92	38.30	57.60	80.84	107.99	139.08
26	0.37	4.03	11.63	23.14	38.59	57.96	81.26	108.48	139.63
27	0.40	4.12	11.79	23.37	38.88	58.32	81.68	108.97	140.18
28	0.43	4.22	11.95	23.60	39.17	58.68	82.10	109.46	140.74
29	0.46	4.32	12.11	23.82	39.46	59.03	82.52	109.95	141.29
30	0.49	4.42	12.27	24.05	39.76	59.40	82.95	110.44	141.85
31	0.52	4.52	12.43	24.28	40.05	59.75	83.38	110.93	142.40
32	0.56	4.62	12.60	24.51	40.35	60.11	83.81	111.43	142.96
33	0.59	4.72	12.76	24.74	40.65	60.47	84.23	111.92	143.52
34	0.63	4.82	12.93	24.98	40.95	60.84	84.66	112.41	144.08
35	0.67	4.92	13.10	25.21	41.25	61.20	85.09	112.90	144.64
36	0.71	5.03	13.27	25.45	41.55	61.57	85.52	113.40	145.20
37	0.75	5.13	13.44	25.68	41.85	61.94	85.95	113.90	145.76
38	0.79	5.24	13.62	25.92	42.15	62.31	86.39	114.40	146.33
39	0.83	5.34	13.79	26.16	42.45	62.68	86.82	114.90	146.89
40	0.87	5.45	13.96	26.40	42.76	63.05	87.26	115.40	147.46
41	0.91	5.56	14.13	26.64	43.06	63.42	87.70	115.90	148.03
42	0.96	5.67	14.31	26.88	43.37	63.79	88.14	116.40	148.60
43	1.01	5.78	14.49	27.12	43.68	64.16	88.57	116.90	149.17
44	1.06	5.90	14.67	27.37	43.99	64.54	89.01	117.41	149.74
45	1.10	6.01	14.85	27.61	44.30	64.91	89.45	117.92	150.31
46	1.15	6.13	15.03	27.86	44.61	65.29	89.89	118.43	150.88
47	1.20	6.24	15.21	28.10	44.92	65.67	90.33	118.94	151.45
48	1.26	6.36	15.39	28.35	45.24	66.05	90.78	119.45	152.03
49	1.31	6.48	15.57	28.60	45.55	66.43	91.23	119.96	152.61
50	1.36	6.60	15.76	28.85	45.87	66.81	91.68	120.47	153.19
51	1.42	6.72	15.95	29.10	46.18	67.19	92.12	120.98	153.77
52	1.48	6.84	16.14	29.36	46.50	67.58	92.57	121.49	154.35
53	1.53	6.96	16.32	29.61	46.82	67.96	93.02	122.01	154.93
54	1.59	7.09	16.51	29.86	47.14	68.35	93.47	122.53	155.51
55	1.65	7.21	16.70	30.12	47.46	68.73	93.92	123.05	156.09
56	1.71	7.34	16.89	30.38	47.79	69.12	94.38	123.57	156.67
57	1.77	7.46	17.08	30.64	48.11	69.51	94.83	124.09	157.25
58	1.83	7.60	17.28	30.90	48.43	69.90	95.29	124.61	157.84
59	1.89	7.72	17.47	31.16	48.76	70.29	95.74	125.13	158.43

TABLE X (*Continued*)

$\frac{2 \sin^2 \frac{1}{2} t}{\sin 1''}$

t	9^m	10^m	11^m	12^m	13^m	14^m	15^m	16^m
s	"	"	"	"	"	"	"	"
0	159.02	196.32	237.54	282.68	331.74	384.74	441.63	502.46
1	159.61	196.97	238.26	283.47	332.59	385.65	442.62	503.50
2	160.20	197.63	238.98	284.25	333.44	386.56	443.60	504.55
3	160.80	198.28	239.70	285.04	334.29	387.48	444.58	505.60
4	161.39	198.94	240.42	285.83	335.15	388.40	445.56	506.65
5	161.98	199.60	241.14	286.62	336.00	389.32	446.55	507.70
6	162.58	200.26	241.87	287.41	336.86	390.24	447.54	508.76
7	163.17	200.92	242.60	288.20	337.72	391.16	448.53	509.81
8	163.77	201.59	243.33	289.00	338.58	392.09	449.51	510.86
9	164.37	202.25	244.06	289.79	339.44	393.01	450.50	511.92
10	164.97	202.92	244.79	290.58	340.30	393.94	451.50	512.98
11	165.57	203.58	245.52	291.38	341.16	394.86	452.49	514.03
12	166.17	204.25	245.25	292.18	342.02	395.79	453.48	515.09
13	166.77	204.92	246.98	292.98	342.88	396.72	454.48	516.15
14	167.37	205.59	247.72	293.78	343.75	397.65	455.47	517.21
15	167.97	206.26	248.45	294.58	344.62	398.58	456.47	518.27
16	168.58	206.93	249.19	295.38	345.49	399.52	457.47	519.34
17	169.19	207.60	249.93	296.18	346.36	400.45	458.47	520.40
18	169.80	208.27	250.67	296.99	347.23	401.38	459.47	521.47
19	170.41	208.94	251.41	297.79	348.10	402.32	460.47	522.53
20	171.02	209.62	252.15	298.60	348.97	403.26	461.47	523.60
21	171.63	210.30	252.89	299.40	349.84	404.20	462.48	524.67
22	172.24	210.98	253.63	300.21	350.71	405.14	463.48	525.74
23	172.85	211.66	254.37	301.02	351.58	406.08	464.48	526.81
24	173.47	212.34	255.12	301.83	352.46	407.02	465.49	527.89
25	174.08	213.02	255.87	302.64	353.34	407.96	466.50	528.96
26	174.70	213.70	256.62	303.46	354.22	408.90	467.51	530.03
27	175.32	214.38	257.37	304.27	355.10	409.84	468.52	531.11
28	175.94	215.07	258.12	305.09	355.98	410.79	469.53	532.18
29	176.56	215.75	258.87	305.90	356.86	411.73	470.54	533.26
30	177.18	216.44	259.62	306.72	357.74	412.68	471.55	534.33
31	177.80	217.12	260.37	307.54	358.62	413.63	472.57	535.41
32	178.43	217.81	261.12	308.36	359.51	414.59	473.58	536.50
33	179.05	218.50	261.88	309.18	360.39	415.54	474.60	537.58
34	179.68	219.19	262.64	310.00	361.28	416.49	475.62	538.67
35	180.30	219.88	263.39	310.82	362.17	417.44	476.64	539.75
36	180.93	220.58	264.15	311.65	363.07	418.40	477.65	540.83
37	181.56	221.27	264.91	312.47	363.96	419.35	478.67	541.91
38	182.19	221.97	265.68	313.30	364.85	420.31	479.70	543.00
39	182.82	222.66	266.44	314.12	365.75	421.27	480.72	544.09
40	183.46	223.36	267.20	314.95	366.64	422.23	481.74	545.18
41	184.09	224.06	267.96	315.78	367.53	423.19	482.77	546.27
42	184.72	224.76	268.73	316.61	368.42	424.15	483.79	547.36
43	185.35	225.46	269.49	317.44	369.31	425.11	484.82	548.45
44	185.99	226.16	270.26	318.27	370.21	426.07	485.85	549.55
45	186.63	226.86	271.02	319.10	371.11	427.04	486.88	550.64
46	187.27	227.57	271.79	319.94	372.01	428.01	487.91	551.73
47	187.91	228.27	272.56	320.78	372.92	428.97	488.94	552.83
48	188.55	228.98	273.34	321.62	373.82	429.93	489.97	553.93
49	189.19	229.68	274.11	322.45	374.72	430.90	491.01	555.03
50	189.83	230.39	274.88	323.29	375.62	431.87	492.05	556.13
51	190.47	231.10	275.65	324.13	376.52	432.84	493.08	557.24
52	191.12	231.81	276.43	324.97	377.43	433.82	494.12	558.34
53	191.76	232.52	277.20	325.81	378.34	434.79	495.15	559.44
54	192.41	233.24	277.98	326.66	379.26	435.76	496.19	560.55
55	193.06	233.95	278.76	327.50	380.17	436.73	497.23	561.65
56	193.71	234.67	279.55	328.35	381.08	437.71	498.28	562.76
57	194.36	235.38	280.33	329.19	381.99	438.69	499.32	563.87
58	195.01	236.10	281.12	330.04	382.90	439.67	500.37	564.98
59	195.66	236.82	281.90	330.89	383.82	440.65	501.41	566.08

GREEK ALPHABET

Letters.	Name.	Letters.	Name.
Α, α,	Alpha	Ν, ν,	Nu
Β, β,	Beta	Ξ, ξ,	Xi
Γ, γ,	Gamma	Ο, ο,	Omicron
Δ, δ,	Delta	Π, π,	Pi
Ε, ε,	Epsilon	Ρ, ρ,	Rho
Ζ, ζ,	Zeta	Σ, σ, ς,	Sigma
Η, η,	Eta	Τ, τ,	Tau
Θ, θ, ϑ,	Theta	Υ, υ,	Upsilon
Ι, ι,	Iota	Φ, φ,	Phi
Κ, κ,	Kappa	Χ, χ,	Chi
Λ, λ,	Lambda	Ψ, ψ,	Psi
Μ, μ,	Mu	Ω, ω,	Omega

ABBREVIATIONS USED IN THIS BOOK

A.C.T.	Atlantic civil time
alt.	altitude
A.M.	ante meridiem
Colat.	colatitude
decl.	declination
E	east
E.D.S.T.	Eastern daylight saving time
Eq. T.	equation of time
E.S.T.	Eastern standard time
G.A.T.	Greenwich apparent time
G.C.T.	Greenwich civil time
G.H.A.	Greenwich hour angle
G.S.T.	Greenwich sidereal time
H.A.	hour angle
H.O.	Hydrographic Office (Navy Department)
I.C.	index correction
L.A.T.	local apparent time
lat.	latitude
L.A.N.	local apparent noon
L.C.	lower culmination
L.C.T.	local civil time
L.H.A.	local hour angle
long.	longitude
L.S.T.	local sidereal time
M.A.	meridian angle
N	north
N.J.G.C.S.	New Jersey Geodetic Control Survey
obs. alt.	observed altitude
P.M.	post meridiem
R.A.	right ascension
R.A.M.S.	right ascension of mean sun
refr.	refraction
S	(1) star or celestial body; (2) south
s.d.	semidiameter
S.H.A.	sidereal hour angle

Sid. int.	sidereal interval
s.t.s.d.p.	sidereal time of semidiameter passing meridian
U.C.	upper culmination
W	west
Z.D.	zenith distance

SYMBOLS USED IN THIS BOOK

h	altitude
h_m	meridian altitude
p	polar distance
S	(1) sidereal time in general; (2) parallax or parallactic angle
T	civil time in general
T_e	time of elongation
t	hour angle
V	vernal equinox
Z	zenith or angle of the astronomical triangle at the zenith, hence azimuth
α	right ascension
α_s	right ascension of mean sun
δ	declination
ζ	zenith distance
λ	longitude
Υ	vernal equinox
ϕ	latitude

APPENDIX

APPENDIX

Spherical Trigonometry

This appendix is not intended to constitute a complete text on spherical trigonometry. It has been written with a view to providing the civil engineering student with a working knowledge of the subject sufficient to enable him to cope successfully with the work in practical astronomy.

Spherical trigonometry deals chiefly with the relationships existing between the sides and angles of a spherical triangle and with the computation of unknown parts of such a triangle from parts which are known.

Definitions

A *spherical surface* is a surface all points of which are equidistant from a point called the center.

A *sphere* is a solid bounded by a spherical surface. The surface of a sphere is generated by the revolution of a semicircle about its diameter.

A plane section of a sphere is a figure the boundary of which is a circle formed by the intersection of a plane with the spherical surface. If a plane is passed through the center of a sphere the resulting plane section is a *great circle.* Where the plane does not pass through the center of the sphere it will cut the surface in a *small circle.*

Through any two points upon the surface of a sphere a great circle can be drawn, since a plane can be passed through the two points and the center of the sphere, and this plane will cut the surface in a great circle. In general, only one great circle can be passed through any two points on a sphere. If, however, the two points lie at the ends of a diameter of the sphere, an infinite number of great circles

can be drawn through them. It can be shown that the shortest distance between any two points on a spherical surface is the arc of a great circle, the length of the arc being not greater than a semicircle.

A *spherical triangle* is that portion of the surface of a sphere bounded by the arcs of three *great circles.* It is at once apparent that three intersecting great circles will form eight spherical triangles on the surface of a sphere. In this work we shall consider only those spherical triangles in which no angle or side is greater than 180°. While a spherical triangle may have an angle or side greater than 180°, its unknown parts may always be deduced by solving an adjacent triangle cut on the surface of the sphere by the same three great circles which form the larger triangle. This adjacent triangle will have no angle or side greater than 180° and can be solved by the formulae given in these pages.

The angles of a spherical triangle, like those of a plane triangle, are expressed in circular measure. The sides, unlike those of a plane triangle, are also expressed in circular measure since they are great circular arcs rather than straight lines. The lengths of the sides may be stated in radians but are more commonly given in degrees, minutes, and seconds.

Relationship between Spherical Triangles and Triedral Angles

Two non-parallel planes intersect in a straight line, and the angle between them at their intersection is known as a *diedral angle.* In Fig. 87 the two planes $ABCD$ and $BCEF$ intersect in the line BC and form the diedral angle $ECBA$. The planes AC and BE form the *faces* and BC the *edge* of the diedral angle.

If GH is drawn perpendicular to BC in the plane AC, and GJ is drawn perpendicular to BC in the plane BE, the angle JGH is called a plane angle of the diedral angle. Since all plane angles of a diedral angle must be equal, the plane angle is a measure of the diedral angle.

When three or more planes meet in a common point a *solid angle* or *polyedral angle* is formed. The point in which the planes meet is called the vertex of the solid angle; the intersections of the planes are called its *edges;* those portions of the planes between the edges are spoken of as its *faces;* the plane angles formed by the edges are the *face angles;* and the diedral angles formed at the edges by the planes are the *diedral* or *edge angles* of the solid angle. A solid angle formed by three planes intersecting at a point is known as a *triedral angle.*

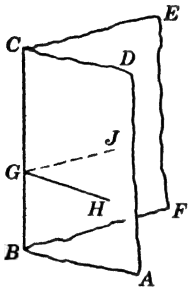


Fig. 87

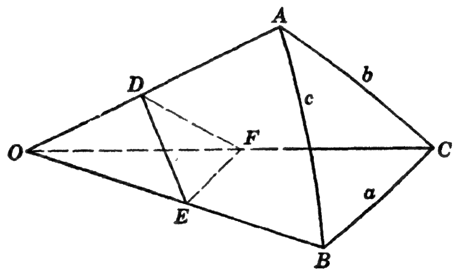


Fig. 88

In Fig. 88 we have such a triedral angle. In this case it is formed by three planes intersecting at the center O of a sphere. The planes cut the spherical triangle whose angles are conventionally represented as A, B, and C and the sides opposite them as a, b, and c. ED and FD are drawn perpendicular to OA.

The angle between two intersecting circular curves is equal to the angle between tangents to the two curves at the point of intersection. In Fig. 88, ED and FD are parallel to tangents to c and b which define the angle A at their intersection. Angle EDF therefore measures angle A and also measures one of the edge or diedral angles of the triedral angle $O\text{-}ABC$ since it is a plane angle of the driedral angle $DOE\text{-}DOF$. Similarly the angles B and C are measured by the corresponding edge angles of the solid angle.

Since a circular arc is measured by its corresponding central angle, the sides a, b, and c of the spherical triangle are measured by the plane angles EOF, DOF, and DOE respectively which are the face angles of the solid angle.

Hence, finding the relations existing between the face angles and edge angles of a triedral angle is identical with finding the relations existing between the sides and angles of the spherical triangle which is formed by the intersecting planes of the solid angle on the surface of any sphere whose center is at the vertex of the triedral angle.

This correspondence between the sides and angles of a spherical triangle on the one hand and between the face angles and edge angles of the triedral angle subtended at the center of the sphere by the triangle on the other hand is most important for the deduction of the relations between the spherical angles and sides and in solving practical problems. From any property of triedral angles an analogous property of spherical triangles can be inferred, or vice versa.

For any triedral angle a spherical triangle may be formed by assuming that the center of the sphere is at the vertex of the angle and assigning any arbitrary value to the radius. The three faces (planes) cut out arcs of great circles which form the sides of the triangle. The solution of the spherical triangle is really at the same time the solution of the solid angle since the six parts of one equal the six corresponding parts of the other. Any three lines passing through a common point define a triedral angle. For example, the earth's axis of rotation, the plumb line at any place on the surface of the earth, and a line in the direction of a celestial body may be conceived to intersect at the earth's center. The relation among the three face angles and the three edge angles of this triedral angle may be calculated by the formulae of spherical trigonometry. The sphere employed, however, is merely an imaginary one.

Fundamental Formulae of Spherical Trigonometry

The fundamental spherical trigonometric formulae, from which we obtain the basic equations of practical astronomy

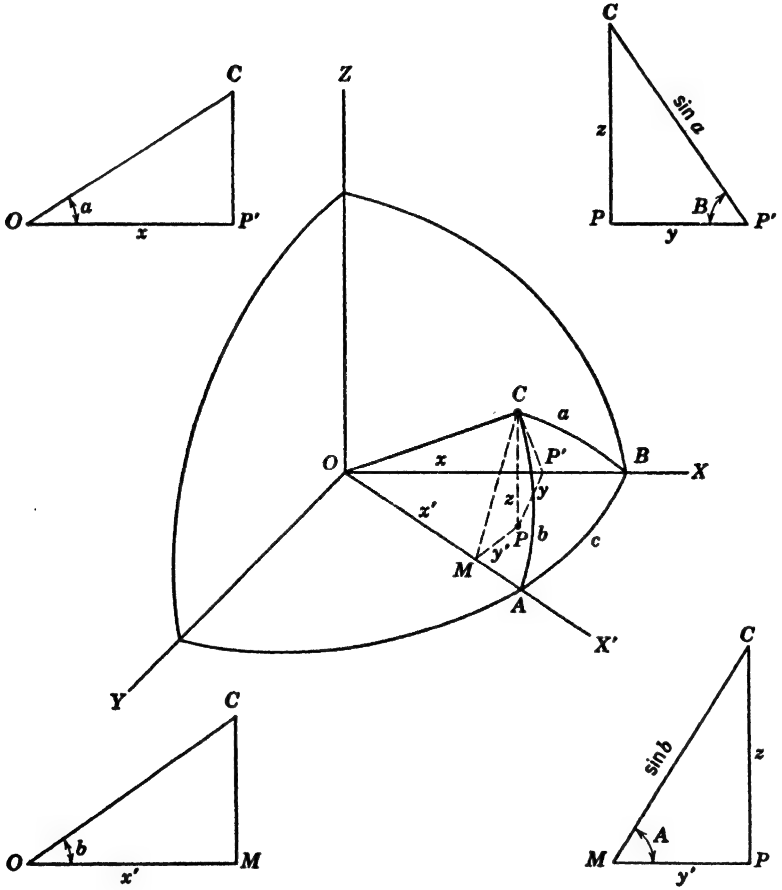


Fig. 89

given in Chapter 4, may be derived by applying the principles of analytic geometry to the spherical triangle. In Fig. 89 the radius of the sphere is assumed to be unity. The

line CP has been dropped from C perpendicular to the XY plane; CP' has been drawn from C perpendicular to OB; and CM has been drawn from C perpendicular to OA. Then angle COP' equals side a of the spherical triangle ABC; angle $P'OM$ equals side c; and angle COM equals side b. Furthermore, angle $CP'P$ equals angle B of the spherical triangle; angle CMP equals angle A.

For radius unity a study of the small triangles shown surrounding Fig. 89 will indicate that

$$x = \cos a \qquad x' = \cos b$$

$$y = \sin a \cos B \qquad y' = \sin b \cos A$$

$$z = \sin a \sin B = \sin b \sin A$$

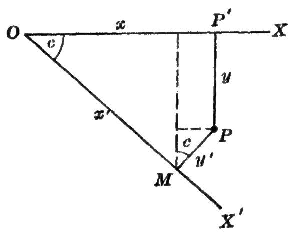


Fig. 90

From Fig. 90 the formulae for transformation are

$$x = x' \cos c + y' \sin c$$

$$y = x' \sin c - y' \cos c$$

By substitution,

$$\cos a = \cos b \cos c + \sin b \sin c \cos A \tag{1}$$

$$\sin a \cos B = \cos b \sin c - \sin b \cos c \cos A \tag{2}$$

$$\sin a \sin B = \sin b \sin A \tag{3}$$

Corresponding formulae may be written involving angles B and C in the right-hand member of each of the above

expressions. Equation 1 is known as the *law of cosines* and may be regarded as the fundamental formula of spherical trigonometry because all others may be derived from it and by means of it all problems in spherical trigonometry may be solved, although not always so conveniently as with other special forms.

Writing the corresponding forms of Eq. 3 and rearranging we have

$$\frac{\sin A}{\sin a} = \frac{\sin B}{\sin b} = \frac{\sin C}{\sin c} \tag{4}$$

which is known as the *law of sines.*

In the derivation of the fundamental formulae from Fig. 89, the sides of the spherical triangle were taken as less than

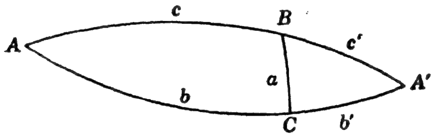


Fig. 91

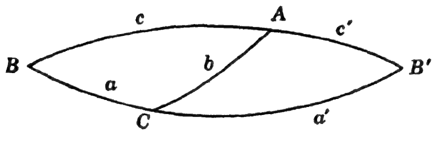


Fig. 92

90° to permit the construction of the right triangles OCP', OCM, etc. To show that the law of cosines applies in all cases, let us assume that both b and c (Eq. 1) are greater than 90° and less than 180°. Then, in Fig. 91, produce c and b to meet at A', forming a lune. In the triangle $A'BC$, b' and c' are less than 90°. The law of cosines therefore holds for this triangle. But, in a lune,

angle A' equals angle A. Hence

$$\cos a = \cos b' \cos c' + \sin b' \sin c' \cos A$$

but

$$b' = 180° - b \text{ and } c' = 180° - c$$

Then

$$\cos a = \cos (180° - b) \cos (180° - c) + \sin (180° - b) \sin (180° - c) \cos A$$

or

$$\cos a = \cos b \cos c + \sin b \sin c \cos A$$

Again, assume b as less than 90° and c between 90° and 180°. In Fig. 92 produce a and c to meet at B', forming a lune. In the triangle ACB', b and c' are less than 90° so that the law of cosines applies. We may then write

$$\cos a' = \cos b \cos c' + \sin b \sin c' \cos CAB'$$

But $a' = 180° - a$, $c' = 180° - c$, and $CAB' = 180° - A$. Hence

$$\cos (180° - a) = \cos b \cos (180° - c) + \sin b \sin (180° - c) \cos (180° - A)$$

or

$$\cos a = \cos b \cos c + \sin b \sin c \cos A$$

This proves the law of cosines for all cases.

The Polar Triangle

Other convenient formulae may be obtained by utilizing the principle of the polar triangle. In Fig. 93 let ABC be a spherical triangle, the angles being denoted by capital letters and the side opposite each angle being denoted by the corresponding lower case letter. This convention will be used throughout. When dealing with right spherical triangles the angle C will always be taken as equal to 90°. The side a of the triangle ABC has two poles A' and A'', that

is, two points on the sphere which are 90° from all points on side a. Take A' as that one of these two which lies on the same side of a as A. Similarly, take B' as that pole of the arc b which lies on the same side of it as B, and C' as that pole of c which lies on the same side of it as C. Joining these points by great circular arcs, we have a spherical triangle $A'B'C'$ which is called the *polar triangle* of ABC. Its sides are denoted by a', b', and c'.

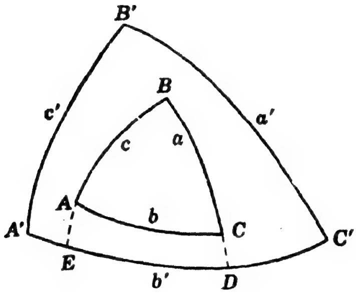


Fig. 93

By construction, C' is at a distance of 90° from all points on c, and B' is at the same distance from all points on b. Since A lies on both b and c, B' and C' are both at a distance of 90° from A. Hence, if we draw a great circle with its pole at A, it will pass through B' and C' and will therefore coincide with a'. Furthermore, since A and A' both lie on the same side of a, it may be seen that they also lie on the same side of a'. A, therefore, is that pole of a' which lies on the same side of it as A'. Similarly B and C are the corresponding poles of b' and c'. It thus develops that ABC is the polar triangle of $A'B'C'$ and that a reciprocal relationship exists between the two triangles.

If we let D and E be the points where the sides a and c (produced if need be) meet b' (or b' produced) then, since A' and C' are 90° distant from all points on a and c respec-

tively, $A'D = 90°$ and $C'E = 90°$. Adding, we have

$$A'D + C'E = 180°$$

or

$$A'C' + ED = 180°$$

But $A'C' = b'$, and ED, a great circular arc 90° from B, is a measure of the angle B. We may therefore write

$$b' + B = 180° \tag{5}$$

Similarly,

$$a' + A = 180° \tag{6}$$

And

$$c' + C = 180° \tag{7}$$

We may then state, as a general proposition, that any angle of a spherical triangle is the supplement of the corresponding side of the polar triangle.

Use of the Polar Triangle

The concept of the polar triangle may be used to write sets of formulae like Eqs. 1 and 2 in which each lower case letter is replaced by a capital letter and vice versa. Since Eq. 1 applies to any triangle it will hold for the polar triangle of ABC. For this polar triangle $A'B'C'$ we may write

$$\cos a' = \cos b' \cos c' + \sin b' \sin c' \cos A'$$

But $a' = 180° - A$, $b' = 180° - B$, etc. Hence

$$\cos (180° - A) = \cos (180° - B) \cos (180° - C) + \sin (180° - B) \sin (180° - C) \cos (180° - a)$$

or

$$-\cos A = \cos B \cos C - \sin B \sin C \cos a$$

Then

$$\cos A = \sin B \sin C \cos a - \cos B \cos C \tag{8}$$

Similarly, Eq. 2 may be transformed to

$$\sin A \cos b = \cos B \sin C - \sin B \cos C \cos a \tag{9}$$

Equations corresponding to 8 and 9 may be written for angles B and C.

The polar triangle is useful in making a rapid solution of one form of oblique spherical triangle, the quadrantal triangle (one having a side equal to a quadrant, or 90°). Since the side of any triangle is the supplement of the corresponding angle of its polar triangle, the polar of a quadrantal triangle is right. We may therefore solve the polar triangle by the convenient right triangle formulae and take the supplements of the angles or sides thus found as the unknown sides or angles of the quadrantal triangle. Other applications of the polar triangle will be found in the pages to follow.

Sum of Sides and Sum of Angles of Spherical Triangles

In general, there is a close parallel between the formulae and the underlying principles of spherical trigonometry and those of plane trigonometry. Where this parallel exists it should be noted by the student as it will prove of material value in solving problems. On the other hand he must be on the alert to discover those conditions where no parallel exists. We have already encountered one departure from this parallelism, namely, in the use of circular measure rather than linear measure in expressing the lengths of the triangle sides. Another is found when we consider the sum of the angles.

In a plane triangle the three angles must add up to 180°. In a spherical triangle this relation does not hold. The sum of the angles is always greater than 180° and always less than 540°. The amount by which the sum of the angles of a spherical triangle exceeds 180° is known as the *spherical excess* of the triangle. This spherical excess, in degrees, may be shown to equal 180 times the area of the triangle divided by π times the square of the radius of the sphere.

In small triangles as drawn on a spherical blackboard the spherical excess may be very large. In the triangles encountered in many practical applications of the subject, such

as those measured in geodetic surveys on the earth's surface, the spherical excess is very small since the sides of the triangle are normally short when compared with the radius of the earth. In this case a spherical excess of one second of angle is found for a triangle whose area is about 75 square miles.

In plane trigonometry no relation existed for the sum of the sides nor does a definite relationship exist in spherical trigonometry. It can be shown, however, that the sum of the sides must be less than 360°. This does not furnish a definite check on a solution, but it may indicate the presence of an error.

Formulae for Right Spherical Triangles

By placing C equal to 90° in our fundamental equations we obtain a number of formulae for solving right spherical triangles.

Equation 1, rewritten to give $\cos c$, becomes

$$\cos c = \cos a \cos b \tag{10}$$

Equation 8, rewritten to give $\cos C$, becomes

$$0 = \sin A \sin B \cos c - \cos A \cos B$$

from which

$$\cos c = \cot A \cot B \tag{11}$$

Equation 8, when written in terms of $\cos A$ and of $\cos B$ becomes

$$\cos A = \sin B \cos a \tag{12}$$

and

$$\cos B = \sin A \cos b \tag{13}$$

Equation 4 may be written

$$\frac{\sin A}{\sin a} = \frac{\sin B}{\sin b} = \frac{1}{\sin c}$$

whence

$$\sin a = \sin A \sin c \tag{14}$$

and

$$\sin b = \sin B \sin c \tag{15}$$

From Eq. 12,

$$\cos A = \sin B \cos a$$

But, from Eq. 15,

$$\sin B = \frac{\sin b}{\sin c}$$

therefore

$$\cos A = \frac{\sin b}{\sin c} \cos a$$

But, from Eq. 10,

$$\cos a = \frac{\cos c}{\cos b}$$

Hence

$$\cos A = \frac{\sin b \cos c}{\sin c \cos b}$$

or

$$\cos A = \frac{\tan b}{\tan c} \tag{16}$$

Similarly

$$\cos B = \frac{\tan a}{\tan c} \tag{17}$$

As in plane trigonometry,

$$\tan A = \frac{\sin A}{\cos A}$$

But, from Eqs. 14 and 16,

$$\sin A = \frac{\sin a}{\sin c} \text{ and } \cos A = \frac{\tan b}{\tan c}$$

$$\tan A = \frac{\sin a \tan c}{\sin c \tan b} = \frac{\sin a}{\tan b \cos c}$$

$$= \frac{\sin a}{\tan b \cos a \cos b} \text{ (from Eq. 10)}$$

Hence

$$\tan A = \frac{\tan a}{\sin b} \tag{18}$$

and

$$\tan B = \frac{\tan b}{\sin a} \tag{19}$$

The above formulae are here grouped for convenient reference.

$\cos c = \cos a \cos b$	(10)	$\cos c = \cot A \cot B$	(11)
$\cos A = \sin B \cos a$	(12)	$\cos B = \sin A \cos b$	(13)
$\sin a = \sin A \sin c$	(14)	$\sin b = \sin B \sin c$	(15)
$\cos A = \dfrac{\tan b}{\tan c}$	(16)	$\cos B = \dfrac{\tan a}{\tan c}$	(17)
$\tan A = \dfrac{\tan a}{\sin b}$	(18)	$\tan B = \dfrac{\tan b}{\sin a}$	(19)

Solution of Right Spherical Triangles

In a right triangle, since one angle is known, only two other parts need to be known to effect a solution. The general rule to follow in solving a right spherical triangle by the formulae given is to select that equation which involves the two known parts and one unknown part. The solution of three such equations will give the three unknown parts of the triangle.

It is essential to note carefully the algebraic signs of the functions in order to fix the sign of the resulting function

and thereby determine the angle. If the part to be found is obtained from a cosine, tangent, or cotangent, no ambiguity can result for, if these functions are positive, the side or angle will have a value less than 90°. If the sign is negative the correct value of the side or angle is the supplement of the value found in the tables.

If, on the other hand, the unknown part is determined by the sine, the sine being positive for all angles from 0° to 180°, the true value of the side or angle may be either that obtained from the tables or its supplement. Both values should be given unless the ambiguity can be removed by the application of the following rules.

1. An angle and its opposite side are in the same quadrant.

2. If the sides adjacent to the right angle are in the same quadrant, the hypotenuse is less than 90°; if they are in different quadrants, the hypotenuse is greater than 90°.

Law (1) may be proved as follows. From Eq. 18,

$$\sin b = \frac{\tan a}{\tan A}$$

Sin b is necessarily positive since no triangle having a side greater than 180° is considered, and the sine of an angle is positive for all angles less 180°. It therefore follows that tan a and tan A are both positive or both negative. Therefore a and A are each less than 90° or each greater than 90°.

We may also demonstrate that law (2) applies. From Eq. 10,

$$\cos c = \cos a \cos b$$

If both a and b are less than 90°, $\cos a = +$, $\cos b = +$, and $\cos c = (+)\ (+) = +$. Therefore c is less than 90°. If both a and b are greater than 90°, $\cos a = -$, $\cos b = -$, and $\cos c = (-)\ (-) = +$ so that c is again less than 90°. If, on the other hand, a and b are in different quadrants, their cosines will have opposite signs, the product of the cosines will be negative, cosine c will be negative, and c will be greater than 90°.

These two rules should be applied only to right spherical triangles.

Cases for Solution of Oblique Spherical Triangles

If any three parts of a spherical triangle are known, the three unknown parts may be deduced and checked by the formulae of spherical trigonometry. Six different cases or combinations of known parts present themselves for consideration.

1. Given the three sides, a, b, c.
2. Given the three angles, A, B, C.
3. Given two sides and the included angle, a, b, C.
4. Given two angles and the included side, A, B, c.
5. Given two sides and the angle opposite one of them, a, b, A.
6. Given two angles and the side opposite one of them, A, B, a.

Each of these cases may be solved by applying the law of cosines, using equations of the type of 1 and 8. This method of solution is not particularly convenient however, unless tables of natural functions and a calculating machine are available. The law of cosines is not well adapted to logarithmic computation.

Again, we may, as in the solution of plane triangles, divide an oblique spherical triangle into two right spherical triangles and by solving these deduce the unknown parts of the oblique triangle. This is convenient for an isosceles triangle since a great circle joining the vertex with the mid-point of the base will be perpendicular to the base and will bisect the angle opposite. This perpendicular divides the triangle into two symmetrical right triangles having identical parts. The general method may be applied to any oblique triangle, but the labor involved is frequently greater than that necessary for a direct solution by other convenient formulae which may be derived from the basic equations.

Before deriving special oblique triangle formulae it will

be well to call attention to three trigonometric relations which commonly receive scant consideration in courses in plane trigonometry but which are of value in surveying and navigation. Certain of these relations will appear in the following pages. The three are the versed sine, haversine (half versed sine), and coversed sine. They may be expressed as follows:

$$\text{vers } a = 1 - \cos a \tag{20}$$

$$\text{hav } a = \tfrac{1}{2}(1 - \cos a) \tag{21}$$

$$\text{covers } a = 1 - \sin a \tag{22}$$

Formulae for the Half-Angles and Half-Sides

From Eq. 1,

$$\cos A = \frac{\cos a - \cos b \cos c}{\sin b \sin c}$$

Therefore

$$1 - \cos A = 1 - \frac{\cos a - \cos b \cos c}{\sin b \sin c}$$

$$\text{vers } A = 1 - \cos A = \frac{\cos b \cos c + \sin b \sin c - \cos a}{\sin b \sin c}$$

But, from plane trigonometry,

$$\cos b \cos c + \sin b \sin c = \cos(b - c)$$

Hence

$$1 - \cos A = \frac{\cos(b - c) - \cos a}{\sin b \sin c}$$

Furthermore, from plane trigonometry,

$$1 - \cos A = 2 \sin^2 \frac{A}{2}$$

and

$$\cos (b - c) - \cos a = -2 \sin \frac{(b - c + a)}{2} \sin \frac{(b - c - a)}{2}$$

Therefore,

$$\text{hav } A = \sin^2 \frac{A}{2} = \frac{\sin \frac{(a-b+c)}{2} \sin \frac{(a+b-c)}{2}}{\sin b \sin c}$$

If we let $a + b + c = 2s$ then

$$\frac{(a-b+c)}{2} = s - b,$$

$$\frac{(a+b-c)}{2} = s - c, \quad \text{and} \quad \frac{(b+c-a)}{2} = s - a$$

We may then write

$$\text{hav } A = \sin^2 \frac{A}{2} = \frac{\sin (s-b) \sin (s-c)}{\sin b \sin c} \tag{23}$$

or

$$\sin \frac{A}{2} = \sqrt{\frac{\sin (s-b) \sin (s-c)}{\sin b \sin c}} \tag{24}$$

Similar equations may be written for hav B, hav C, sin $B/2$ and sin $C/2$.

In the same manner, starting with Eq. 1, we have

$$1 + \cos A = \frac{\sin b \sin c - \cos b \cos c + \cos a}{\sin b \sin c}$$

$$= \frac{\cos a - \cos (b+c)}{\sin b \sin c}$$

Hence

$$\cos^2 \frac{A}{2} = \frac{\sin \frac{(a+b+c)}{2} \sin \frac{(b+c-a)}{2}}{\sin b \sin c}$$

$$= \frac{\sin s \sin (s-a)}{\sin b \sin c}$$

Or

$$\cos\frac{A}{2} = \sqrt{\frac{\sin s \sin (s - a)}{\sin b \sin c}} \tag{25}$$

Similar equations may be obtained for $\cos B/2$ and $\cos C/2$. Dividing Eq. 24 by Eq. 25 we obtain,

$$\tan\frac{A}{2} = \sqrt{\frac{\sin (s - b) \sin (s - c)}{\sin s \sin (s - a)}} \tag{26}$$

Multiplying numerator and denominator by $\sin (s - a)$ we have

$$\tan\frac{A}{2} = \frac{1}{\sin (s - a)} \sqrt{\frac{\sin (s - a) \sin (s - b) \sin (s - c)}{\sin s}}$$

Placing

$$k = \sqrt{\frac{\sin (s - a) \sin (s - b) \sin (s - c)}{\sin s}}$$

we have

$$\tan\frac{A}{2} = \frac{k}{\sin (s - a)} \tag{27}$$

Similarly

$$\tan\frac{B}{2} = \frac{k}{\sin (s - b)} \tag{28}$$

and

$$\tan\frac{C}{2} = \frac{k}{\sin (s - c)} \tag{29}$$

These furnish equations for the sine, cosine, and tangent of the half angles.

To obtain similar equations for the half sides we may start with Eq. 8.

$$\cos a = \frac{\cos A + \cos B \cos C}{\sin B \sin C}$$

Finding $1 - \cos a$ and $1 + \cos a$, combining and simplifying in the manner followed for the half angles, and placing

$$A + B + C = 2S,$$

and

$$K = \sqrt{\frac{-\cos S}{\cos (S - A) \cos (S - B) \cos (S - C)}}$$

we may obtain

$$\sin \frac{a}{2} = \sqrt{\frac{-\cos S \cos (S - A)}{\sin B \sin C}} \tag{30}$$

$$\cos \frac{a}{2} = \sqrt{\frac{\cos (S - B) \cos (S - C)}{\sin B \sin C}} \tag{31}$$

$$\tan \frac{a}{2} = \sqrt{\frac{-\cos S \cos (S - A)}{\cos (S - B) \cos (S - C)}} \tag{32}$$

$$\tan \frac{a}{2} = K \cos (S - A) \tag{33}$$

Similar expressions may be written for $\sin b/2$, $\sin c/2$, $\cos b/2$, $\cos c/2$, $\tan b/2$, and $\tan c/2$.

Napier's Analogies

Dividing Eq. 27 by Eq. 28, we obtain

$$\frac{\tan \frac{A}{2}}{\tan \frac{B}{2}} = \frac{\sin (s - b)}{\sin (s - a)}$$

From this

$$\frac{\tan \frac{A}{2} + \tan \frac{B}{2}}{\tan \frac{A}{2} - \tan \frac{B}{2}} = \frac{\sin (s - b) + \sin (s - a)}{\sin (s - b) - \sin (s - a)}$$

And, by plane trigonometry,

$$\frac{\dfrac{\sin A/2}{\cos A/2} + \dfrac{\sin B/2}{\cos B/2}}{\dfrac{\sin A/2}{\cos A/2} - \dfrac{\sin B/2}{\cos B/2}} = \frac{\sin \dfrac{(2s - a - b)}{2} \cos \dfrac{(a - b)}{2}}{\cos \dfrac{(2s - a - b)}{2} \sin \dfrac{(a - b)}{2}}$$

or

$$\frac{\sin \dfrac{(A + B)}{2}}{\sin \dfrac{(A - b)}{2}} = \frac{\tan \dfrac{(2s - a - b)}{2}}{\tan \dfrac{(a - b)}{2}} = \frac{\tan \dfrac{c}{2}}{\tan \dfrac{(a - b)}{2}}$$

since $(2s - a - b) = c$. Hence

$$\tan \frac{(a - b)}{2} = \frac{\sin \dfrac{(A - B)}{2}}{\sin \dfrac{(A + B)}{2}} \tan \frac{c}{2} \tag{34}$$

To obtain a similar equation for $\tan (A - B)/2$ we may use the polar triangle of ABC, substituting $(180° - A)$ for a, $(180° - a)$ for A, etc. Then

$$\frac{a - b}{2} = \frac{(180° - A - 180° + B)}{2} = -\frac{(A - B)}{2}$$

$$\frac{A - B}{2} = \frac{(180° - a - 180° + b)}{2} = -\frac{(a - b)}{2}$$

$$\frac{A + B}{2} = \frac{(180° - a + 180° - b)}{2} = 180° - \frac{(a + b)}{2}$$

$$\frac{c}{2} = 90° - \frac{C}{2}$$

Substituting in Eq. 34, we obtain

$$\tan\frac{(A-B)}{2} = \frac{\sin\dfrac{(a-b)}{2}}{\sin\dfrac{(a+b)}{2}}\cot\frac{C}{2} \tag{35}$$

Formulae for $\tan (a + b)/2$ and $\tan (A + B)/2$ are derived as follows: From Eqs. 27 and 28

$$\tan\frac{A}{2}\tan\frac{B}{2} = \frac{k^2}{\sin(s-a)\sin(s-b)}$$

Then

$$\frac{\sin\dfrac{A}{2}\sin\dfrac{B}{2}}{\cos\dfrac{A}{2}\cos\dfrac{B}{2}} = \frac{\sin(s-c)}{\sin s}$$

By composition and division,

$$\frac{\cos\dfrac{A}{2}\cos\dfrac{B}{2} + \sin\dfrac{A}{2}\sin\dfrac{B}{2}}{\cos\dfrac{A}{2}\cos\dfrac{B}{2} - \sin\dfrac{A}{2}\sin\dfrac{B}{2}} = \frac{\sin s + \sin(s-c)}{\sin s - \sin(s-c)}.$$

Whence, by plane trigonometry,

$$\frac{\cos\dfrac{(A-B)}{2}}{\cos\dfrac{(A+B)}{2}} = \frac{\tan\dfrac{(a+b)}{2}}{\tan\dfrac{c}{2}}$$

since $2s - c = a + b$, or

$$\tan\frac{(a+b)}{2} = \frac{\cos\dfrac{(A-B)}{2}}{\cos\dfrac{(A+B)}{2}}\tan\frac{c}{2} \tag{36}$$

Making use of the polar triangle as before and substituting in Eq. 36 in terms of the corresponding parts of the polar triangle we obtain

$$\tan\frac{(A+B)}{2} = \frac{\cos\dfrac{(a-b)}{2}}{\cos\dfrac{(a+b)}{2}}\cot\frac{C}{2} \tag{37}$$

Equations 34, 35, 36, and 37 are known as Napier's analogies. There is a general parallel between them and the theorem of plane trigonometry which states that, in any triangle, the sum of two sides is to their difference as the tangent of half the sum of the angles opposite the two sides is to the tangent of half their difference. Equations similar to these for the other parts of the triangle may be written directly from the above by proper changes of A, B, C, a, b, and c in the formulae.

Delambre's Analogies or Gauss's Equations

Since

$$\sin\frac{(A+B)}{2} = \sin\frac{A}{2}\cos\frac{B}{2} + \cos\frac{A}{2}\sin\frac{B}{2}$$

$$\sin\frac{(A+B)}{2} = \frac{\sin(s-b)+\sin(s-a)}{\sin c}\sqrt{\frac{\sin s\sin(s-c)}{\sin a\sin b}}$$

from Eqs. 24 and 25. Hence

$$\frac{\sin\dfrac{(A+B)}{2}}{\cos\dfrac{C}{2}} = \frac{\sin(s-b)+\sin(s-a)}{\sin c}$$

$$= \frac{2\sin\dfrac{c}{2}\cos\dfrac{(a-b)}{2}}{2\sin\dfrac{c}{2}\cos\dfrac{c}{2}}$$

and

$$\sin \frac{(A + B)}{2} = \frac{\cos \frac{(a - b)}{2}}{\cos \frac{c}{2}} \cos \frac{C}{2} \tag{38}$$

In a similar manner we may obtain

$$\sin \frac{(A - B)}{2} = \frac{\sin \frac{(a - b)}{2}}{\sin \frac{c}{2}} \cos \frac{C}{2} \tag{39}$$

$$\cos \frac{(A + B)}{2} = \frac{\cos \frac{(a + b)}{2}}{\cos \frac{c}{2}} \sin \frac{C}{2} \tag{40}$$

$$\cos \frac{(A - B)}{2} = \frac{\sin \frac{(a + b)}{2}}{\sin \frac{c}{2}} \sin \frac{C}{2} \tag{41}$$

Equations 38, 39, 40, and 41 are known as Delambre's analogies or as Gauss's equations.

Solution of Oblique Spherical Triangles

The methods of solution of oblique spherical triangles will be discussed for each of the six cases previously listed.

Case 1. Given the three sides. Problems of this type may be solved by the half-angle formulae. Equations 27, 28, and 29 permit direct logarithmic calculation of A, B, and C. The angles should be checked by applying the law of sines which is also well adapted for logarithmic computation. In using any equation to check results, care should be taken to be sure that it is one which will provide a check

on the numerical values of the unknown parts themselves rather than a check of logarithmic functions alone.

Case 2. Given the three angles. Problems of this type may be solved by the same formulae used for case 1 by taking the sides of the polar triangle a', b', c', solving for the angles A', B', C', and then taking their supplements to obtain the sides a, b, c of the original triangle. The problem may also be solved directly by the half-side equations similar to Eq. 33. In any case the law of sines should be used as a check.

Case 3. Given two sides and the included angle. Napier's analogies may be used to solve problems of this nature. Equations 35 and 37 will furnish the values for A and B. Then Eq. 34 or 36 will give the missing side c. Delambre's analogies may also be used. The law of sines again furnishes a check.

Case 4. Given two angles and the included side. Napier's analogies may again be used, Eqs. 34 and 36 furnishing the unknown sides a and b, and then Eq. 35 or 37 will give the missing angle C. Careful attention should be paid to algebraic signs in determining the proper value for the unknown angle. The law of sines will again provide a check.

Case 5. Given two sides and the angle opposite one of them. We may solve problems of this nature by the law of sines together with Napier's analogies. Thus, if a, b, and A are given, by the law of sines

$$\sin B = \frac{\sin b \sin A}{\sin a}$$

This equation, it at once appears, does not definitely determine B since the angle may be either acute or obtuse. In this case there may be two solutions, one solution, or none. It may be shown, however, that if two sides or angles of a spherical triangle are unequal, the angles or sides opposite are unequal, and the greater angle or side lies opposite the greater side or angle. This fact enables us to determine

which values of the angle are possible. Thus, if b is less than a, B must be less than A. If b is greater than a, B must be greater than A. Both values of B, as determined by the law of sines, may be greater or less than A, in which case there are two solutions. Again, only one value of B may be greater or less than A, and only one solution is obtained. If the sine of B is greater than unity, that is, if log sin B is positive, no solution is possible. Similar arguments obviously apply to case 6 as well. Having thus determined either one or two values for B we have two sides and the corresponding two angles known and may use Napier's analogies to determine the third side and angle. It should be noted that for two possible values of B there will be two entirely independent solutions for both side c and for angle C.

Case 6. Given two angles and the side opposite one of them. Assuming A, B, and a to be given, we may solve by the law of sines for side b. We then determine which, if any, values of b are possible, and solve for the remaining side and angle by use of Napier's analogies.

Radians — Degrees, Minutes, and Seconds

While the lengths of the sides of a spherical triangle are commonly stated in degrees, minutes, and seconds it is convenient for certain purposes to express the values in radians. If the length of an arc is divided by the radius the result expresses the central angle in radians. The number obtained is the corresponding length of arc on a circle whose radius is unity. The unit of measurement of angles in this system is the radius of the circle; that is, an angle of 1 radian is an angle whose arc equals the radius and therefore contains about 57°.3.

Since the ratio of the semi-circumference to the radius is π, there are π radians in 180° of the circumference.* The conversion of angles from degrees into radians (or π meas-

* $\pi = 3.14159, 26535$.

ure, or arc measure) is effected by multiplying by the ratio of these two.

$$\text{Angle in degrees} = \text{Angle in radians} \times \frac{180°}{\pi}$$

and

$$\text{Angle in radians} = \text{Angle in degrees} \times \frac{\pi}{180°}$$

To convert an angle in radians into minutes multiply by $(180 \times 60)/\pi = 3437'.77$; or divide by $\pi/(180 \times 60) = 0.0002909$. This latter number, the arc $1'$, is nearly equal to $\sin 1'$ or $\tan 1'$. To convert an angle expressed in radians into seconds multiply by $(180 \times 60 \times 60)/\pi = 206264''.8$; or divide by the reciprocal, 0.00000, 48481, 36811, the arc $1''$; this number is identical with $\sin 1''$ or $\tan 1''$ for 16 decimal places.

Area of a Spherical Triangle

It may be shown that the area of a spherical triangle is given by the following expression in which Δ is the area of the triangle, A, B, and C are the angles, and r is the radius of the sphere.

$$\Delta = \frac{(A + B + C - 180°)\pi r^2}{180°} \tag{42}$$

The term $(A + B + C - 180°)$ is obviously the spherical excess of the triangle. Putting this equal to e we have

$$\Delta = \frac{e°\pi r^2}{180°} \tag{43}$$

If e'' is the spherical excess in seconds we may write

$$\Delta = \frac{e''r^2}{206264.8} \tag{44}$$

Spherical Excess

The spherical excess may be obtained from a direct computation of the angles of a triangle. If the area of the triangle is known Eq. 44 may be rewritten

$$c'' = \frac{\Delta \times 206264.8}{r^2} \tag{45}$$

the constant being the number of seconds in one radian.

Problems

Solve for the missing data in the following spherical triangles. Show checks.

1. $A = 98° 10'$, $a = 140° 40'$, $C = 90° 00'$.
2. $A = 159° 25'$, $a = 136° 25'$, $c = 90° 00'$.
3. $a = 54° 21'$, $b = 72° 54'$, $c = 54° 21'$.
4. $a = 125° 18'$, $b = 151° 35'$, $c = 75° 52'$.
5. $A = 53° 13'$, $B = 42° 34'$, $C = 108° 53'$.
6. $a = 90° 52'$, $B = 120° 10'$, $c = 117° 48'$.
7. $A = 120° 40'$, $b = 74° 30'$, $C = 46° 30'$.
8. $b = 100° 10'$, $c = 32° 40'$, $C = 159° 30'$.
9. $B = 142° 32'$, $C = 71° 18'$, $c = 39° 36'$.
10. $A = 59° 57'$, $B = 142° 12'$, $b = 151° 21'$.

11. Find the distance and bearing of Bengasi, Libya, lat. 32° 08′ N, long. 20° 05′ E; from Taranto, Italy, lat. 40° 30′ N, long. 17° 15′ E. A nautical mile is one minute of angle.

12. An airplane flies from Botwood, Newfoundland, lat. 49° 10′ N, long. 55° 30′ W, to Liverpool, lat. 53° 24′ N, long. 02° 58′ W, along a great-circle track at an average altitude of 3 miles and at a cruising speed of 250 miles per hour. Using the mean radius of the earth as 3960 statute miles, determine the flying time for the trip.

13. Determine the area of the spherical triangle on the surface of the earth the angles of which are 43°, 67°, and 72°. Take the radius of the earth as 3960 miles.

14. A ship sails from Boston, lat. 42° 21′ N, long. 71° 06′ W, to Gibraltar, lat. 36° 06′ N, long. 05° 21′ W. If the ship sailed a great-circle track, find the distance sailed, the course on leaving Boston, and the course approaching Gibraltar.

15. A ship sails from Rio de Janeiro, lat. 22° 54′ S, long. 43° 10′ W, to Columbo, Ceylon, lat. 07° 20′ N, long. 79° 30′ E. To clear the Cape of Good Hope she first sails to a point off Cape Town in lat. 36° 10′ S, long. 18° 20′ E, thence to Columbo. Determine the distance sailed in nautical miles, the course at Rio, and the course leaving Cape Town.

16. A ship sails from Philadelphia to Lisbon. Cape Henlopen, at the mouth of Delaware Bay and her landfall at the mouth of the Tagus are both in lat. 38° 45′ N. The former is in long. 75° 05′ W, the long. of the latter is 09° 11′ W. Determine the great-circle distance between these points, the course sailed on clearing Delaware Bay, and the course approaching the Tagus.

Index